POLYNOMIALS

Polynomials :- The algebraic expression in which the degree of each term is whole number is called polynomial. It is denoted by p(x).

The general form of a polynomial in x is

$$p(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots + a_nx^n$$

where,

$a_0, a_1, a_2, a_3, \dots\dots a_n$ are real numbers.

- The highest of veriable in polynomial is called its degree.

Based on the degree of polynomial, it has 3 types

i) Linear Polynomial
ii) Quadratic Polynomial
iii) Cubic Polynomial

① Linear Polynomial :- If the degree of polynomial is 1, then it is called as linear polynomial.

The general form of linear polynomial is

$p(x) = ax + b$; $a \neq 0$ & a & b are real no.s

For e.g. $p(x) = x+3$, $p(x) = 4x-5$ etc.

(2) Quadratic Polynomial :- If the degree of polynomial is 2, then it is called as quadratic polynomial & its general form is

$$p(x) = ax^2 + bx + c \; ; \quad a \neq 0 \text{ \& } a, b, c \text{ are real no.s}$$

For e.g. $p(x) = x^2 + 5x + 6$,
$p(x) = x^2 - 4$ etc

(3) Cubic Polynomial :- If degree of polynomial is three then it is called as cubic polynomial & its general form is

$$p(x) = ax^3 + bx^2 + cx + d \; , \quad a \neq 0 \text{ \& } a, b, c, d \text{ are real nos}$$

For e.g. :- $p(x) = x^3 - x^2 + 2x + 3$
$p(x) = 4x^3 + 4x^2 + 8$ etc

* Zeros of Polynomial *

The values of variables for which the value of polynomial p(x) becomes zero is called zero of polynomial.

For a linear polynomial $p(x) = ax + b$, it has only 1 zero,

For a quadratic polynomial, it has 2 zeroes & for cubic polynomial it has 3 zeroes.

1. NOTE :- Graph of linear polynomial is a straight line & it intersect X - axis in one & only one point.

2. NOTE :- Graph of quadratic polynomial is curve shape i.e. parabola

If $a > 0$ then parabola opens upwards & $a < 0$ then parabola opens downwards.

Relation between zeros & coefficents of Polynomial

① If alpha (α) is the zero of linear polynomial

$$p(x) = ax + b$$

then, $\alpha = \dfrac{-b}{a}$

② If alpha (α) & beta (β) are the zeros of quadratic polynomial

$$p(x) = ax^2 + bx + c$$

$$\underline{\alpha + \beta = -b/a \quad \& \quad \alpha\beta = c/a}$$

③ If alpha (α), beta (β) & gamma (γ) are zeros of cubic polynomial,

$$p(x) = ax^3 + bx^2 + cx^{\cancel{2}} + d$$

$\alpha + \beta + \gamma = -b/a$,

$\alpha\beta + \beta\gamma + \gamma\alpha = c/a$,

$\alpha\beta\gamma = -d/a$

NOTE :- If alpha & beta are the zeros of quadratic polynomial p(x) ~~then~~ p(x) is given by

$p(x) = k[x^2 - (\alpha + \beta)x + \alpha\beta]$

* Division Algorithm for Polynomials

If $p(x)$ & $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that,

$$p(x) = g(x) \times q(x) + r(x),$$

where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$

This result is known as the **Division Algorithm for polynomials.**

(i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

$$
\begin{array}{r|l}
 & x - 3 \\
x^2 - 2 & \cancel{x^3} - 3x^2 + 5x - 3 \\
 & \underline{\ominus x^3 \pm 0x^2 \oplus 2x} \\
 & -\cancel{3x^2} + 7x - 3 \\
 & \underline{\overset{-}{\oplus} 3x^2 \pm 0x \overset{+}{\ominus} 6} \\
 & 7x - 9
\end{array}
$$

$\therefore$ Quotient $= x - 3$

Remainder $= 7x - 9$

(iii) $p(x) = x^4 - 5x + 6, \quad g(x) = 2 - x^2$

$\rightarrow$

$$
\begin{array}{r|l}
 & -x^2 - 2 \\
-x^2+2 & x^4 + 0x^3 + 0x^2 - 5x + 6 \\
 & \underline{\overset{(-)}{x^4} \overset{(-)}{+} 0x^3 \overset{(+)}{-} 2x^2} \\
 & \qquad 2x^2 - 5x + 6 \\
 & \qquad \underline{\overset{(+)}{2x^2} \overset{(-)}{+} 0x \overset{(+)}{-} 4} \\
 & \qquad\quad -5x + 10
\end{array}
$$

2] Check whether the first polynomial is a factor of second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2 - 3, \quad 2t^4 + 3t^3 - 2t^2 - 9t - 12$

$\rightarrow$

$$
\begin{array}{r|l}
 & 2t^2 + 3t + 4 \\
t^2-3 & 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\
 & \underline{\overset{(-)}{2t^4} \overset{(-)}{+} 0t^3 \overset{(+)}{-} 6t^2} \\
 & \qquad 3t^3 + 4t^2 - 9t - 12 \\
 & \qquad \underline{\overset{(-)}{3t^3} \overset{(-)}{+} 0t^2 \overset{(+)}{-} 9t} \\
 & \qquad\qquad 4t^2 - 12 \\
 & \qquad\qquad \underline{\overset{(-)}{4t^2} \overset{(+)}{-} 12} \\
 & \qquad\qquad\quad 0
\end{array}
$$

Yes ∴ Hence Proved

(ii) $\deg q(x) = \deg r(x)$

→ Let us assume the division of $x^6 + x^2$ by x^4.

Here $p(x) = x^6 + x^2$

$g(x) = x^4$

$q(x) = x^2$ & $r(x) = x^2$

By division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$\Rightarrow x^6 + x^2 = (x^4) \times x^2 + x^2$

$x^6 + x^2 = x^6 + x^2$

$\therefore \deg q(x) = \deg r(x)$

(iii) $\deg r(x) = 0$

→ Let us assume division of $x^3 + 1$ by x^2.

Here, $p(x) = x^3 + 1$

$g(x) = x^2$, $q(x) = x$ & $r(x) = 1$

Clearly, the degree of $r(x) = 0$

(as r $1x^0$)