TUESDAY
classmate
Date 17-9-19
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Kinetic theory of Gases

Introduction:

↳ John Dalton discovered atom in 1803

↳ Kinetic theory in 1873

↓

Atoms / molecules constantly moves (MOTION)

Given by Maxwell & Boltzmann

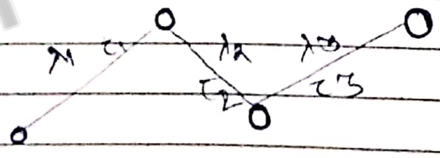
Assumption:

i) Intermolecular force is absent in ideal gases

* We study avg. properties of gases

Mean Free Path:

Avg. distance travelled by a molecule b/w 2 successive coll^n s.



$$\lambda_m = \frac{\lambda_1 + \lambda_2 + \dots + \lambda_n}{n}$$

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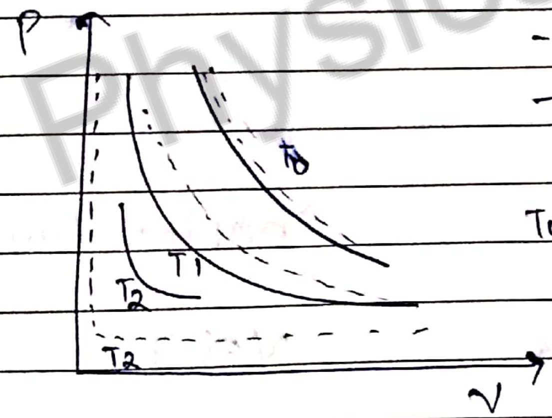
Avg relaxation time (τ)
Avg time period b/w 2 successive coll^{n}s.

$$\tau_m = \frac{\tau_1 + \tau_2 + \text{------} \tau_n}{n}$$

IDEAL GAS Laws:

I] Boyle's law: (1661)
At, constant temp of gas, [isothermal]

$$V \propto \frac{1}{P}$$



------ → experimental
—— → law/theoreotical.

$T_0 > T_1 > T_2$

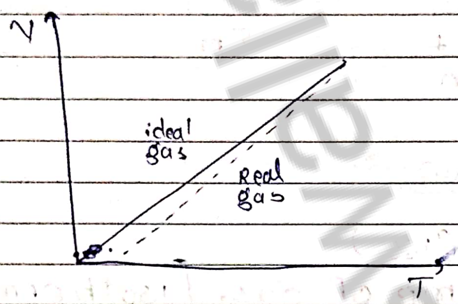
High temp: (Real gas → ideal gas)
Gas law agrees experiment.

VP = constant
$\boxed{P_1 V_1 = P_2 V_2}$ = constant.

II) 1783: Charles Law

At constant P

$V \propto T$



At high temp. law holds for all gases.

$\frac{V}{T} = \text{constant}$

$\frac{V_1}{T_1} = \frac{V_2}{T_2}$

* Boyle's Law

$V \propto \frac{1}{P}$

T = constant

Charles Law

$V \propto T$

P = constant

$V \propto \frac{T}{P}$

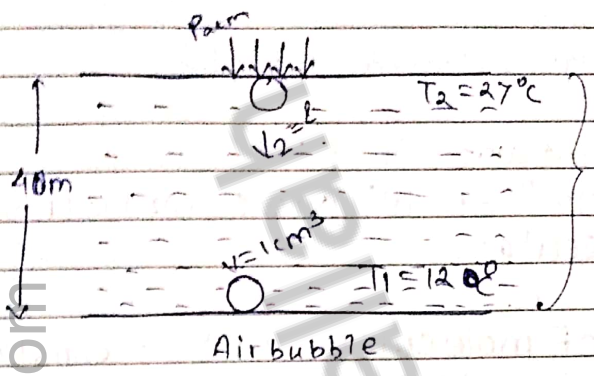
$\frac{PV}{T} = \text{constant}$

$$\boxed{\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}}$$

$P_{atm} = 1\,atm$

$= 1.01 \times 10^5 \frac{N}{m^2}$

Q



$P_1 = \rho g h + P_{atm}$

$\cancel{P_1V_1 = P_2V_2}$

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$\frac{(1.01 \times 10^5 + 10^3 \times 10 \times 40) \times 1cm^3}{285} = \frac{(1.01 \times 10^5) \times V_2}{300}$

$\frac{10^5 + 4 \times 10^5}{285} = \frac{10^5 \times V_2}{300}$

$10^5(5)$

$V_2 = \frac{300}{60}$

$V_2 = 5\,cm^3$

III) Avogadro's hypothesis (1812)

At constant T & P:
Equal vol^m of all gases contains equal no. of molecules.

$V \propto$ no. of molecules (P & T → constant)

$V \propto$ no. of moles

$\boxed{V \propto n}$

Ideal Gas Equation:

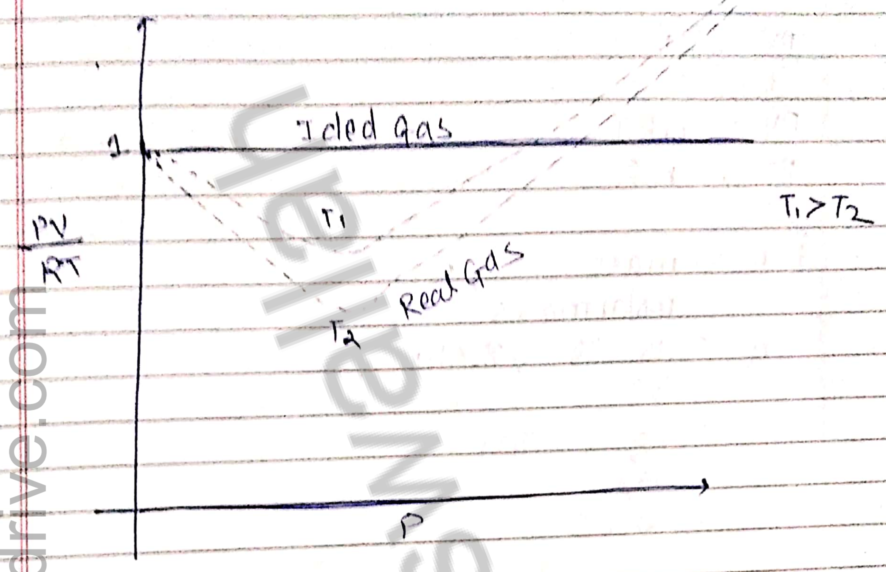
$V \propto \frac{1}{P}$ $\quad V \propto T$ $\quad V \propto n$

$V \propto \frac{nT}{P}$

$V = \frac{RnT}{P}$

$\boxed{PV = nRT}$

Where,
- P = Pressure
- V = Vol^m
- R = Gas constant : 8.314 J/mol·K.
- T = Temperature (in kelvin)
- n = no. of moles.

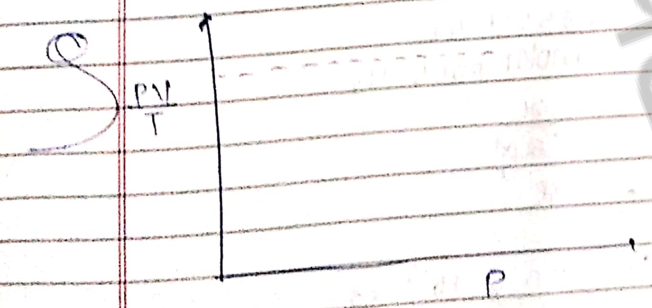


Real gas $\Rightarrow$ Ideal gas $\Rightarrow$ Low Temp Pressure
High Temp

1 mole of any Gas: $PV = RT$

ideal Gas $= \frac{PV}{RT} = 1$ (Low P & High T)

High temp ↑↑ real gas & ideal gas get closer



10^{-3} kg of O_2

Ideal gas

At all pressure fall Temp.

$R = 8.14$ J/mol K.

$\frac{PV}{T} = ?$

where ···· cut y axis.

$\frac{PV}{T} = P$

$PV = nRT$

$\frac{PV}{T} = nR$

$n = \frac{mass}{molar\,mass} \times R$

$n = \frac{10^{-3}\,kg}{32\,g} \times 8.314$

$= \frac{1\,g}{32\,g} \times 8.314$

$= 0.25$

For what mass of H_2 gas the value of $\frac{PV}{T}$ will be same as $\frac{PV}{T}$ for 10^{-3} kg of O_2 gas

$PV = n_{O_2} RT$

$PV = n_{H_2} RT$

$\left(\frac{PV}{T}\right)_{O_2} = \left(\frac{PV}{T}\right)_{H_2}$

$n_{O_2} \cancel{R} = n_{H_2} \cancel{R}$

$\frac{mass\ of\ O_2}{molar\ mass\ O_2} = \frac{mass\ of\ H_2}{molar\ mass\ H_2}$

$\frac{10^{-3}\,kg}{\cancel{32}_{16}\,g} = \frac{x}{\cancel{2}\,g}$

$\frac{1}{16} \times 10^{-3}\,kg = x$

$x = \frac{100}{16} \times 10^{-5}\,kg$

$= 6.25 \times 10^{-5}\,kg$

$m_{H_2} = 6.25 \times 10^{-5}\,kg$

Q. Find the ratio of $\frac{\text{molecular vol}^m}{\text{Actual vol}^m \text{ occupied}}$ by O_2 molecules.

(diameter of O_2 molecule is $3A°$

→ 1 mole of O_2 at STP

$= 22.4L$

$= 22.4 \times 10^{-3} m^3$

6.022×10^{23} molecules of O_2

$V = \frac{4}{3} \pi r^3 \times 6.022 \times 10^{-23}$

$\therefore \frac{\text{Molecular vol}^m}{\text{Actual vol}^m \text{ occupied}} = \frac{4/3 \pi r^3 \times 6.022 \times 10^{23}}{22.4 \times 10^{-3} m^3}$

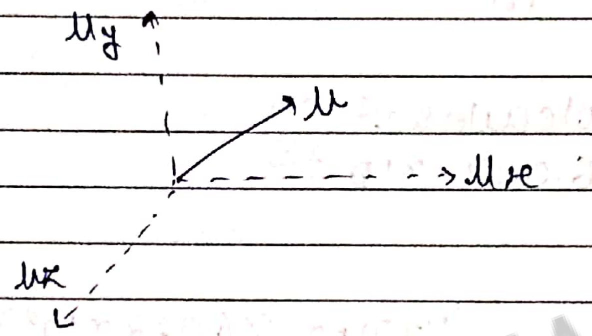
Kinetic Theory of Gases:

Postulates:

i] Gas is made up of atoms & molecules.

ii] Molecules of same gases are identical in all respects. (mass, shape, size)

iii] Molecules are constanly in random motion along straight line.



$$u = u_x\hat{i} + u_y\hat{j} + u_z\hat{k}$$

$$u_x = u_y = u_z$$

$$|u| = \sqrt{u_x^2 + u_y^2 + u_z^2}$$

$$u^2 = u_x^2 + u_y^2 + u_z^2 = 3u_x^2$$

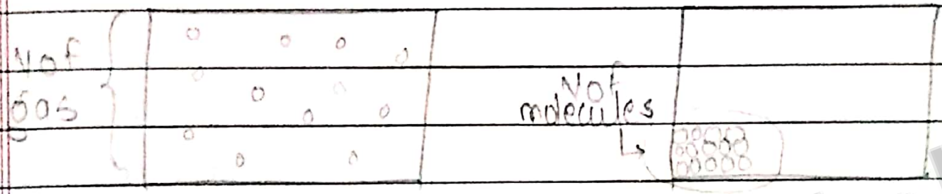
iv] All the coll^{n}s of gas molecules among themselves & wall of container is elastic in nature

Momentum } No loss.
Kinetic energy

v] The pressure of a gas is due to coll^{n}s of molecules with wall of container.

$P \propto$ No. of coll^{n}s of molecules per unit area vi] The kinetic energy of gas depends only & only upon temp (absolute) (doesn't depend on nature of gas)

vii] The vol^m occupied by gas molecules is negligible when compared to vol^m of gas.



viii] There is no inter molecular force of attraction among molecules.

ix] Gravity is neglected.

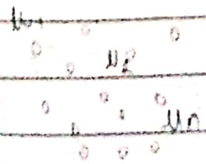
* Kinetic gas eq^n :

$$\boxed{PV = \frac{1}{3} m n u_{rms}^2}$$

m → mass of one molecule of gas

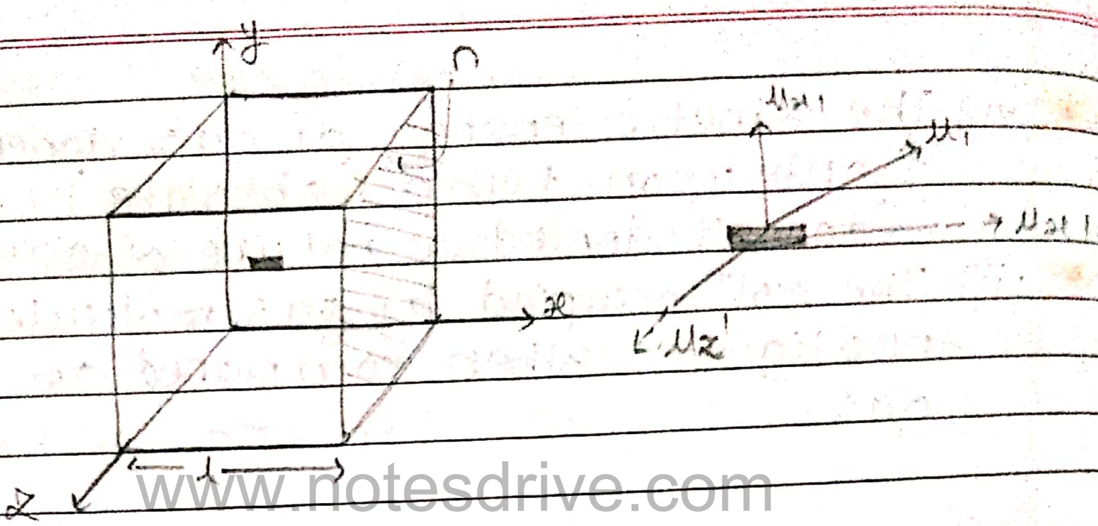
n → no. of molecules

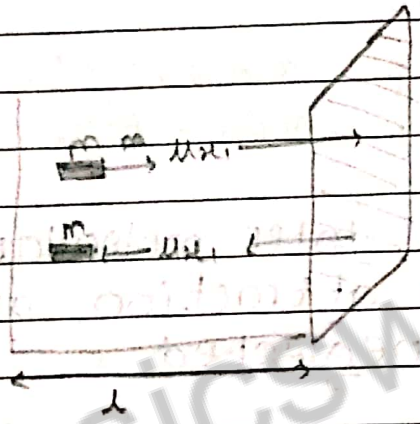
u_{rms} → Root mean square velocity of molecules



$$\boxed{u_{rms} = \sqrt{\frac{u_1^2 + u_2^2 + u_3^2 + \dots u_n^2}{n}}}$$

$$\boxed{\text{K.E of n moles of gas} = n \times \frac{3}{2} RT}$$





$P_i = m u_{x1}$

$P_f = -m u_{x1}$

$\Delta P = 2 m u_{x1}$

$$\boxed{\text{time taken for each coll}^n = \frac{2l}{u_{x1}}}$$

Force exerted on wall $= \dfrac{\Delta P}{\Delta t}$

$$\vec{F} = \frac{-2 m u_{x1}}{\dfrac{2l}{u_{x1}}}$$

$$\boxed{\vec{F} = \frac{m u_{x1}^2}{l}}$$

$$* \ u = u_x \hat{i} + u_y \hat{j} + u_z \hat{k}$$
$$u^2 = 3u_x^2$$

$$\vec{F} = \frac{m}{l} \frac{1}{3} u_1^2$$

$\vec{F}$ due to one molecule $= \frac{m}{l} u_{x_1}^2 = \frac{m}{l} \frac{1}{3} u_1^2$

F due to 'n' molecules:

$$= \frac{1}{3} \frac{m}{l} u_1^2 + \frac{1}{3} \frac{m}{l} u_2^2 \dots + \frac{1}{3} \frac{m}{l} u_n^2$$

$$F_{net} = \frac{1}{3} \frac{m}{l} \left[\frac{u_1^2 + u_2^2 + \dots u_n^2}{n} \right] \times n$$

$$F_{net} = \frac{1}{3} \frac{mn}{l} u_{rms}^2$$

$$\text{Pressure} = \frac{F}{A} = \frac{1}{3} \frac{mn u_{rms}^2}{l \times l^2} = \frac{1}{3} \frac{mn u_{rms}^2}{l^3}$$

$$P = \frac{1}{3} \frac{mn u_{rms}^2}{V}$$

Kinetic Gas eq^n

$$PV = \frac{1}{3} mn u_{rms}^2$$

$PV = nRT$

0.0821 atm L/mol K

$K.E = \frac{3}{2}RT$

8.314 J/mol K

Kinetic energy:

$K.E = \frac{1}{2} m u_1^2$
(of 1 molecule)

K.E of n molecules $= \frac{1}{2} m u_1^2 + \frac{1}{2} m u_2^2 + \dots + \frac{1}{2} m u_n^2$

$= \frac{1}{2} m (u_1^2 + u_2^2 + u_3^2 + \dots + u_n^2)$

$\boxed{\text{K.E of n molecules} = \frac{1}{2} m u^2_{rms} \times n}$

$\frac{K.E}{PV} = \frac{\frac{1}{2} m u^2_{rms} \times n}{\frac{1}{3} m n u^2_{rms}}$

$\boxed{PV = \frac{2}{3} K.E}$

$\boxed{K.E = \frac{3}{2} PV}$ for 1 mole of gas (ideal) $[PV = RT]$

$\boxed{K.E = \frac{3}{2} RT}$

$\boxed{PV = \frac{1}{3} m n u^2_{rms}}$

m: mass of 1 molecule
n: no. of molecules

$\boxed{\text{K.E of 1 mole of ideal gas} = \frac{3}{2} RT}$

R: Gas constant 8.314 J/mol K

$\frac{R}{N_A}$ = Boltzmann constant

$$\boxed{\text{K.E of } n \text{ moles of ideal gas} = n \times \frac{3}{2} RT}$$

$$\boxed{\text{K.E of 1 molecule} = \frac{3}{2}\frac{RT}{N_A} = \frac{3}{2} KT}$$

Boltzman constant

$$\boxed{K = 1.38 \times 10^{-23} \; J/K}$$

Q.1 Find out K.E of 8g of CH_3 at 27°C.

→ $K.E = n \times \frac{3}{2} RT$

$= 0.5 \times \frac{3}{2} \times 8.314 \times 300 \; J$

no. of moles = $\frac{\text{mass}}{\text{molarmass}}$

$= \frac{8}{16} = 0.5$

Q 2] Find out K.E of 1 molecule of oxygen(O_2) gas at 127°C.

→ $K.E = \frac{3}{2} KT$

$= \frac{3}{2} \times 1.38 \times 10^{-23} \times 400$

$= 600 \times 1.38 \times 10^{-23} \; J$

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Q.3 Find the change in K.E of 1 mole of ideal gas when temp changes by $50^\circ C$

→ $\Delta K.E = KE_f - K.E_i$

$$= \frac{3}{2}RT_2 - \frac{3}{2}RT_1$$

Me

$$= \frac{3}{2}R[\quad]$$

Sir

$$KE_i = \frac{3}{2}RT$$

$$K.E_f = \frac{3}{2}R(T+50)$$

$$\Delta KE = KE_f - K.E_i$$

$$= \frac{3}{2}R(T+50-T)$$

$$= \frac{3}{2}R \times 50$$

$$= \frac{3}{2} \times 8.314 \times 50 \text{ J}$$

Q4 At what temp, the KE will be half of its value at ~~$127^\circ C$~~ $127^\circ C$.

→ $K.E_{127^\circ C} = n \times \frac{3}{2} \times R \times 400$

$$K.E_T = \frac{K.E_{127^\circ C}}{2}$$

$$\cancel{n} \times \frac{\cancel{3}}{\cancel{2}} \times \cancel{R} \times T = \frac{\cancel{n} \times \cancel{3/2} \times \cancel{R} \times 400}{2}$$

$$\boxed{T = 200K}$$

$1 amu = 1.67 \times 10^{-27} kg$

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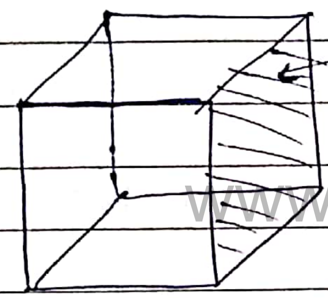
Short cut

$\boxed{KE \propto T}$

Q5



They make 500 coll^{n}s with the wall in each sec.

www.notesdrive.com

Find the temp.

$\rightarrow$ Time for 1 coll^n = $\frac{1}{500}$ s

time taken for 1 coll^n = $\frac{2a}{u_{rms}}$

$$\frac{1}{500} = \frac{2a}{u_{rms}}$$

$$\frac{1}{500} = \frac{2 \times 2 \times 10^{-2}}{u_{rms}}$$

$$u_{rms} = 20 m/s$$

K.E of 1 molecule $= \frac{1}{2} m u_{rms}^2$

$$= \frac{3}{2} kT$$

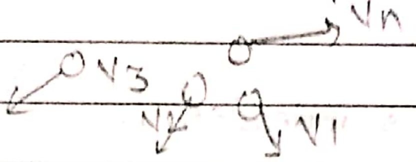
$$\frac{1}{2} m u_{rms}^2 = \frac{3}{2} kT$$

1 molecule of He = 4 amu
$= 4 \times 1.67 \times 10^{-27}$ kg

Physicswallah

$M = m N_A$
(molar mass) (1 molecule mass)

RMS Velocity
[Root Mean Square]

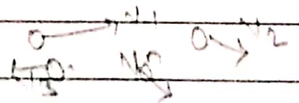


$$V_{RMS} = \sqrt{\frac{V_1^2 + V_2^2 + V_3^2 + \dots + V_n^2}{n}}$$

$$\boxed{V_{RMS} = \sqrt{\frac{3RT}{M}}}$$

V_{RMS} (m/s)

R = 8.314 J/mol K
T = in Kelvin
M = Mass i.e molar mass of gas (in kg)



$$K.E_{Total} = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \dots + \frac{1}{2}mv_n^2$$

$$= \frac{1}{2}m\left(\frac{v_1^2 + v_2^2 + \dots v_n^2}{n}\right) \times n$$

$$= \frac{1}{2}m\, v_{rms}^2 \times n$$

K.E of 1mole of gas $= \frac{1}{2}mv_{rms}^2 \times N_A$

$$\frac{3}{2}RT = \frac{1}{2}Mv_{rms}^2$$

$$\boxed{V_{rms} = \sqrt{\frac{3RT}{M}}}$$

(in kg)

Q. V_{rms} for H_2 molecules at $27^\circ C$.

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$= \sqrt{\frac{3 \times 8.314 \times 300}{2 \times 10^{-3}}}$$

$$= \sqrt{3 \times 4 \times 300 \times 10^{3}}$$

$$= \sqrt{36 \times 10^{5}}$$

$$= \sqrt{3.6 \times 10^{6}}$$

$$= 1.3 \times 10^{3}$$

Q. if V_{rms} for H_2 is x at a given Temp. Find V_{rms} for O_2 at given Temp.

$$\frac{V_{O_2}}{V_{H_2}} = \frac{\sqrt{\frac{\cancel{3RT}}{M_{O_2}}}}{\sqrt{\frac{\cancel{3RT}}{M_{H_2}}}} = \sqrt{\frac{M_{H_2}}{M_{O_2}}}$$

$$\frac{V_{O_2}}{x} = \sqrt{\frac{2 \times 10^{-3}}{32 \times 10^{-3}}} = \sqrt{\frac{1}{16}}$$

$$V_{O_2} = \frac{x}{4}$$

$$\boxed{V_{RMS} \propto \sqrt{T}}$$

Q At 300°K, V_{rms} of H_2 is same as V_{rms} of O_2 at T (in kelvin). Find T.

→ $V_{rms\,H_2} = V_{rms\,O_2}$

$$\sqrt{\frac{3RT}{M_{H_2}}} = \sqrt{\frac{3RT}{M_{O_2}}}$$

$$\sqrt{\frac{300}{\cancel{2}_1 \times \cancel{10^{-3}}}} = \sqrt{\frac{T}{\cancel{32}_{16} \times \cancel{10^{-3}}}}$$

$$\sqrt{300} = \sqrt{\frac{T}{16}}$$

$$300 = \frac{T}{16}$$

$$T = 4800\,K$$

Pressure

$$V_{RMS} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{\rho}} \text{ (S.I unit)}$$

Derivation:

K.E of 1 mole of Gas $= \frac{1}{2} M V_{rms}^2$

$$\frac{3}{\cancel{2}} RT = \frac{1}{\cancel{2}} M V_{rms}^2$$

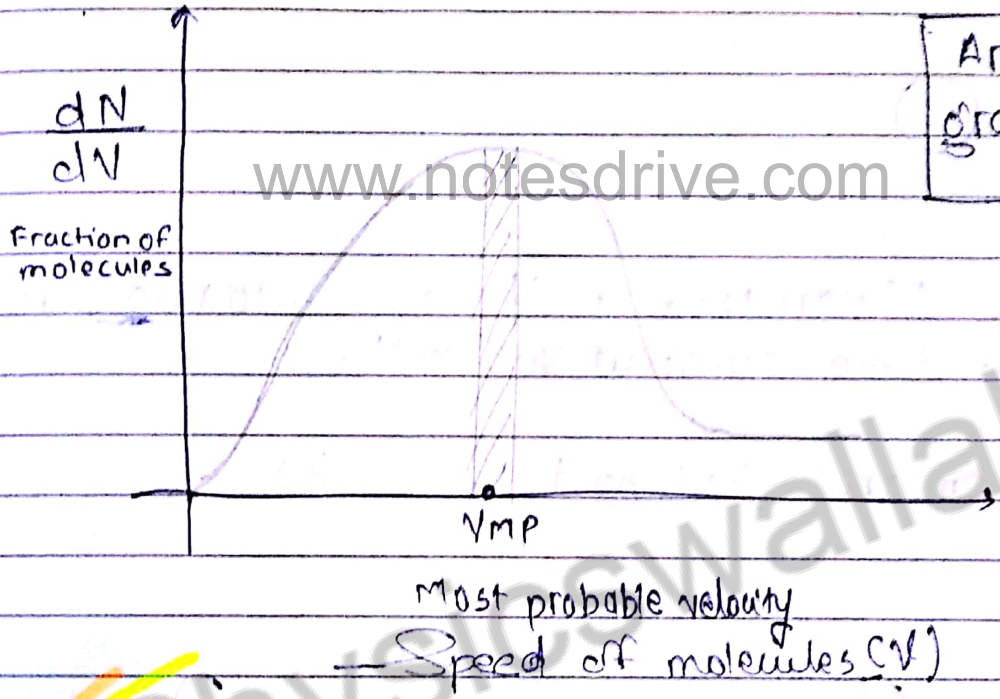
$$\frac{3}{\cancel{2}} PV = \frac{1}{\cancel{2}} M V_{rms}^2$$

$$V_{rms} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3P}{(M/V)}} = \sqrt{\frac{3P}{\rho}}$$

At constant temp

$P \uparrow \quad V_{rms} = \text{constant}$

Maxwell's distribution of velocities



Area of graph = Total no. of molecules

$$\boxed{V_{MP} = \sqrt{\frac{2RT}{M}}}$$

$$\boxed{V_{RMS} = \sqrt{\frac{3RT}{M}}}$$

(speed) $\boxed{V_{avg} = \sqrt{\frac{8RT}{\pi M}}}$

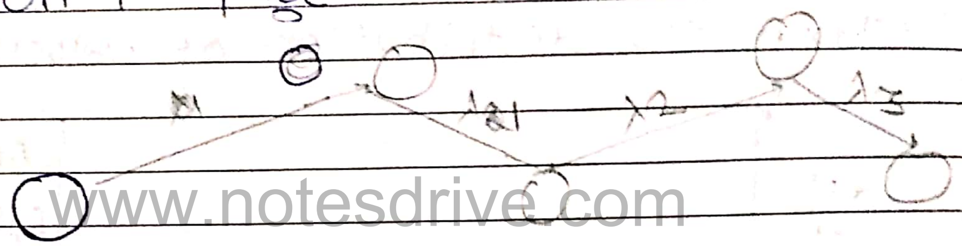
(velocity) $\boxed{\vec{V}_{avg} = 0}$

R > A > M

$\boxed{V_{RMS} > V_{avg} > V_{MP}}$

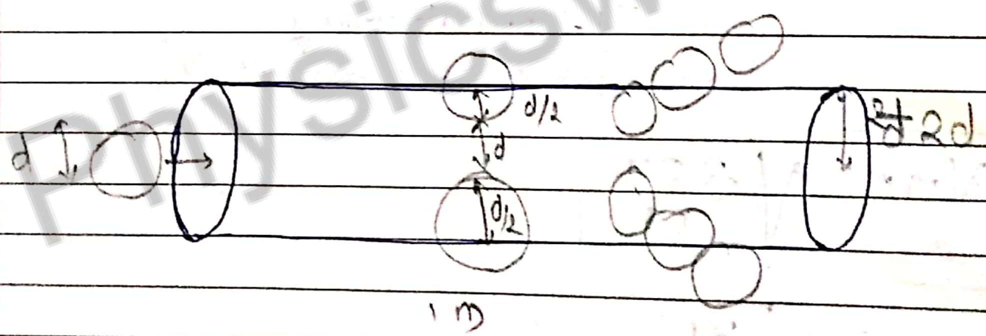
Mean Free Path:

On 1st page.



Mean free path is average distance b/w two consecutive coll^{n}s.

$$\lambda_{mean} = \frac{\lambda_1 + \lambda_2 + \cdots \lambda_n}{n}$$



no. of molecules per unit vol^m = x

$1m^3 = x$

$2m^3 = 2x$

$V m^3 = Vx$

no of molecules inside cylinder = $\pi (d)^2 \times 1 \times x$

no of coll^{n}s in 1 m travelling distance = $\pi (d)^2 x$

$\pi d^2 x$ $coll^n s \longrightarrow 1\ m$ distance

$1\ coll^n \longrightarrow \dfrac{1}{\pi d^2 x}$ distance.

$\boxed{\lambda = \dfrac{1}{\pi d^2 x}}$ $\boxed{\lambda = \dfrac{RT}{\pi d^2 P N_A}}$

Avogadro no

Pressure in SI unit

diameter of molecule

$x \rightarrow$ no of molecules per unit vol^m

$$PV = nRT$$

$$\frac{n}{V} = \frac{P}{RT}$$

$$x = \frac{n}{V} \times N_A$$

$$x = \frac{PN_A}{RT}$$