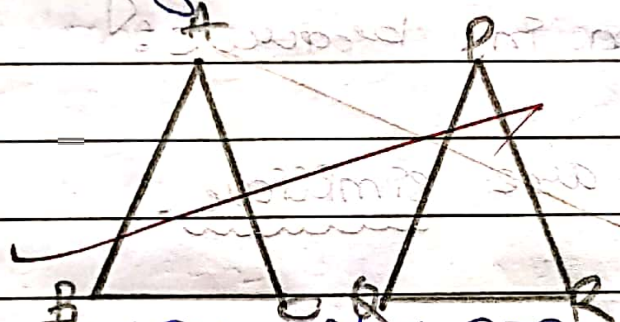
TRIANGLES

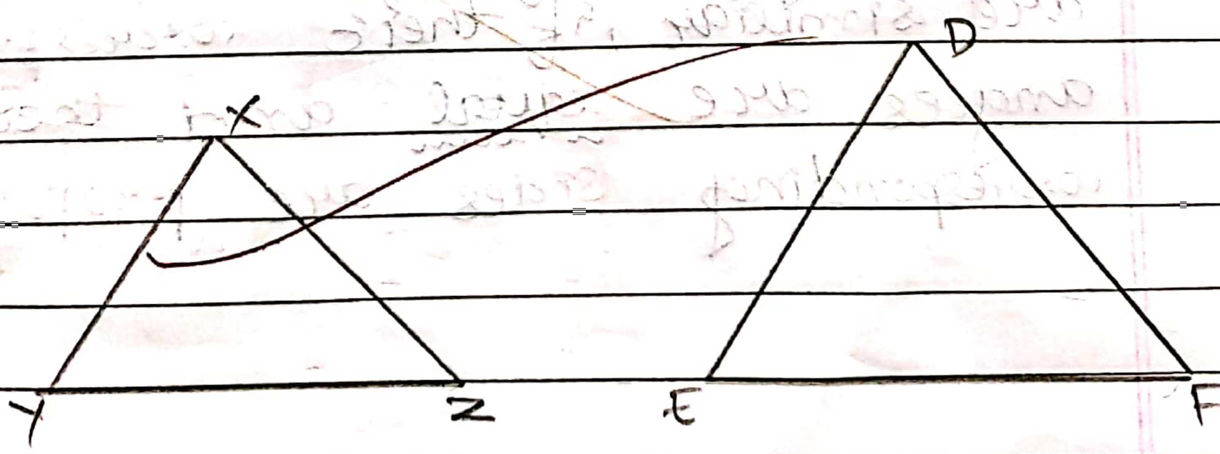
- Similar Figures & Congruent Figures

Two figures are said to be Congruent if they have same ~~an~~ shape and same size.

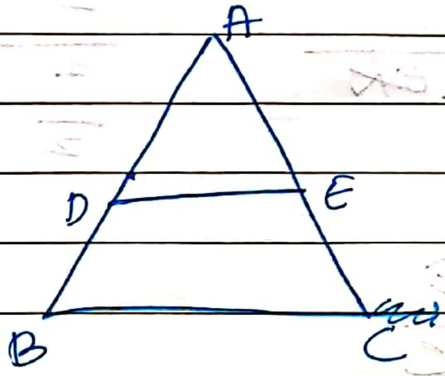


$\Delta ABC \cong \Delta PQR$

Two figures are said to be similar if they have same shape but different size.



Basic Proportionality Theorem (BPT)
If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, then the other two sides are divided in same ratio.



$$\frac{AD}{DB} = \frac{AE}{EC}$$

Proof :-

Given $DE \parallel BC$

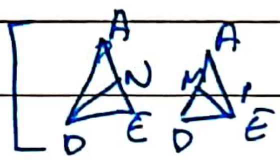
To find $\frac{AD}{DB} = \frac{AE}{EC}$

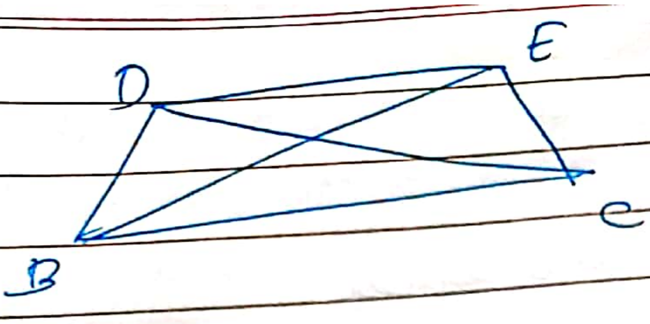
Proof :-

A of $\Delta = 1/2 \times b \times h$

A of $\Delta ADE = 1/2 \times AD \times EN$

& $= 1/2 \times AE \times DM$





A of ΔBDE = A of Δ CDE

$\frac{1}{2} \times DB \times EN = \frac{1}{2} \times EC \times DM$

$$\frac{A(\Delta ADE)}{A(\Delta BDE)} = \frac{A(\Delta ADE)}{A(\Delta CDE)}$$

$$\frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM}$$

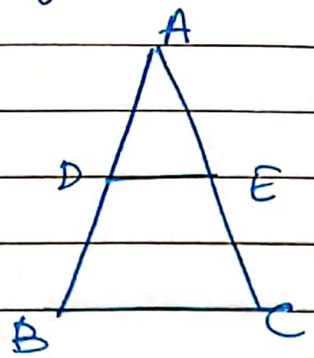
$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence proved

Converse of BPT

~~Proof~~

If a line divides any two sides of a triangle in the same ratio, then the line will be parallel to the third side.



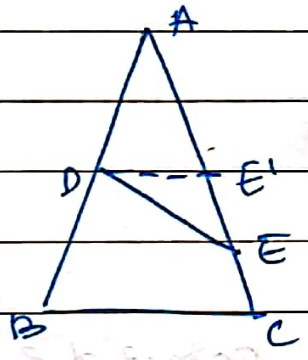
If

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Then,

DE || BC

Proof,



Given

$$\frac{AD}{DB} = \frac{AE}{EC}$$

To prove DE || BC

Cons^n → Draw DE' || BC

Proof

Assume $DE \nparallel BC$

Since $DE' \parallel BC$

$$\frac{AD}{DB} = \frac{AE'}{E'C} \quad \text{(From BPT)}$$

$$\frac{AE}{EC} = \frac{AE'}{E'C}$$

$$\frac{AE}{EC} + 1 = \frac{AE'}{E'C} + 1$$

$$\frac{AE + EC}{EC} = \frac{AE' + E'C}{E'C}$$

$$\frac{AC}{EC} = \frac{AC}{E'C}$$

$$EC = E'C$$

This implies E and E' must coincide

$\therefore DE \parallel BC$

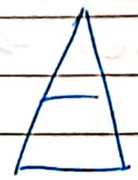
Hence proved

Examples

In ΔABC, if $DE \parallel BC$, prove that $\frac{AB}{AD} = \frac{AC}{AE}$

→ Given, $DE \parallel BC$

$\frac{AB}{AD} = \frac{AC}{AE}$ To prove



Proof,

$DE \parallel BC$

$\frac{AD}{DB} = \frac{AE}{EC}$ (From BPT)

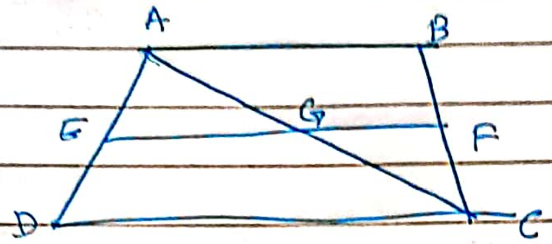
$\frac{DB}{AD} + 1 = \frac{EC}{AE} + 1$

$\frac{DB + AD}{AD} = \frac{EC + AE}{AE}$

$\therefore \frac{AB}{AD} = \frac{AC}{AE}$

Hence proved

② ABCD is a trapezium, with AB ∥ DC and EF ∥ AB.
Prove that $\frac{AE}{ED} = \frac{BF}{FC}$



Given → AB ∥ DC
EF ∥ AB

To prove → $\frac{AE}{ED} = \frac{BF}{FC}$

Cons → Join A & C

Proof
Since AB ∥ DC & EF ∥ AB
EF ∥ DC

In ΔADC,
$\frac{AE}{ED} = \frac{AG}{GC}$ (From BPT) —— ①

In ΔCAB
$\frac{CG}{GA} = \frac{CF}{FB}$ (From BPT) —— ②

Compare ① & ②

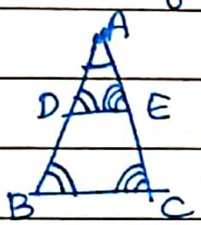
$$\frac{AE}{ED} = \frac{BF}{FC}$$

Hence proved

CRITERIA FOR SIMILARITY

① AAA

Proof,



Given,

$\angle A = \angle A$

$\angle B = \angle D$

$\angle C = \angle E$

To prove

$\Delta ABC \sim \Delta ADE$

Proof

In ΔABC & ΔADE;

$\because \angle D = \angle B$ & $\angle E = \angle C$

$DE \parallel BC$ ($\angle B$ & $\angle D$ are corresp

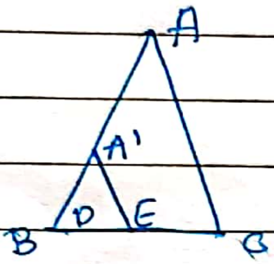
By BPT

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{DB}{AD} + 1 = \frac{EC}{AE} + 1$$

$$\frac{DB + AD}{AB} = \frac{EC + AE}{AC}$$

$$\frac{AD}{AB} = \frac{AE}{AC} \quad \text{———} \quad (2)$$



$A'E \parallel AC$ ($\angle A'$ & $\angle A$ are corresponding angles)

By BPT,

$$\cancel{\frac{AE}{AC}} \quad \frac{A'B}{AB} = \frac{BE}{BC}$$

$$\frac{AD}{AB} = \frac{DE}{BC} \quad (\because BA' = AD \text{ \& } BE = DE) \text{ ——— } (2)$$

A From (1) & (2)

$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

$\Delta ABC \sim \Delta ADE$

Hence proved

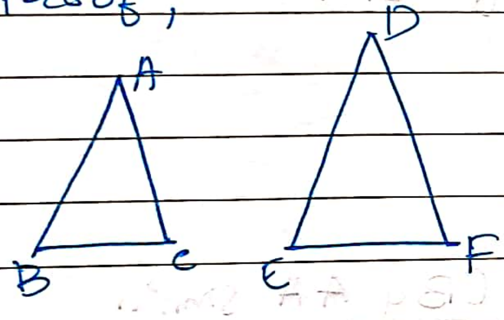
AA Similarity

If two angles of one triangle are equal to the corresponding two angles of another triangle, then the two triangles are similar.

SSS Similarity

In two Δ's, if sides of 1 Δ are proportional to the sides of other Δ, then their corresponding angles are equal and hence the two triangles are similar.

Proof,



The sides are in same ratio, i.e.

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

To prove

$\angle A = \angle D$

$\angle B = \angle E$

$\angle C = \angle F$

Cons

Mark P on DE such $DP = AB$

Mark Q on DF such $DQ = AC$

Join PQ

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$$\frac{AB}{DE} = \frac{AC}{DF}$$

$$\frac{DP}{DE} = \frac{DQ}{DF}$$

$$\frac{DP+PE}{DF} = \frac{DQ+QF}{DQ}$$

$$\frac{DP}{PE} = \frac{DQ}{QF}$$

$PQ \parallel EF$ (converse of BPT)

$\angle P = \angle E$

$\angle Q = \angle F$

$\therefore \triangle DPQ \sim \triangle DEF$ (By AA simil.

Hence, sides are proportional

$$\frac{DP}{DE} = \frac{DQ}{DF} = \frac{PQ}{EF} \quad \text{—— (1)}$$

$AB = DP$ & $AC = DQ$ (By cons

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} \quad \text{(Given) —— (2)}$$

From (1) & (2)

$BC = PQ$

$\therefore \Delta ABC \cong \Delta DPQ$ (By SSS criterion)

$\therefore \angle A = \angle D$

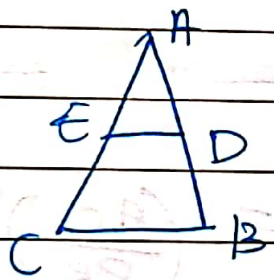
$\angle B = \angle P = \angle E$

$\angle C = \angle Q = \angle F$

$\therefore \Delta ABC \sim \Delta DEF$

SAS criterion

If one angle of a triangle is equal to one angle of other triangle & the sides including these angles are proportional, then the two triangles will be similar.



$$\frac{AD}{AB} = \frac{AE}{AC}$$

Proof,

Given,

$$\frac{AD}{AB} = \frac{AE}{AC}$$

$\angle A = \angle A$

To prove $\Delta ABC \sim \Delta ADE$

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Proof,

DE || BC (converse of BPT)

$\angle D = \angle B$ & $\angle E = \angle C$

$\Delta ABC \sim \Delta ADE$ (By AA similarity)

Hence proved

The ratio of the area of two similar triangles is equal to the square of the ratios of their corresponding sides.

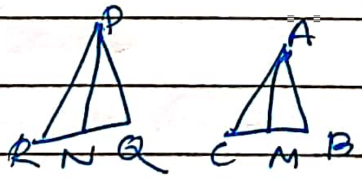
Proof,

Given :- $\Delta ABC \sim \Delta PQR$

To prove

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

Constn $\rightarrow$ $AM \perp BC$ & $PN \perp QR$



Proof

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN}$$

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC \times AM}{QR \times PN} \text{ ——— } ①$$

In ΔABM & ΔPQN

$\angle B = \angle Q$ ($\Delta ABC \sim \Delta PQR$)

$\angle M = \angle N$ (Right angle

$\therefore \Delta ABM \sim \Delta PQN$ (By AA s——)

BPT

$$\frac{AM}{PN} = \frac{AB}{PQ} \text{ ——— } ②$$

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \text{ ——— } ③$$

Replace ② & ③ in 1

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB \times AB}{PQ \times PQ} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

Hence proved

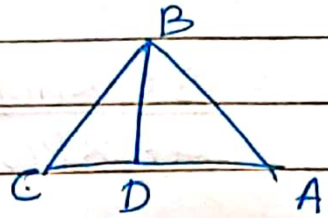
~~Dst~~

In a right angle triangle, if a perpendicular is drawn from the vertex of the right angle to the hypotenuse, then the triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

IMP

Given $\angle B = 90°$

$BD \perp AC$



To prove,

$\Delta ADB \sim \Delta ABC$

$\Delta BDC \sim \Delta ABC$

$\Delta ADB \sim \Delta BDC$

Proof

In ΔADB & ΔABC,

$\angle A = \angle A$ (Common)

$\angle ADB = \angle ABC$ (Right angle

$\therefore \Delta ADB \sim \Delta ABC$ (AA sim)

In ΔBDC & ΔABC,

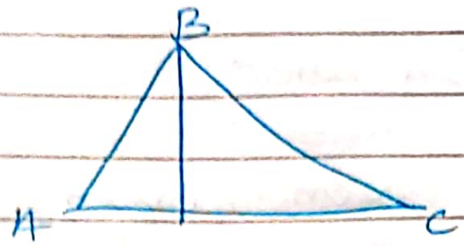
$\angle C = \angle C$ (Common)

$\angle BDC = \angle ABC$ (Right angle)

$\therefore \Delta BDC \sim \Delta ABC$ (AA sim

$\Rightarrow \Delta ADB \sim \Delta BDC$ (Inner Δ are similar)

Pythagoras Theorem



Given.

$\angle B = 90^\circ$

T.P $AB^2 + BC^2 = AC^2$

Cons Draw $BD \perp AC$

Proof,

$\Delta ADB \sim \Delta ABC$

$$\frac{AD}{AB} = \frac{AB}{AC}$$

$$AD \times AC = AB^2 \quad \text{—— (1)}$$

$\Delta BDC \sim \Delta ABC$

$$\frac{CD}{BC} = \frac{BC}{AC} \quad \text{—— (2)}$$

$$CD \times AC = BC^2 \quad \text{—— (2)}$$

from (1) & (2)

Add,

$$AB^2 + BC^2 = AC(AD + CD)$$

$$AB^2 + BC^2 = AC^2$$

Hence proved

In ΔABC, if $\angle B = 90°$ & $BD \perp AC$, the $BD^2 = AD \times DC$

Given $BD \perp AC$

~~To prove~~ $\angle ABC = 90°$

To prove $BD^2 = AD \times DC$

Proof

$\Delta ADB \sim \Delta ABC$

$\Delta BDC \sim \Delta ABC$

Hence, $\Delta ADB \sim \Delta BDC$

$\Rightarrow \dfrac{AD}{BD} = \dfrac{BD}{DC}$

$AD \times DC = BD^2$

Hence proved

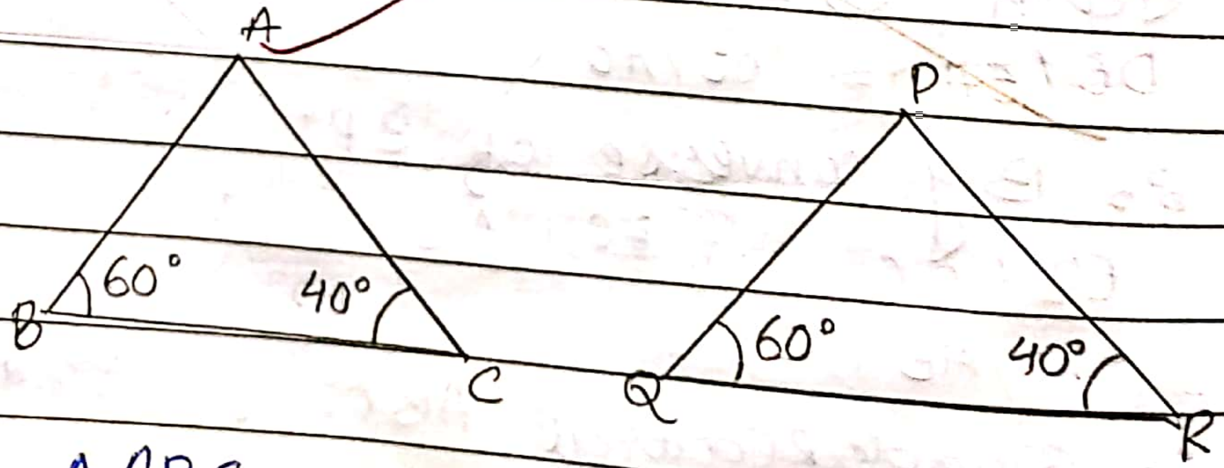
CRITERIA FOR SIMILARITY OF TRIANGLES

If two triangles are similar then, there corresponding sides are in proportion and corresponding angles are equal.

There are three criterias for similarity of triangles :-

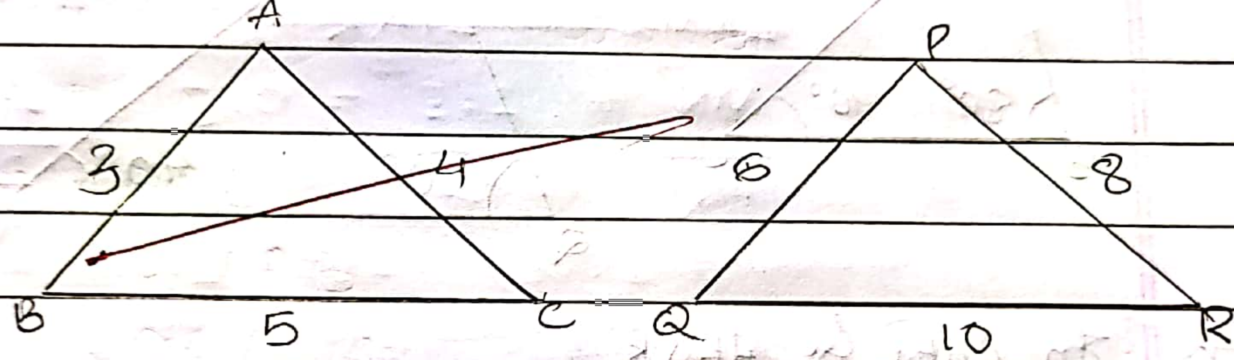
① AA criteria
② SSS criteria
③ SAS criteria

THEOREM 6.3 : If in two triangles, corresponding angles are equal, then their corresponding sides are in same ratio (or proportion) and hence the two triangles are similar.



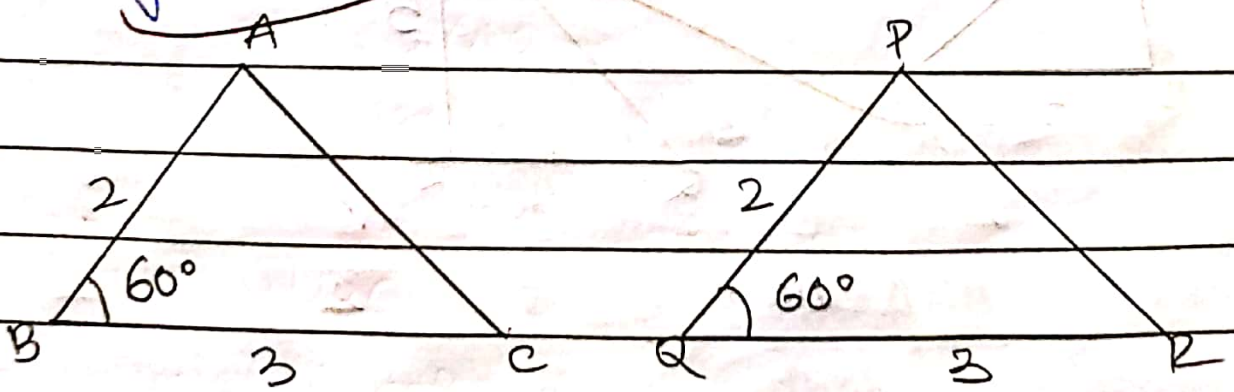
$\Delta ABC \sim \Delta PQR$

THEOREM 6.4 :- If in two triangles, sides of one triangle are proportional to (i.e. in same ratio) the sides of other triangle, then their corresponding angles are equal and hence the two triangles are similar.



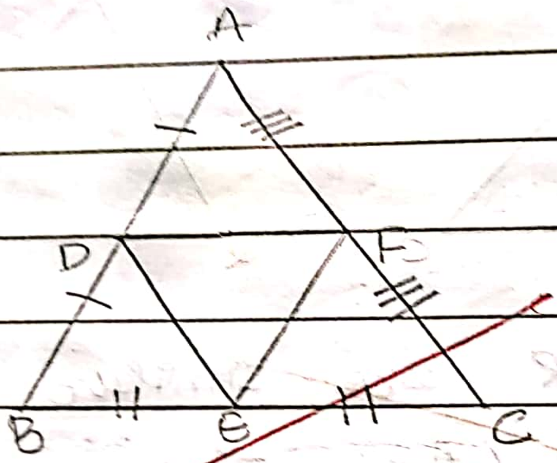
$\Delta ABC \sim \Delta PQR$

THEOREM 6.5 :- If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.



~~Thus, $\Delta ABC \cong \Delta PQR$~~
~~By SSS criterion~~

D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of areas of ΔDEF and ΔABC.



~~D, E and F are mid points of AB, BC and AC of ΔABC.~~

By Mid-point theorem,

$DF = 1/2 \ BC$, $DE = 1/2 \ AC$, $EF = 1/2 \ AB$

$\frac{DF}{BC} = \frac{1}{2}$, $\frac{DE}{AC} = \frac{1}{2}$, $\frac{EF}{AB} = \frac{1}{2}$

$\therefore \frac{DF}{BC} = \frac{DE}{AC} = \frac{EF}{AB}$ ——— (1)

In ΔDFE & ΔABC,

$\frac{DF}{BC} = \frac{DE}{AC} = \frac{EF}{AB}$ ——— from (2)

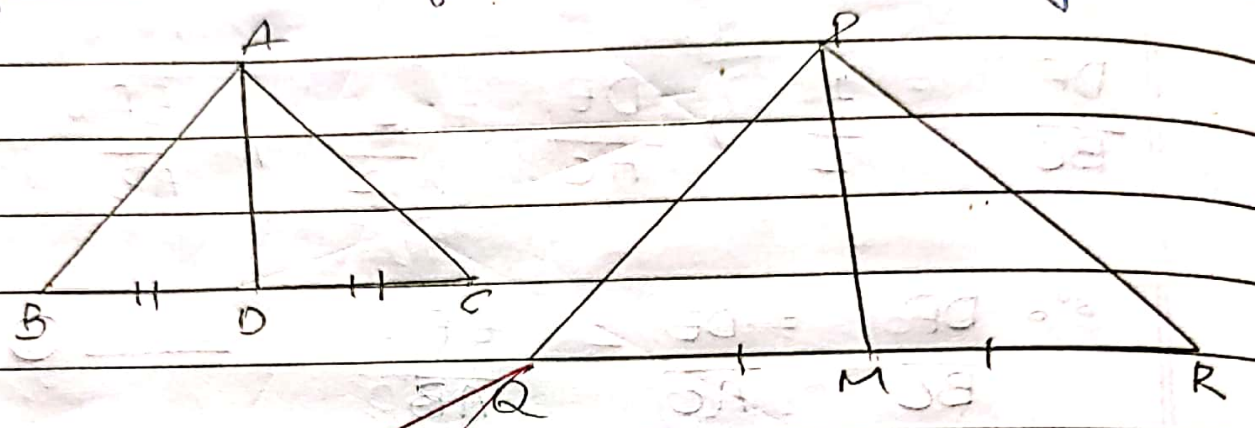
$\therefore \Delta DFE \sim \Delta ABC$

$\therefore \frac{ar(DFE)}{ar(ABC)} = \frac{DF^2}{BC^2} = \frac{(1)^2}{(2)^2} = \frac{1}{4}$

$\therefore ar(DFE) : ar(ABC)$

$1 : 4$

6) Prove that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding medians.



Given :- $\Delta ABC \sim \Delta PQR$

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \quad \text{(1)}$$

$$\angle A = \angle P, \ \angle B = \angle Q, \ \angle C = \angle R$$

$$\frac{AB}{PQ} = \frac{\cancel{2}BD}{\cancel{2}QM}$$

$$\frac{AB}{PQ} = \frac{BD}{QM} \ \text{——} \ (2)$$

In ΔABD & ΔPQM,

$$\frac{AB}{PQ} = \frac{BD}{QM} \quad \text{— from (2)}$$

$$\angle B = \angle Q \quad \text{— from (1)}$$

$\Rightarrow \Delta ABD \sim \Delta PQM$ By SAS criteria

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM} \quad \text{— (3)}$$

$$\frac{ar(ABC)}{ar(PQR)} = \frac{AB^2}{PQ^2} = \left(\frac{AB}{PQ}\right)^2$$

$$= \left(\frac{AD}{PM}\right)^2 \quad \text{— from (3)}$$

$$= \frac{AD^2}{PM^2}$$

Hence proved.

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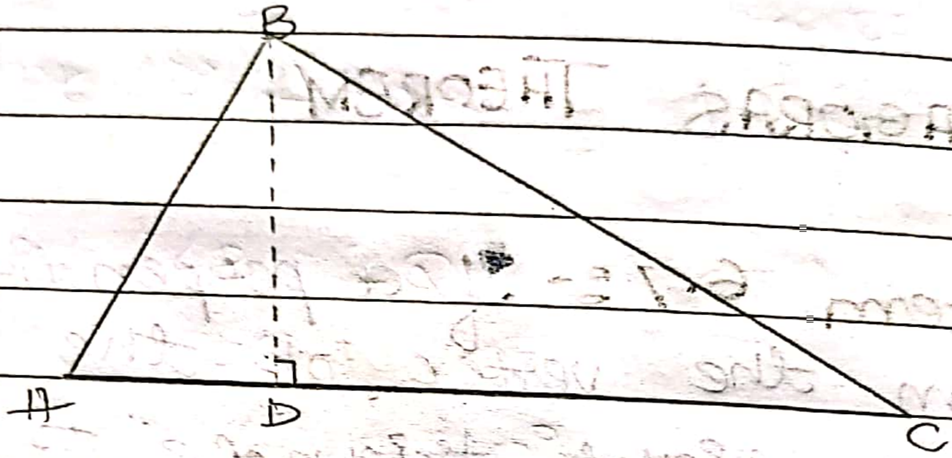
THEOREM 6.8:-

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



Given, $\angle B = 90^\circ$

To prove $AC^2 = AB^2 + BC^2$

Draw $BD \perp AC$

$\Delta ADB \sim \Delta ABC$ $(BD \perp AC)$

So,

$$\frac{AD}{AB} = \frac{AB}{AC}$$

$$AD \times AC = AB^2 \quad \text{—— (1)}$$

$\Delta BDC \sim \Delta ABC$ $(BD \perp AC)$

$$\frac{CD}{BC} = \frac{BC}{AC}$$

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$CD \times AC = BC^2$ —— ②

Adding ① & ②

$AD \times AC + CD \times AC = AB^2 + BC^2$

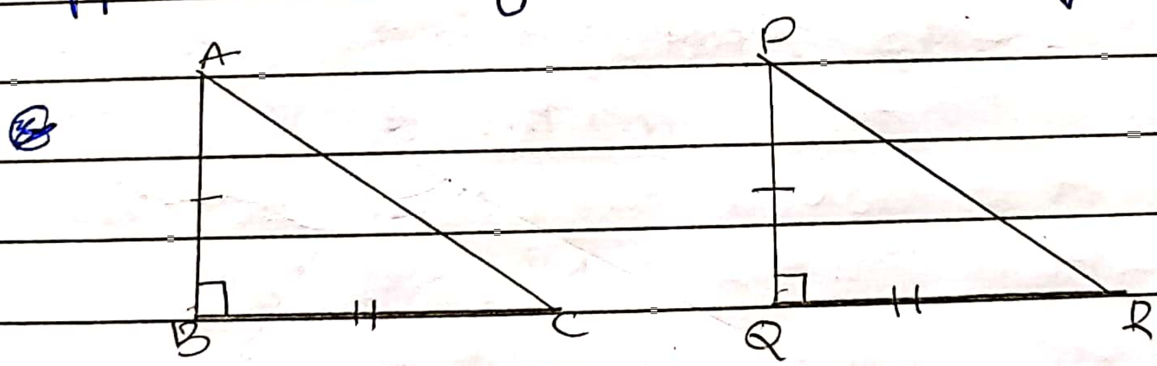
$AC\,(AD + CD) = AB^2 + BC^2$

$AC \times AC = AB^2 + BC^2$

$AC^2 = AB^2 + BC^2$

THEOREM 6.9 :-

In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.



Given :- $AC^2 = AB^2 + BC^2$

To prove :- $\angle B = 90°$

By Pythagoras theorem,

In ΔPQR

$PR^2 = PQ^2 + QR^2$

$RP^2 = AB^2 + BC^2$

$AC^2 = AB^2 + BC^2$ (Given) —— (2)

So, $AC = PR$ —— (3)

In $\triangle ABC$ & $\triangle PQR$

$AB = PQ$ (construction)

$BC = RQ$ (construction)

$AC = PR$ (Proved above (3))

$\therefore \triangle ABC \cong \triangle PQR$ by SSS criteria

$\angle B = \angle Q$ (CPCT)

$\angle Q = 90^\circ$

$\angle B = 90^\circ$

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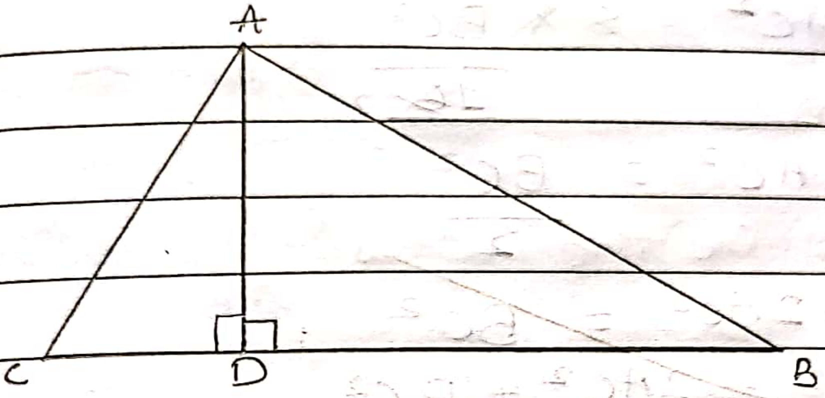
14) The perpendicular from A on side BC of a ΔABC intersects BC at D such that $DB = 3CD$. Prove that $2AB^2 = 2AC^2 + BC^2$.



→ $DB = 3CD$

$BC = CD + DB$

$BC = CD + 3CD$ (given)

$BC = 4CD$

In ΔADB & ΔADC,

By Pythagoras theorem,

$AB^2 = AD^2 + DB^2$ —— (1)

$AC^2 = AD^2 + CD^2$ = (2)

Subtract

$AB^2 - AC^2 = DB^2 - CD^2$

$AB^2 - AC^2 = (3CD)^2 - CD^2$ (given)

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$$AB^2 - AC^2 = 9CD - CD^2$$

$$AB^2 - AC^2 = 8CD^2$$

$$AB^2 - AC^2 = 8 \times \left(\frac{BC}{4}\right)^2$$

$$AB^2 - AC^2 = \cancel{8} \times \frac{BC^2}{\cancel{16}\,2}$$

$$AB^2 - AC^2 = \frac{BC^2}{2}$$

$$2AB^2 - 2AC^2 = BC^2$$

$$2AB^2 = 2AC^2 + BC^2$$