6. Work, Power & Energy.

- Work - It is a scalar product of Force and displacement.

$$\boxed{W = \vec{F} \cdot \vec{S}} = |F||S|\cos\theta$$

Unit = Nm → Joule.

- When work done is positive then the system gets energy.

- When work done is negative then the system loses energy.

* For variable force

$$\boxed{W = \int_{x_1}^{x_2} F\,dx}$$

* The area of Force & displacement gives work done in graph

* It is not necessary that if displacement is zero, total work done is also zero.

* For forces in multiple directions

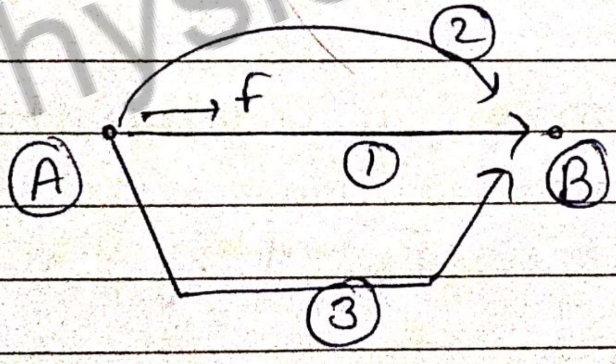
$$W = \int F \, d\vec{r}$$

where $d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$.

* Conservative force

- The work done by such force

1. does not depend upon path
2. Depends only & only on initial & final position.



$W_1 = W_2 = W_3$

*) Ex - gravitational force, spring force, Electrostatic force.

* Work done by conservative force in a closed path is always zero.

* Non conservative forces are forces which are not conservative.

Conservative force	Non conservative force
1) $W_I = W_{II}$	$W_I \neq W_{II}$
• does not depend upon path	• does depend on path
• Closed path $W = 0$	Closed path $W \neq 0$

* If a Force is conservative

then $F = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

Check i) $$\frac{\delta F_x}{\delta y} = \frac{\delta F_y}{\delta x}$$

ii) $$\frac{\delta F_x}{\delta z} = \frac{\delta F_z}{\delta x}$$

iii) $$\frac{\delta F_y}{\delta z} = \frac{\delta F_z}{\delta y}$$

Qu) Find whether given force $x\hat{i} + y\hat{j}$ is conservative or non-conservative..

(1) $\frac{dF_x}{dy} = \frac{dF_y}{dx} \Rightarrow 0 = 0$

[F_x will be constant w.r.t y]

[F_y will be constant w.r.t x]

(2) $\frac{dF_y}{dz} = \frac{dF_z}{dy} \Rightarrow 0 = 0$

(3) $\frac{dF_x}{dz} = \frac{dF_z}{dx} \Rightarrow 0 = 0$

$\Rightarrow$ $\boxed{\text{Given force is conservative.}}$

★ Work Energy Theorem.

- Kinetic energy comes into action when body is in motion..
- It depends upon frame of refference.

- work energy theorem.

⇒ Work done by all the forces on a body = Change in K.E. of the body.

→ internal + external force.

$$\boxed{W_{\text{by all force}} = K.E_f - K.E_i}$$

$$dW_{\text{all forces}} = d(K.E.)$$

★ Potential Energy

ΔU = - Work done by Conservative force.

$$\Delta U = -W_c$$

$$dU = -dW_c$$

* Potential energy is not defined for non-conservative force.

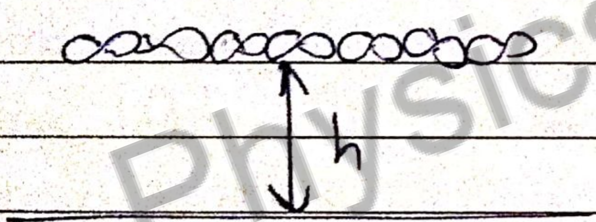
$$\int dU = -\int dW_c$$

$$\int_{U_i}^{U_f} dU = -\int F_c \cdot dx$$

* Gravitational Potential Energy. (mgh)

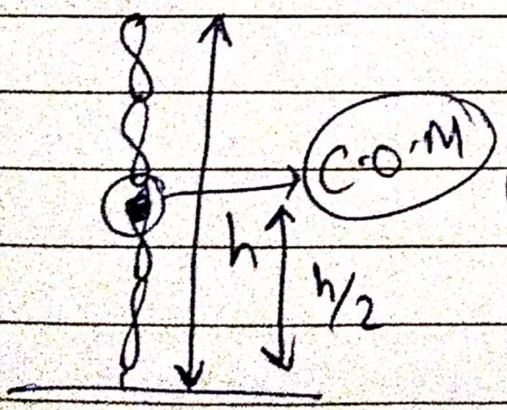
• Check for center of mass of for h

(i)



mgh

(ii)

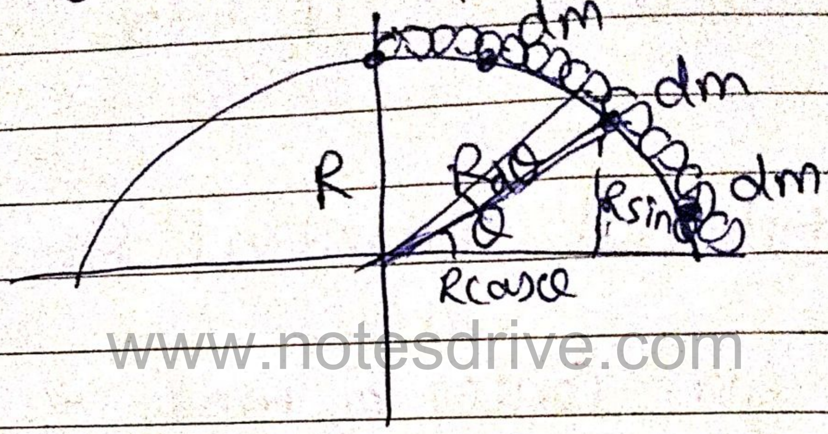


$mg\,h/2$

(iii)

0.

* To find Potential Energy of given arrangement of chain.

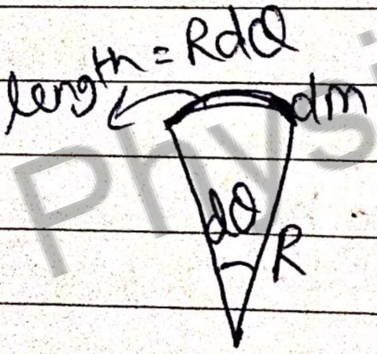


$dU = dmgh$

$\Rightarrow dU = dmg\,R\sin\theta$

$\Rightarrow \int dU = \int dmg\,R\sin\theta$

[Here variable is 'Θ' ~~so~~ not dm, hence make Θ variable]



Mass (m) for length $\frac{\pi R}{2}$

then mass ~~dm~~? for length RdΘ

$\Rightarrow \dfrac{m\,R\,d\theta}{\frac{\pi R}{2}} = ?$

$$\boxed{dm \Rightarrow \frac{2m\,d\theta}{\pi}}$$

* Potential Energy is invisible form of energy. It is imparted to the system, that energy is stored as potential energy.

$$\int dU = \int_0^{\pi/2} \frac{2m}{\pi} d\theta\, g R\sin\theta$$

$$= \frac{2m}{\pi} gR \int [\sin\theta]_0^{\frac{\pi}{2}}$$

$$\Rightarrow \frac{2m}{\pi} gR \left[-\cos\theta\right]_0^{\frac{\pi}{2}}$$

$$\Rightarrow \frac{2m}{\pi} gR \left[-\cos 90 - (-\cos 0)\right]$$

$$\Rightarrow \boxed{\frac{2mgR}{\pi}}$$

* Spring Potential Energy.

$$U = \frac{1}{2} Kx^2$$

• Work done by spring force

$$= \frac{1}{2} K (x_1^2 - x_2^2)$$

* Potential Energy of spring is zero at its natural length.

* $U_2 - U_1 = \frac{1}{2}Kx_2^2 - \frac{1}{2}Kx_1^2$

- Either you have stretched x, or compressed x ~~for~~ from natural length the potential energy stored will always be equal to $\frac{1}{2}Kx^2$.

- $\boxed{\text{Potential Energy of Spring is always positive.}}$

- Gravitational & Electrostatic potential energy can be negative.

* $F_x = -\frac{dU}{dx}$

* $F_y = -\frac{dU}{dy}$

* $F_z = -\frac{dU}{dz}$

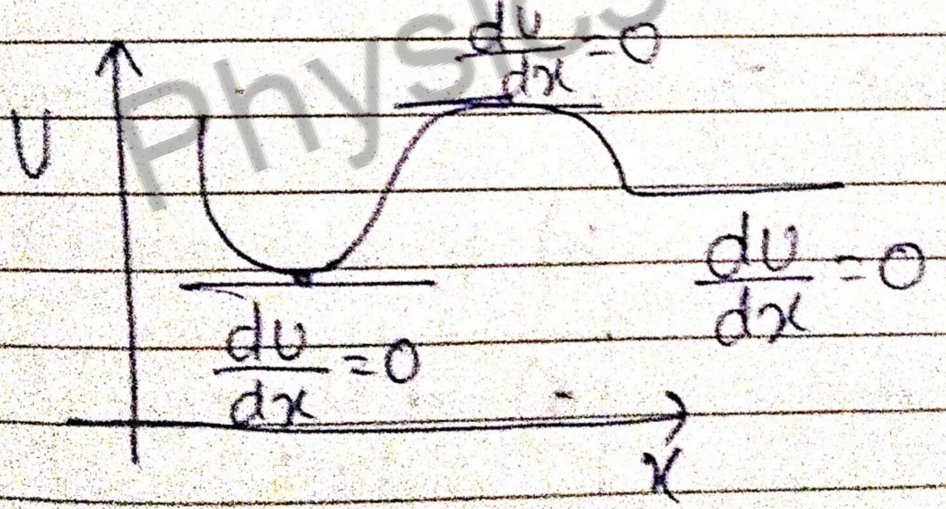
* Slope of U vs x graph is conservative force.

* Equilibrium.

- If total force acting on body is zero, then body is in Equilibrium.

Stable | Unstable | Neutral

* $F_x = -ve$ (Slope U v/s x)



* Stable Equilibrium ⇒ U → minimum
* Unstable Equilibrium ⇒ U → maximum
* Neutral Equilibrium ⇒ U → constant.

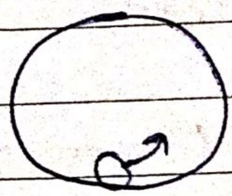
1. Stable Equilibrium [U = minimum]

a) $F_{net} = 0$

b) U - x slope = 0

c) $\frac{dU}{dx} = 0$

d) $\boxed{\frac{d^2U}{dx^2} > 0}$



If we displace the body slightly it, returns back to stable equilibrium.

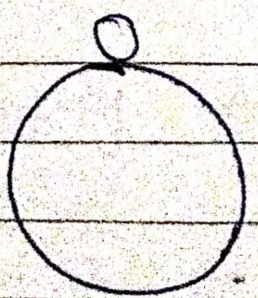
2. Unstable Equilibrium [U - maximum]

a) $F_{net} = 0$

b) U - x slope = 0

c) $\frac{dU}{dx} = 0$

d) $\boxed{\frac{d^2U}{dx^2} < 0}$



If we displace the body slightly it will move towards ~~Static~~ Stable equilibrium.

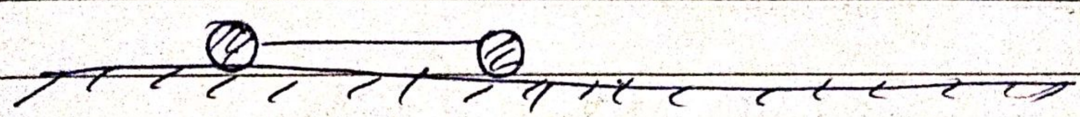
(3) Neutral Equilibrium [U - constant]

a) $F_{net} = 0$

b) U - x slope = 0

c) $\frac{dU}{dx} = 0$

d) $\boxed{\frac{d^2U}{dx^2} = 0}$



will remain in again in neutral equilibrium position [new position].

★ Conservation of Mechanical Energy.

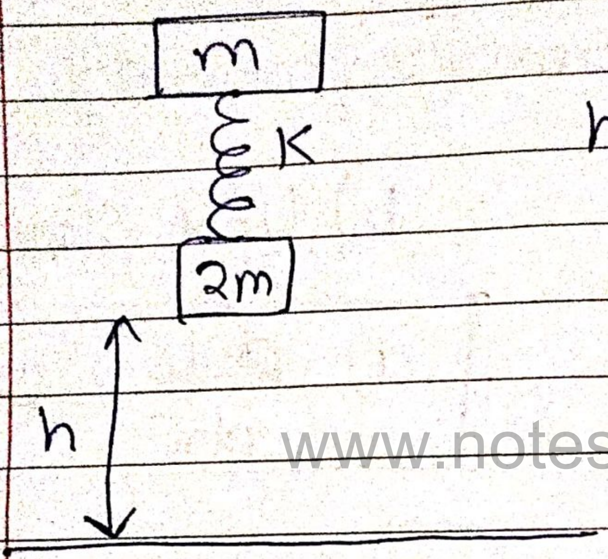
- $W_e + W_{N.C.} = (K_f + U_f) - (K_i + U_i)$

⇒ when external force = 0
Non - conservative force = 0

then, $K_i + U_i = K_f + U_f$

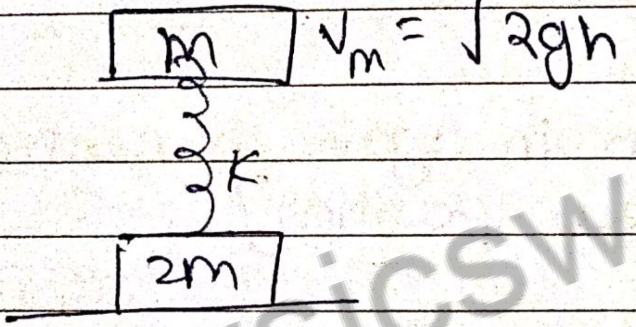
Pr ADV

Que

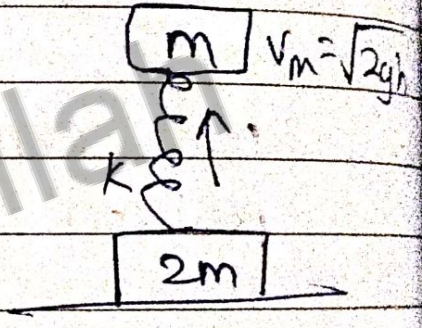


Find the value of h such that 2m just manages to rise above ground after collision.

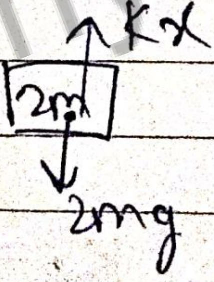
⇒



$v_m = \sqrt{2gh}$



$v_m = \sqrt{2gh}$

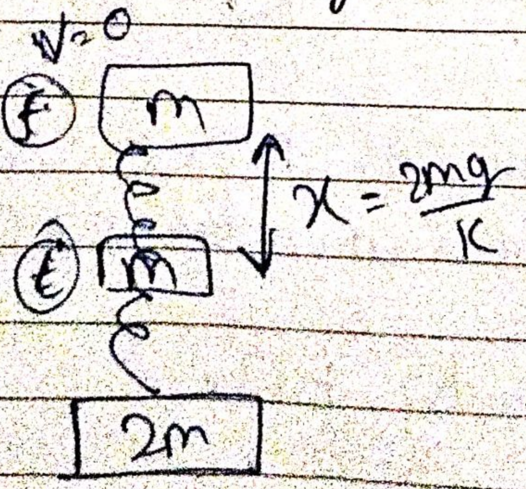


Kx should just more than 2mg

⇒ We can take

$Kx = 2mg$

$x = \frac{2mg}{K}$



$x = \frac{2mg}{K}$

Using Conservation of mechanical Energy

$$K_f + U_f = K_i + U_i$$

$$0 + mgx + \frac{1}{2}kx^2 = \frac{1}{2}m \cdot 2gh + 0$$

$$\Rightarrow mgx + \frac{1}{2}kx^2 = \frac{1}{\cancel{2}} \cdot \cancel{2}mgh$$

$$\Rightarrow \frac{2mgx + kx^2}{2} = mgh$$

$$\Rightarrow 2mgx + kx^2 = 2mgh$$

$$\Rightarrow kx^2 + 2mgx - 2mgh = 0$$

$$x = \frac{2mg}{k}$$

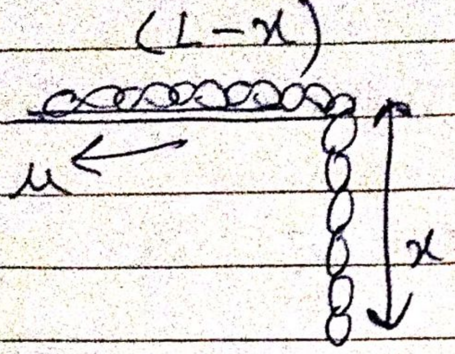
$$\Rightarrow \cancel{k} \cdot \frac{4m^2g^2}{k^{\cancel{2}}} + 2mg\,\frac{2mg}{k} - 2mgh = 0$$

$$\Rightarrow \frac{4m^2g^2}{k} + \frac{4m^2g^2}{k} = 2mgh$$

$$\Rightarrow \frac{8m^2g^2}{2k} = 2mgh$$

$$\Rightarrow \frac{4m^2g^2}{k} = mgh \Rightarrow \boxed{h = \frac{4mg}{k}}$$

Que



A chain of length 'L' and mass 'M' is made to sustain on the table find the maximum length it can hang so that body doesn't falls.

⇒ Let the chain to be hanged be of length x.

$\therefore$ chain on table $= (L-x)$

If L length of chain weighs M

the x length of chain weighs $\boxed{\dfrac{Mx}{L}}$

As $(L-x)$ length of chain weight $\boxed{\dfrac{M(L-x)}{L}}$

Now frictional force $= \mu N$

$\Rightarrow \mu(mg)$

$\Rightarrow \mu \dfrac{M(L-x)}{L} g$

force acting on falling chain $= \dfrac{Mxg}{L}$

⇒ As body has to stay

$$\therefore \mu \frac{\cancel{M}(L-x)\cancel{g}}{\cancel{L}} = \frac{\cancel{M}x\cancel{g}}{\cancel{L}}$$

$$\Rightarrow \mu(L-x) = x$$

$$\Rightarrow \mu L - \mu x = x$$

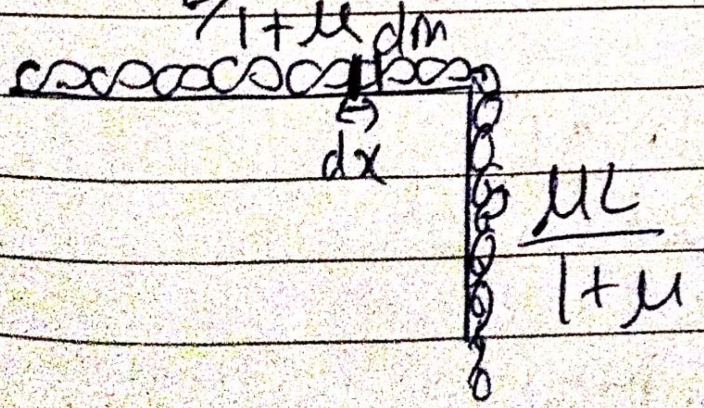
$$\Rightarrow \mu L = x + \mu x$$

$$\Rightarrow \mu L = x(1+\mu)$$

$$\Rightarrow \boxed{\frac{\mu L}{1+\mu} = x}$$

Que) In the same Que above find the total work done by frictional force if the chain is pulled horizontal down.

⇒



If $L \rightarrow M$

$dx \rightarrow \boxed{\frac{M\,dx}{L}}$

$$f = \mu m g = \mu \frac{M\,dx}{L} g$$

$$dw = fx$$

$$\int dw = \int \frac{\mu M dx\, g\, x}{L}$$

$$\Rightarrow \int dw = \frac{\mu M g}{L} \int x\, dx$$

Vary x from $\frac{L}{1+\mu}$ to 0.

$$\therefore \int dw = \frac{\mu M g}{L} \int_0^{L/1+\mu} x\, dx$$

$$\Rightarrow \frac{\mu M g}{L} \left[\frac{x^2}{2}\right]_0^{\frac{L}{1+\mu}}$$

$$= \frac{\mu M g}{2L} \left[\left(\frac{L}{1+\mu}\right)^2\right]$$

$$\Rightarrow \frac{\mu M g L^{\cancel{2}}}{2\cancel{L}(1+\mu)^2}$$

$$\Rightarrow \frac{\mu M g L}{2(1+\mu)^2}.$$