MECHANICAL PROPERTIES OF SOLID.

ELASTICITY

(Molecule of Solid)

The tendency of an object to regain its original position, its original configuration when Deforming force is removed. Up to its Elastic Limit.

The Tendency to oppose any change in its shape, size by a body.

Q Which is More elastic?? Steel or Rubber??

Ans. Steel, because it come very speedly to its original position.

Q Why Elasticity?

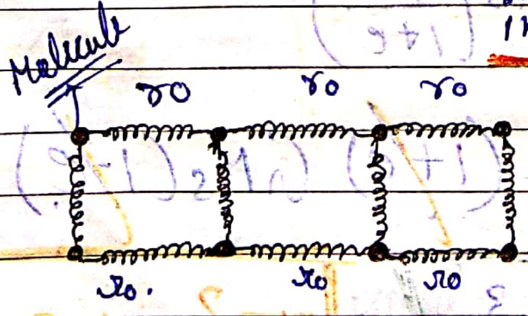
Molecule → Deforming force (external)

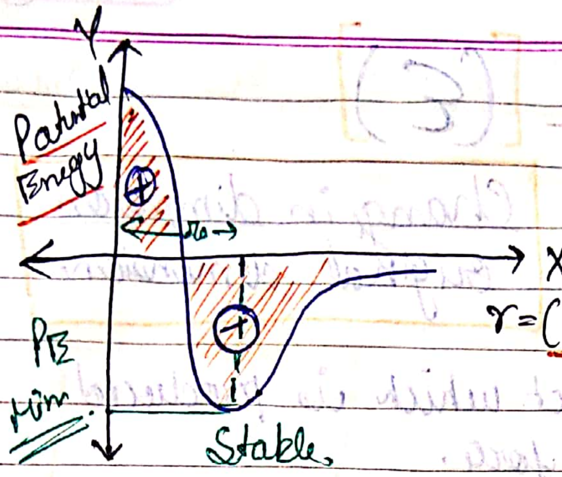
⇓

Restoring Force
intermolecular Force

⇓

Elasticity



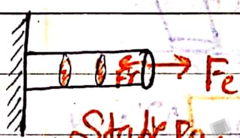


r = (inter molecular distance)

P.E (minimum Stable)

STRESS :-

The internal restoring force per unit cross sectional Area.



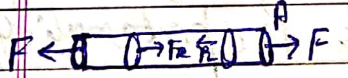
Restoring Force = Deforming Force

$\boxed{F_R = F_{ext}}$ 99% case

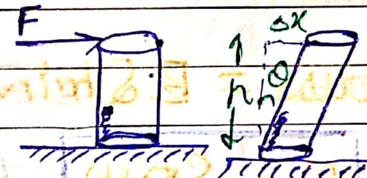
$$\boxed{\sigma = \frac{F_R}{A_{Cross\,Sect.}}}$$ $\frac{N}{m^2}$ ~~Scaler q.~~ (Tensor)

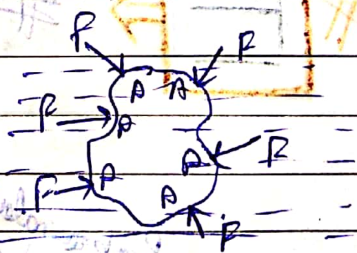
TYPES :-

LONGITUDINAL | **SHEARING / TANGENTIAL** | **VOLUME STRESS**



Tensile Stress.

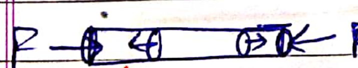




$$\boxed{\frac{F_t}{A} = \sigma}$$

$$\boxed{\sigma = \frac{F}{A} = P}$$

$$\boxed{\sigma = \Delta P}$$



Compressive.

$$\boxed{\sigma = \frac{F}{A_{Cross\,Sect}}}$$

$(P - P_0)$ 2 pressure

Ex. Plastic Ball

$\sin\theta \rightarrow \text{small} \rightarrow \theta$

$\tan\theta \rightarrow \text{small} \rightarrow \theta$

Always +ve STRAIN :- $[(\varepsilon)]$

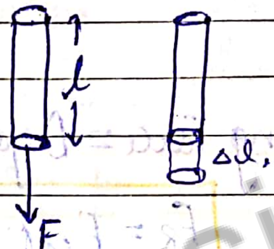
$$\boxed{\frac{\text{Change in dimension}}{\text{original dimension}}}$$

No unit

\# It is an effect which is produced by deforming force.

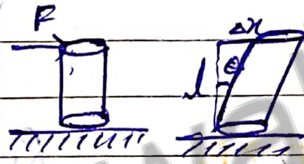
TYPES :-

LONGITUDINAL



$$\boxed{\varepsilon = \frac{\Delta l}{l}}$$

SHEAR STRAIN



$$\boxed{\varepsilon = \frac{\Delta x}{l}}$$ Δx very small

$$\boxed{\frac{\Delta x}{l} = \tan\theta} \rightarrow \theta \text{ (radian)}$$

$$\boxed{\varepsilon = \frac{\Delta x}{l} = \theta}$$

VOLUME STRAIN

$$\boxed{-\frac{\Delta V}{V} = \varepsilon}$$

Because it comes to be negative (−ve)

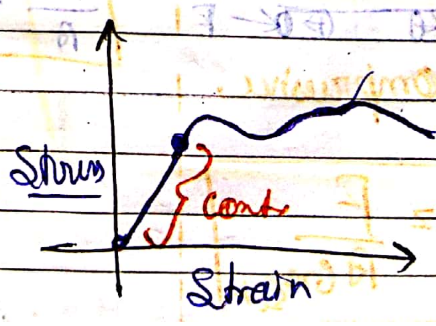
$\boxed{E}$ ⇒ # Stress ∝ Strain (limit of Proportionality)

Stress = E Strain

Proportionality constant ← $\boxed{E = \frac{\text{Stress}}{\text{Strain}}}$ $\frac{N}{m^2}$

E ⇒ Modulus of Elasticity

E → Property of Material.



(i) YOUNG'S MODULUS OF ELASTICITY

$$Y = \frac{\text{Longitudinal Stress}}{\text{Longitudinal Strain}}$$

$$\boxed{Y = \frac{Fl}{A \Delta l}}$$

(ii) SHEAR MODULUS OF ELASTICITY

$$G = \frac{\text{Shear Stress}}{\text{Shear Strain}}$$

MODULUS OF RIGIDITY

$$G = \frac{F_t l}{A \Delta x}$$

$$\boxed{G = \frac{F_t}{A\theta}}$$

(iii) BULK MODULUS OF ELASTICITY

$$B = \frac{\text{Volume Stress}}{\text{Volume Strain}}$$

COMPRESSIBILITY

$$B = \frac{F}{A}\left(\frac{V}{-\Delta V}\right)$$

$$\boxed{B = -\frac{\Delta P V}{\Delta V}}$$

$$\boxed{C = \frac{1}{B}}$$

Q Elongation of a steel bar 1 m long and $1.5\,cm^2$ cross sectional area when subjected to a pull of $1.5 \times 10^4\,N$. ($Y_s = 2 \times 10^{11}\,N/m^2$)

a) 0.1 mm b) 0.3 mm c) 0.5 mm ✓ d) 0.2 mm

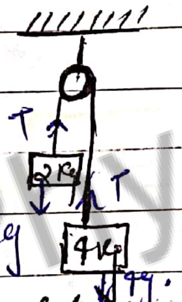
$$Y = \frac{\text{Stress}}{\text{Strain}} = \boxed{\frac{F l}{A \Delta l}}$$

$$\Delta l = \frac{F l}{A Y}$$

$$= \frac{1.5 \times 10^4 \times 1}{1.5 \times 10^{-4} \times 2 \times 10^{11}} = \frac{1}{2} \times 10^{8-11}$$

$$= 0.5 \times 10^{-3}\,m$$

$$\boxed{= 0.5\,mm}$$

Q 

Wire → Breaking stress $= 2 \times 10^5\,N/m^2$

Find the min cross sectional area of wire so that it doesn't break.

a) $2.66\,cm^2$ b) $1.66\,cm^2$ c) $3.33\,cm^2$ d) $1.33\,cm^2$ ✓

$$\text{Stress} = \frac{F}{A}$$

$$\boxed{T = \frac{2 m_1 m_2 g}{m_1 + m_2}}$$ Trick to find Tension

$4g - \cancel{T} = 4a$

$\cancel{T} - 2g = 2a$

$2g = 6a$

$$\boxed{a = g/3}$$

$T = 2a + 2g$

$$\boxed{T = \frac{8g}{3}}$$

$$\text{Stress} = \frac{F}{A} = \frac{T}{A}$$

$$2 \times 10^5 = \frac{8g}{3A}$$

$$A = \frac{8g}{3 \times 2 \times 10^5}$$

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$$A = \frac{8 \times 10^{-5}}{3 \times 2 \times 10^{4} \times 10^{-2}}$$

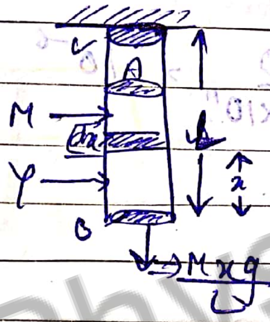
$$A = \frac{4}{3} \times 10^{-4}$$

$$A = 1.33 \times 10^{-4}\ m^2$$

$$\boxed{A = 1.33\ cm^2}$$

Q) Find the extension in a wire of length 'L' Cross Section Area (A), Mass M & Young's Modulus of Elasticity Y under its own weight ??

a) $\frac{MgL}{2AY}$ b) $\frac{MgL}{3AY}$ c) $\frac{2MgL}{3AY}$ d) $\frac{MgL}{AY}$



$$Y = \frac{Fl}{A\Delta l}$$

$$\Delta l = \frac{Fl}{AY}$$

$L \to M$

$x \to \frac{Mx}{L}$

Elongation in Small element 'dx'

$$dl = \frac{F\,dx}{AY}$$

$$\int dl = \int_0^L \frac{Mxg\,dx}{LAY}$$

$$= \frac{Mg}{ALY}\int_0^L x\,dx$$

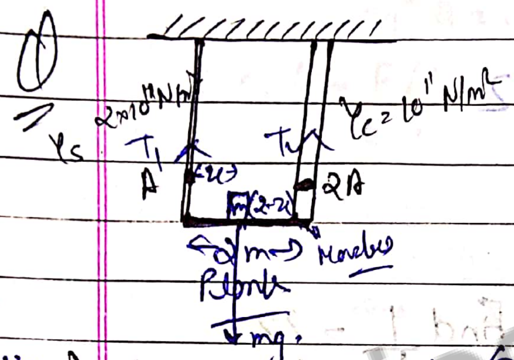
$$\Delta l = \frac{Mg}{ALY}\,\frac{L^2}{2}$$

$$\boxed{\Delta l = \frac{MgL}{2AY}}$$

$$\left(\frac{T_M}{T}\right)^2 = 1 + \frac{Mg}{AY}$$

$$\left(\frac{T_M}{T}\right)^2 - 1 = \frac{Mg}{AY}$$

$$\boxed{\frac{1}{Y} = \left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}}$$

(1) 

Where should a mass 'm' be kept so that in steel & copper wire, we obtain
(i) equal stress
(ii) equal strain

(ii) a) 1.00 m
b) 0.5 m
c) 0.33 m
(i) d) 1.33 m
e) 1.66 m
f) 0.8 m

(i) $(\text{Stress})_s = (\text{Stress})_c$

$$\frac{T_1}{A} = \frac{T_2}{2A}$$

$$\boxed{T_2 = 2T_1}$$

equilibrium $\tau_{net} = 0$, $\tau = r \times F$

$T_1 \times x - T_2(2-x) = 0$

$T_1 x - 2T_1(2-x) = 0$

$x - 4 + 2x = 0$

$3x = 4$

$\boxed{x = \frac{4}{3}}$ $\boxed{x = 1.33}$

$(\text{Strain})_s = (\text{Strain})_c$

$\eta = \frac{\text{Stress}}{\text{Strain}}$

$\left(\frac{\text{Stress}}{\eta}\right)_s = \left(\frac{\text{Stress}}{\eta}\right)_c$

$\left(\frac{T}{\cancel{A}\, \eta_s}\right) = \left(\frac{T_2}{2A\eta_c}\right)$

$\frac{T}{\cancel{2\times10^{11}}} = \frac{T_2}{\cancel{2\times10^{11}}}$

$\boxed{T_1 = T_2}$.

eq. Tnet = 0.

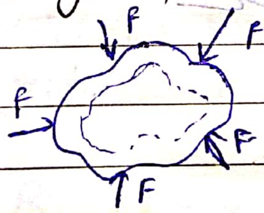
$T_1 x - T_2(2-x) = 0$

$x - 2 + x = 0$

$2x = 2$

$\boxed{x = 1}$.

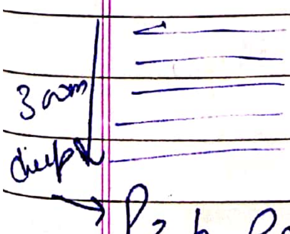
Q2 Avg. depth of Indian Ocean is about 300 m. Calculate the fractional compression $\frac{\Delta V}{V}$ of water at the bottom of ocean ($g = 10\,m/s^2$) $\rho_w = 10^3\,kg/m^3$, Bulk Mod of water $= 2.2\times10^9\,N/m^2$



Volume Stress $= \frac{F}{A} = P$.

Volume Strain $= -\frac{\Delta V}{V}$

$\beta = \frac{\text{Vol. Stress}}{\text{Vol. Strain}} = \frac{-PV}{\Delta V}$

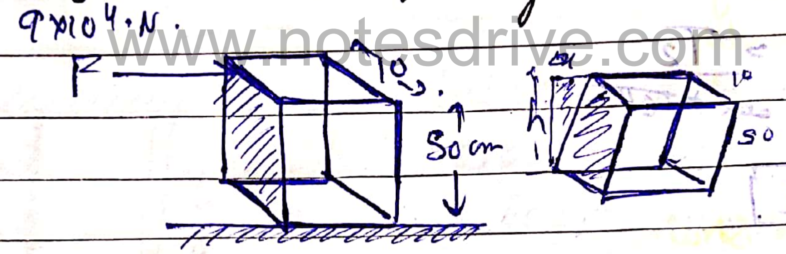


$P = h\rho g$.

$= 300\times10^3\times10 = 3\times10^6\,N/m^2$

$$\frac{\Delta V}{V} = \frac{P}{B} = \frac{3\times10^{6}}{2.2\times10^{9}}$$

Q. A Square lead slab of side 50 cm & Thickness 10 cm is subjected to a shearing Force 9×10^4 N. How Much upper edge will be displaced. If Shear Modulus of elasticity $G = 5.6\times10^{9}\ N/m^2$ for lead.



Shear Stress $= \frac{F}{A} = \frac{9\times10^4}{50\times10\times10^{-4}}\ \frac{N}{m^2}$

Shear Strain $\Rightarrow \frac{\Delta x}{50}$

$$G = \frac{9\times10^{4}}{50\times10\times10^{-4}}$$

$$G = \frac{\quad}{\frac{\Delta x}{50}}$$

$$\Delta x = $$

$\frac{9\times10^4}{50\times10\times10^{-4}}$

$\frac{9\times10^8}{50\times10}$

$\frac{9\times10^6}{8}$

$50\times1.8\times10^6$

9×10^7

$1.8 \times 5 = 9.0$

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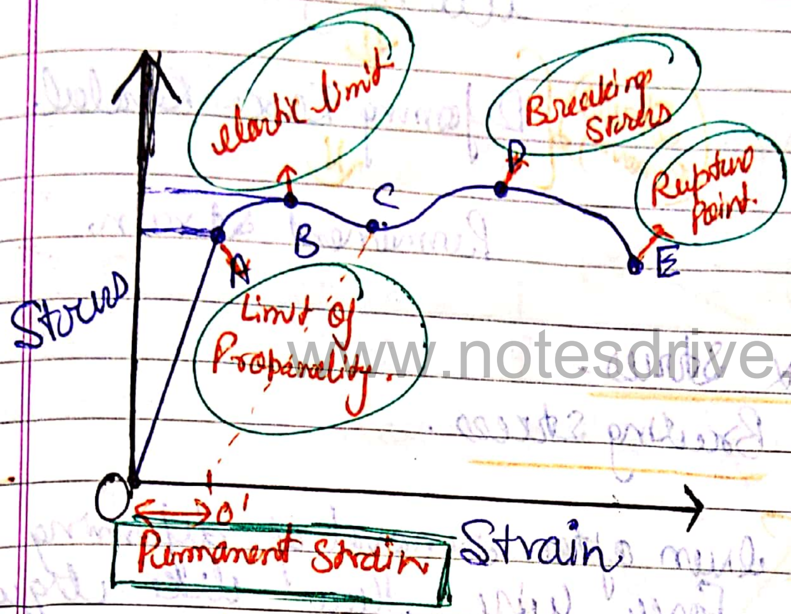
STRESS-STRAIN CURVE.



Stress ∝ Strain
$\frac{F}{A}$ $\frac{\Delta l}{l}$

OA → Stress ∝ Strain.

A (Stress) → limit of proportionality.

Slope = $\frac{\Delta y}{\Delta x}$ = $\frac{\text{Stress}}{\text{Strain}}$ = Modulus of Elasticity.

B (Stress) → elastic limit

(Force is removed uptill B, wire returns to its original state.)

A → B stress $\cancel{\propto}$ strain.

B→D Plastic Region

After B → Small stress → large strain
↓
Elasticity ✗
↓
(Stress Removal) { Deforming Force Removal
↓
Permanent strain

D → Max Stress.
Breaking stress.

✱ { Even after removal of deforming Force wire flows like liquid (thinning of wire ↑↑)

E → Rupture Point

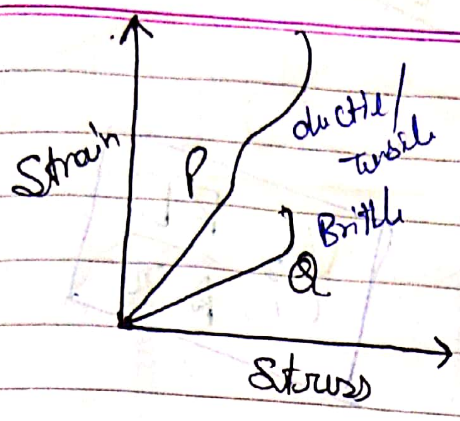
$\boxed{\begin{array}{l}\text{\# OB} \rightarrow \text{Elastic Region} \\ \text{\# BD} \rightarrow \text{Plastic Region.}\end{array}}$

1) DUCTILE / TENSILE.
Plastic Region Large.

2) BRITTLE →
Plastic Region small.

3) ELASTOMER
No Plastic Region.
only have
Elastic Region (Rubber)

Ad (18)
Q
2



a) P has more tensile strength
b) P is more ductile
c) P is more brittle
d) Y of P is more than that of Q.

$$\text{Slope} = \frac{\text{Strain}}{\text{Stress}} = \frac{1}{Y}$$

$$(\text{Slope})_P > (\text{Slope})_Q$$

$$\left(\frac{1}{Y}\right)_P > \left(\frac{1}{Y}\right)_Q$$

$$\boxed{Y_Q > Y_P}$$

ENERGY STORED IN A STRECHED WIRE

$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{volume}$$

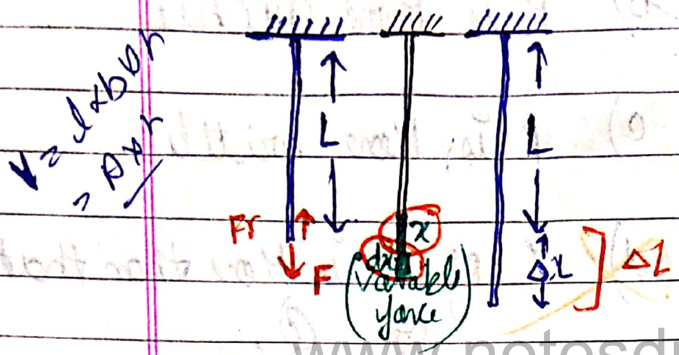
$$\frac{PE}{Vol.} = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$\boxed{E = \frac{\text{Stress}}{\text{Strain}}}$$

$$\boxed{\frac{PE}{Vol.} = \frac{1}{2} \times E \times (\text{Strain})^2}$$

$$\boxed{F = YA\frac{\Delta L}{L}}$$

DERIVATION :-



$$\boxed{Y = \frac{FL}{Ax}}$$

Small work done in stretching wire by 'dx' amount when it is stretched upto 'x' amount

$$dw = F\,dx$$

$$\int dw = \int_0^{\Delta L} \left(\frac{YAx}{L}\right) dx$$

$$W = \frac{YA}{L}\left[\frac{x^2}{2}\right]_0^{\Delta L}$$

$$W = \frac{YA}{L}\,\frac{(\Delta L)^2}{2}$$

$$W = \frac{YA}{L} \times \frac{(\Delta L)^2}{2} \times \frac{L}{L}$$

$$W = \frac{YAL}{L} \times \left(\frac{\Delta L}{L}\right)^2 \times \frac{1}{2}$$

$$\left(\frac{1}{2}\sigma \cdot \epsilon\right)$$

$$W = Y \times \text{volume} \times (\text{Strain})^2 \times \frac{1}{2}$$

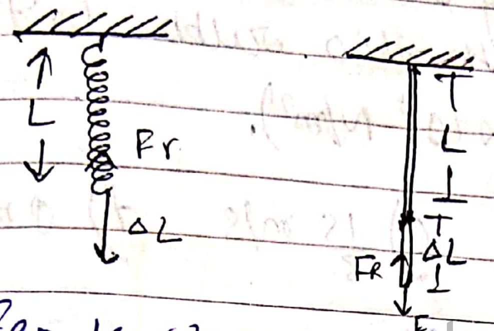
$$W = \frac{1}{2} \times (\text{Strain})\,(\text{Strain}\,Y)\ \text{volume}$$

PE

U

$$\boxed{W = \frac{1}{2} \times \text{Stress} \times \text{strain} \times \text{volume}}$$

TRICK :-



$Y = \frac{FL}{A\,\Delta L}$

$F_R = K\,\Delta L$

$F = \left(\frac{YA}{L}\right)\Delta L$

$K\cancel{\Delta L} = \left(\frac{YA}{L}\right)\cancel{\Delta L}$

$\boxed{K = \frac{YA}{L}}$ **Most**

Energy stored in a streched spring $= \frac{1}{2}K(\Delta x)^2$

$= \frac{1}{2} \times \frac{YA}{L}(\Delta L)^2 \times \frac{L}{L}$

$= \frac{1}{2} \times Y(AL)\left(\frac{\Delta L}{L}\right)^2$

$= \frac{1}{2} \times Y \times \text{vol} \times (\text{strain})^2$

$\boxed{W = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}}$

Q A rubber cord of length 10 cm is stretched upto 12 cm. If cross sectional area of cord is 1 mm², find the velocity of a missile (mass 5g) which is hit upon with this rubber cord ($Y_{Rubb} = 5\times10^{8}$ N/m²).

a) 10 m/s ✓b) 20 m/s c) $\frac{15}{3}$ m/s d) 0 m/s

Tension / Force (T) $= \frac{Y A l}{L} = 100$ N.

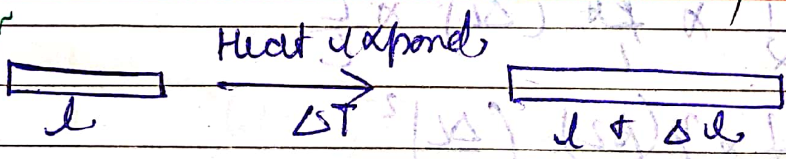
Velocity $V = \sqrt{\frac{F l}{m}}$

$= \sqrt{\frac{100 \times 2 \times 10^{(-2)}}{5 \times 10^{(-3)}}}$

$= 20$ m/s

THERMAL STRESSES & STRAIN

Rod Force to Deform



$\boxed{\Delta l = l \alpha \Delta T}$

(alpha) [Thermal Coefficient of linear expansion]

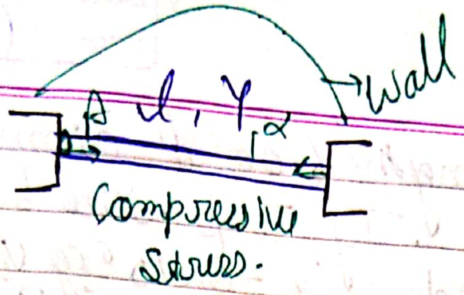
$\alpha \Rightarrow$ Unit temp increase

⇓

Unit length Me

⇓

Kitna Change Hoga



Temp rise $\rightarrow \Delta T$

New length

$= l + \Delta l$

$= \underline{l + l\alpha\Delta T}$

Not free to deform

$$\boxed{\text{Thermal Strain} = \frac{\Delta l}{l}}$$

[original len $\rightarrow$ 'l']

Thermal Stress $= Y \text{ strain} = Y \frac{\Delta l}{l}$

$$\frac{\text{Stress}}{\text{Strain}} = Y$$

$$\text{Stress} = \frac{Y(l\alpha\Delta T)}{l}$$

$$\boxed{\text{Thermal Stress} = Y\alpha\Delta T}$$

How much Force :-

$$\text{Stress} = \frac{F}{A}$$

$$F = \text{Stress} \times \text{Area}$$

$$\boxed{F = Y\alpha A\Delta T}$$

Q. A steel rod of length 6.0 m & diameter 20 mm is fixed as shown. If Temp. rise by 80°C find the stress in rod. $Y = 2\times10^6$ kg/cm²
$\alpha = 12\times10^{-6}$ Per °C param.

$\Delta T = 80°C$

$\Delta l = l\alpha\Delta T$

$Strain = \frac{\Delta l}{l} = \frac{l\alpha\Delta T}{l} = \alpha\Delta T$

$Stress = Y \cdot Strain$

$= Y\alpha\Delta T$

$= 2\times10^6 \frac{kg}{cm^2} \times 12\times10^{-6}\times80$

$= 1920\, kg/cm^2$

Q2. In the previous Q. Agar that :— if it is allowed to yield (expand by 1 mm)

a) 1920
b) More
c) Less

Kitna expand karna chahiye tha.

$\Delta l = l\alpha\Delta T$

$$\boxed{Strain = \frac{l - 1mm}{l}}$$

$Stress = Y\, Strain$

$= 2\times10^6 \left(\frac{l\alpha\Delta T - 1mm}{l}\right)$

$= 2\times10^6 \left(\alpha\Delta T - \frac{0.1 cm}{600 cm}\right)$

$= 2\times10^6 \left(12\times10^{-6}\times80 - \frac{0.1}{600}\right)$,

$$\boxed{\text{Stress} = \gamma \, \alpha \, \Delta T}$$

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$= 1920 - 2 \times 10^6 \times \frac{1}{6000}$

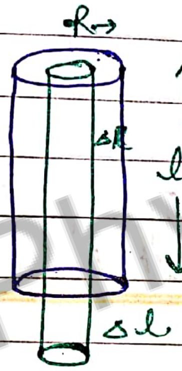
$= 1920 - \frac{2 \times 10^3}{6} = 1920 - \frac{1000}{3}$

$= 1920 - 333.3$

$= 1586.67 \ Kg/cm^2$

POISSON'S RATIO.

$$\boxed{\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}}}$$



$$\boxed{\sigma = - \frac{\Delta R / R}{\Delta l / l}}$$

+ve

Q → It's length increase by 1%.
if $\sigma = \frac{1}{4}$, find the % change in its vol.

a) 1% increase

b) 1% decrease

c) 0.5% increase

d) 0.5% Decrease

$$V = \pi R^2 L$$

inc.

$$\left(\frac{\Delta V}{V}\times 100\right) = 2\left(\frac{\Delta R}{R}\times 100\right) + \left(\frac{\Delta L}{L}\times 100\right)$$

$$= -2\times\frac{1}{4}\left(\frac{\Delta l}{l}\times 100\right) + \left(\frac{\Delta L}{L}\times 100\right)$$

$$\sigma = \frac{-\Delta R/R}{\Delta l/l}$$

$$\boxed{\frac{\Delta R}{R} = -\left(\frac{\Delta l}{l}\,\frac{1}{4}\right)}$$

$$= -2\times\frac{1}{4}(1\%) + 1\%$$

$$= 1 - \frac{1}{2} = 0.5\%$$

Change In Density Of A Liquid

⇓

Due to Pressure.

$$\boxed{\rho \uparrow \Rightarrow P \uparrow}$$

Practical (Non-Ideal liquid) # ideal fluid → Incompressible ↓ [Density is same]

$$\rho = \frac{m}{V}$$

$$\rho \propto \frac{1}{V}$$

$$\rho \rightarrow \rho'$$

$$V \rightarrow V + dV \ \ominus$$

$$\frac{\rho'}{\rho} = \frac{V}{V+dV}$$

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Bulk Mod.

$$B = -\frac{dP}{\left(\frac{du}{u}\right)}$$

$$= \frac{1}{1 + \frac{du}{u}}$$

$$\boxed{\rho' = \rho\left(\frac{1}{1 - \frac{dP}{B}}\right)}$$

$P\uparrow \Rightarrow dP = +ve.$

$\left(1 - \frac{dP}{B}\right)\downarrow \Rightarrow \rho\uparrow.$

FINISH