QUADRATIC EQUATIONS

An equation in which the highest power of variable, i.e., degree of equation is 2, is called quadratic equation.

The general form of quadratic equation is :-

$$ax^2 + bx + c = 0$$

where,

$$a \neq 0$$

a, b, c are real numbers

For e.g.,

$$x^2 + 3x + 2 = 0 \quad ; \quad x^2 - 4 = 0$$

$$(x+3)(x-1) = 0 \quad \text{etc.}$$

i) $(x+1)^2 = 2(x-3)$

$\rightarrow$ $x^2+2x+1 = 2x-6$

$x^2+2x-2x = -6-1$

$x^2 = -7$

$x^2+7=0$

$\therefore$ It is a quadratic equation.

ii) $x^2-2x = (-2)(3-x)$

$\rightarrow$ $x^2-2x = -6+2x$

$x^2-2x-2x = -6$

$x^2-4x+7 = 0$

$\therefore$ It is a quadratic equation

iii) $(x-2)(x+1) = (x-1)(x+3)$

$\rightarrow$ $x^2+x-2x-2 = x^2+3x-x-3$

$x^2-x-2 = x^2+2x-3$

$\cancel{x^2}-\cancel{x^2}-x-2x-2+3=0$

$-3x+1=0$

$\therefore$ It is not a quadratic equation

2) Represent the following situations in the form of quadratic equations :

(i) The area of rectangular plot is $528 m^2$. The length of the plot is one more than twice its breadth. We need to find the

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length and breadth of the plot.

Let the breadth of rectangular plot of be x.

According to given condition

breadth $= x\,m$

length $= (2x+1)\,m$

$\therefore$ Area of Rectangular plot $= l \times b$

$528 = (2x+1) \times x$

$528 = 2x^2 + x$

$\therefore 2x^2 + x - 528 = 0$ is the required ~~deno~~ solutions

The products of two consecutive positive integers is 306. We need to find integers.

Let first integer be x.

$\therefore$ ~~According~~ ~~Second~~ Consecutive integer will be $x+1$

$\therefore$ According to given condition

$x \times (x+1) = 306$

$x^2 + x = 306$

$\therefore x^2 + x - 306 = 0$ will be required solutions

Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

Let Rohan's present age be x yrs.

$\therefore$ His mothers age $= x + 26$ yrs

3 years from now,

Rohan's age $= x + 3$

Age of Rohan's mother will be $= x + 26 + 3$

$= x + 29$

$\therefore$ The product of their ages 3 years from now will be 360 so that,

$(x+3)(x+29) = 360$

$x^2 + 29x + 3x + 87 = 360$

$x^2 + 32x - 273 = 0$

SOLUTION OF QUADRATIC EQUATION BY FACTORISATION

The values of variables which satisfy the quadratic equation is called roots of quadratic equation or its solution.

NOTE :- If a product of two numbers is '0' then either first or second number is '0'

(ii) $2x^2 + x - 6 = 0$

$\rightarrow$ $2x^2 + 4x - 3x - 6 = 0$

$2x(x+2) - 3(x+2) = 0$

$(x+2)(2x-3) = 0$

$x+2 = 0$ or $2x - 3 = 0$

$\therefore$ $x = -2$ or $2x = 3$

$x = 3/2$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\rightarrow$ $\sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$

$x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$

$(\sqrt{2}x + 5)(x + \sqrt{2}) = 0$

$\sqrt{2}x + 5 = 0$ or $x + \sqrt{2} = 0$

$x = -5/\sqrt{2}$ or $x = -\sqrt{2}$

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5) Find two consecutive positive integers, sum of whose squares is 365.

→ Let ~~x~~ two consecutive positive integers be,

x, $(x+1)$

According to given condition,

$$x^2 + (x+1)^2 = 365$$

$$x^2 + x^2 + 2x + 1 - 365 = 0$$

$$2x^2 + 2x - 364 = 0$$

Divide by 2,

$$x^2 + x - 182 = 0$$

$$x^2 + 14x - 13x - 182 = 0$$

$$x(x+14) - 13(x+14) = 0$$

$$(x+14)(x-13) = 0$$

$\therefore$ $x + 14 = 0$ or $x - 13 = 0$

$x = -14$ or $x = 13$

$\therefore$ $x \neq -14$

$\therefore$ $x = 13$

$\therefore$ Two consecutive integers are 13 & 14

SOLUTION OF QUADRATIC EQUATION BY COMPLETING THE SQUARE

) $2x^2 - 7x + 3 = 0$

Given equation is,

$$2x^2 - 7x + 3 = 0$$

$$2(x^2 - 7x/2 + 3/2) = 0$$

$$x^2 - 7x/2 + 3/2 = 0$$ [To get third term]

$$x^2 - 7x/2 = -3/2$$ $(1/2 \times -7/2)^2$

$$x^2 - 7x/2 + 49/16 = -3/2 + 49/16$$

$$(x - 7/4)^2 = 25/16$$

$$x - 7/4 = \pm\sqrt{25/16} = \pm 5/4$$

i.e. $x - 7/4 = 5/4$ or $x - 7/4 = -5/4$

$x = 5/4 + 7/4$ or $x = -5/4 + 7/4$

$x = 3$ & $x = 1/2$

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QE formula,

$ax^2 + bx + c = 0$

$x^2 + (b/a)x + c/a = 0$

$x^2 + (b/2a)x + (b/2a)x + c/a = 0$

$x^2 + 2(b/2a)x + (b/2a)^2 - (b/2a)^2 + c/a = 0$

$(x + b/2a)^2 - (b^2 - 4ac/4a^2) = 0$

If $b^2 - 4ac \geq 0$,

$\Rightarrow x + b/2a = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}} \qquad D = b^2 - 4ac$$

Roots of Equation

If $b^2 - 4ac > 0$ (Positive)

Real & Unequal Roots

If $b^2 - 4ac = 0$

Real & Equal Roots

$b^2 - 4ac < 0$ (Negative)

Roots are not real

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$a\ (x-p)\ (x-q)=0$

$a\ (x^2-(p+q)x+pq)=0$

$-a(p+q)=b$ & $apq=c$

$\Rightarrow\ p+q=\frac{-b}{a}$ → sum of roots

$(\alpha+\beta)$

$pq=c/a$ → products of roots

$(\alpha\beta)$

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Find the roots of the following equations:

$x - 1/x = 3, x \neq 0$

$x - 1/x = 3$

$x^2 - 1 = 3x$ (Multiply by x)

compare equation by

$ax^2 + bx + c = 0$

$\therefore a = 1, \quad b = \cancel{1} -3, \quad c = -1$

By quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2 \times 1}$$

$$x = \frac{3 \pm \sqrt{9 + 4}}{2}$$

$$x = \frac{3 \pm \sqrt{13}}{2}$$

$\therefore x = \frac{3 + \sqrt{13}}{2}$ & $x = \frac{3 - \sqrt{13}}{2}$

NATURE OF ROOTS

If $ax^2+bx+c=0$, be a quadratic equation in x, then, the nature of roots can be determined using

"b^2-4ac" is called as Discriminant (D) its denoted by "Δ" delta

i.e. $\Delta = b^2-4ac$

So, a quadratic equation $ax^2+bx+c=0$ has

1. two distinct real roots, if $b^2-4ac>0$
2. two equal real roots, if $b^2-4ac=0$,
3. no real roots, if $b^2-4ac<0$

1] Find the nature of roots of the following quadratic equation. If real roots exist, find them:

(i) $2x^2 - 3x + 5 = 0$

→ We know,

$\Delta = b^2 - 4ac$

$= (-3)^2 - 4(2)(5)$

$= 9 - 40$

$= -31$

∴ It has no real roots

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

→ $\Delta = b^2 - 4ac$

$= (-4\sqrt{3})^2 - 4(3)(4)$

$= 48 - 48$

$= 0$

∴ It has equal real roots

3) Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m². If so, find its length and breadth.

→ Let length be $2x$ and breadth be x

So Area $= 800 m^2$

$$(x)(2x) = 800$$

$$2x^2 = 800$$

$$x^2 = 400$$

$$x = \pm\sqrt{400}$$

$$x = \pm 20$$

∴ Yes it is possible

So length $= 40m$

breadth $= 20m$