ARITHMETIC PROGRESSION

Sequence :- An arrangement of numbers in ~~an~~ definite order or ~~in~~ specific pattern forms a sequence.

For e.g.

① 2, 4, 6, 8, 10, ...

② 3, 6, 12, 24, 48, ---------

③ 1, 4, 9, 16, 25, ---------

In general the n^{th} term of a sequence is denoted by a_n or t_n and sequence is denoted by $\langle a_n \rangle$ or $\{a_n\}$

Types of Sequences

There are three types of sequences

① Arithmetic Progression (A.P.)

② Geometric Progression (G.P.)

③ Harmonic Progression (H.P.)

(1) Arithmetic Progression :- An arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.

This fixed number is called common difference of the A.P. Remember that it can be positive, negative or zero.

Each no. which forms an A.P. is called term of an A.P.

Example :-

(1) 1, 2, 3, 4, 5, ---------
(2) 100, 70, 40, 10, ---------
(3) 3, 3, 3, 3, 3, ---------

NOTE :- The first term of an A.P. is denoted by 'a' & common difference is denoted by 'd'.

First term = a
Common difference = d

nth term

something which represents any term of a given arithmetic progression.

$$n^{th}\ term = a + (n-1)\,d$$

Second Term − First Term = Third Term − Second Term = 4^{th} Term − 3^{rd} Term
(a + d) (a) (a + 2d) (a + d) (a + 3d) (a + 2d)

So its an AP

Sum of n terms is

$S_n = n/2$ [first term + last term]
OR
$S_n = n/2 \{2a + (n-1) d\}$

) $a = -2$, $d = 0$

We know that,

$a = a_1 = -2$

$a_2 = -2 + 0 = -2$

$a_3 = -2 + 2(0) = -2$

$a_4 = -2 + 3(0) = -2$

$\therefore$ First four terms are $-2, -2, -2, -2$

$a = 4$, $d = -3$

We know that,

$a = a_1 = 4$

$a_2 = 4 - 3 = 1$

$a_3 = 4 + 2(-3) = -2$

$a_4 = 4 + 3(-3) = -5$

$\therefore$ First four terms are $4, 1, -2, -5$

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Which of the following AP's ? If they form an AP, find the common difference d and write three more terms.

2, 4, 8, 16

First term, a, = 2

common difference, d, $= a_2 - a_1$

$= 4 - 2$

$= 2$

$= a_3 - a_2$

$= 8 - 4$

$= 4$

∴ Common difference is not same or equal.

∴ It is not an AP

$-10, -6, -2, 2$

First term, $a = -10$

$d = -6 + 10$

$= 4$

$d = a_3 - a_2$

$= -2 + 6$

$= 4$

$\therefore$ It is an AP.

$\therefore$ Next three terms are,

$a_5 = a + 4d$

$= -10 + 4(4)$

$= -10 + 16 = 6$

$a_6 = a + 5b$

$= -10 + 5(4)$

$= -10 + 20 = 10$

$a_7 = a + 6d$

$= -10 + 6(4) = -10 + 24 = 14$

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(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, ----$

→ $a = \sqrt{3}$

$d = \sqrt{6} - \sqrt{3}$

$= \sqrt{3} \times \sqrt{2} - \sqrt{3}$

$= \cancel{\sqrt{3}(\sqrt{2}-1)}\ \sqrt{3}(\sqrt{2}-1)$

$= a_3 - a_2$

$= \sqrt{9} - \sqrt{6}$

$= \sqrt{3} \times \sqrt{3} - \sqrt{3} \times \sqrt{2}$

$= \cancel{3 - \sqrt{6}}\ \sqrt{3}(\sqrt{3}-\sqrt{2})$

∴ It is not an AP.

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The amount of air present in a cylinder when a vaccum pump removes 1/4 of the air remaining in cylinder at a time.

Let amount of air present in a cylinder at time t be x.

After removing 1/4 of the air from cylinder, it will remain

$$x - \frac{1}{4}x = \frac{3x}{4}$$

After removing 1/4 of ~~air~~ $3x/4$ from cylinder, it remains

$$\frac{3x}{4} - \frac{3x}{16} = \frac{9x}{16}$$

Similarly,

After removing 1/4 of 9x/16 of air from cylinder, it remains

$$\frac{9x}{16} - \frac{9x}{64} = \frac{27x}{64}$$

∴ It is not an AP as its common difference is not similar.

The amount of money in the account every year, when ₹ 10,000 is deposited at compound interest at 8% per annum.

Amount for 1^{st} year is,

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$A = \overset{10000}{P}\left(1 + \frac{8}{100}\right)^1$$

$$A = 1000\cancel{0}\left(\frac{108}{10\cancel{0}}\right)$$

$$A = 10800$$

So, CI $= 10800 - 10000$

$= ₹ 800$

Amount for 2nd year,

$$A = P\left(1 + \frac{8}{100}\right)^2$$

$$A = 10000 \times \frac{108}{100} \times \frac{108}{100}$$

$$A = 11664$$

$$CI = 11664 - 10000$$

$$= 21664$$

Amount for 3rd year,

$$A = 10000\left(1 + \frac{8}{100}\right)^3$$

$$A = 10000 \times \frac{108}{100} \times \frac{108}{100} \times \frac{108}{100}$$

$$A = \frac{108 \times 108 \times 108}{100}$$

$$A = \frac{108 \times 161664}{100}$$

$$A = \frac{12597}{100}$$

$$A = 125.97$$

$\therefore$ It is not an AP

n^{th} TERM OF AN AP

If 'a' is the first term & 'd' is common difference of an AP then the n^{th} term of an AP is given by

$$a_n = a + (n-1)d$$

where,

n = no. of terms

n^{th} term of an AP is also denoted by 'l'.

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(i) $a = 7$, $d = 3$, $n = 8$, $a_n = ?$

$\rightarrow$ $a_n = a + (n-1)d$

$a_8 = 7 + (8-1)(3$

$a_8 = 7 + 21 = 28$

(iii) $a = -18$, $d = ?$, $n = 10$, $a_n = 0$

$\rightarrow$ $a_n = a + (n-1)d$

$0 = -18 + 9d$

$18 = 9d$

$d = 2$

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(ii) □, 13, □, 3

→ $a = ?$, $a_2 = 13$, $a_3 = ?$, $a_4 = 3$

$a_2 = a + d = 13$ —— (1)

$a + 3d = 3$ —— (2)

Subtract (1) & (2)

$$\cancel{a} + d = 13$$
$$-\cancel{a} + 3d = 3$$
$$-2d = 10$$
$$d = -5$$

From eq. (1)

$a - 5 = 13$

$a = 18$

So $a_3 = a + 2d$

$= 18 + 2(-5)$

$= 18 - 10 = 8$

So AP will be 18, 13, 8, 3.

Check whether -150 is a term in each of the following APs : ~~18~~ 11, 8, 5, 2 ....

$a = 11$

$d = -3$

$a_n = -150$

$a_n = a + (n-1)d$

$-150 = 11 + (n-1)(-3)$

$\frac{+161}{+3} = n-1$

$\frac{161}{3} + 1 = n$

$n = \frac{164}{3}$

$\therefore$ It is not a term involve in AP as n^{th} term cannot be in fraction.

Find the 31st term of an AP whose 11th term is 38 and 16th term is 73.

$a_{11} = 38$

$a_{16} = 73$

$a + 10d = 38$ —— (1)

$d + 15d = 73$ —— (2)

Subtract (1) & (2)

$$\cancel{a} + 10d = 38$$
$$-\cancel{a} \underset{(-)}{+} 15d \underset{(-)}{=} 73$$
$$-5d = -35$$
$$d = 7$$

$a + 70 = 38$

$a = 38 - 70$

$a = -32$

So $a_{31} = a + 30d$

$= -32 + 30(7)$

$= -32 + 210$

$= 178$

Ramkali saved ₹5 in the first week of a year and then increase her weekly saving by ₹1.75. If in the n^{th} week, her weekly savings become ₹20.75, find n.

According to given condition,

$a = 5$, $d = 1.75$, $n = ?$, $a_n = 20.75$

$a_n = a + (n-1)d$

$20.75 = 5 + (n-1)1.75$

$15.75 = (n-1)1.75$

$n-1 = 9$

$n = 10$

So $n = 10$

Sum of first n Terms of an AP

If 'a' & 'd' are first term & common difference of an AP then the sum of first n terms of an AP is denoted by S_n & is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

If 'a' is the first term & 'l' is the last term of an AP then sum of first n terms is given by

$$S_n = \frac{n}{2}(a + l)$$

1) Find the sum of following AP's :

(i) 2, 7, 12, ..., to 10 terms

→ Here, $a = 2$, $d = 5$, $n = 10$

we know,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{10} = 5[4 + 45]$$

$$S_{10} = 5 \times 49$$

$$S_{10} = 245$$

∴ Sum of ten terms is 245.

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$\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots$, to 11 terms

$a = \frac{1}{15}$, $d = \frac{1}{60}$, $n = 11$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{11} = \frac{11}{2}[2 \times 1/15 + (11-1) 1/60]$

$S_{11} = \frac{11}{2}[2/15 + 1/6]$

$S_{11} = \frac{11}{2}[9/30]$

$S_{11} = \frac{11}{2} \times \frac{9}{30}$

$S_{11} = \frac{99}{60} = \frac{33}{20}$

$\therefore$ Sum of 11 terms is $\frac{33}{20}$

(i) $7 + \cancel{10}\ 10\frac{1}{2} + 14, \text{----}, + 84$

$\rightarrow$ $a = 7$, $d = \frac{7}{2}$, $a_n = 84$

$a_n = a + (n-1)d$

$84 = 7 + (n-1)\frac{7}{2}$

$77 = (n-1)\frac{7}{2}$

$\frac{\overset{11}{\cancel{77}} \times 2}{\cancel{7}} = (n-1)$

$22 = n-1$

$n = 23$

(iv) given $a_3 = 15$, $S_{10} = 125$, find d & a_{10}

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{10} = \frac{10}{2}[2(15-2d) + 9d]$$

$$125 = 5[30 - 4d + 9d]$$

$$25 = 30 + 5d$$

$$-5 = 5d$$

$$d = -1$$

$$a = 15 - 2d$$

$$a = 15 - (2)(-1)$$

$$a = 15 + 2$$

$$a = 17$$

$$a_n = a + (n-1)d$$

$$a_{10} = 17 + (10-1)(-1)$$

$$a_{10} = 17 + (9)(-1)$$

$$a_{10} = 17 - 9$$

$$a_{10} = 8$$

(vi) given $a = 2$, $d = 8$, $S_n = 90$, find n and a_n

→ $S_n = \frac{n}{2}[2a + (n-1)d]$

$90 = \frac{n}{2}[4 + (n-1)8]$

$90 = \frac{n}{2}[4 + 8n - 8]$

$180 = n[4 + 8n - 8]$

$180 = 4n + 8n^2 - 8n$

$180 = 8n^2 - 4n$

$\cancel{8n^2 - 4n - 180 = 0}$

$4(2n^2 - n) = 180$

$2n^2 - n = 45$

$2n^2 - n - 45 = 0$

$2n^2 + 9n - 10n - 45 = 0$

$n(2n+9) - 5(2n+9) = 0$

$(2n+9)(n-5) = 0$

$2n + 9 = 0$ & $n - 5 = 0$

$n = -9/2$ & $n = 5$

so n cannot be in fraction

so $n = 5$

$a_n = a + (n-1)d$

$a_5 = 2 + (5-1)8$

$a_5 = 2 + (4)8$

$a_5 = 2 + 32$

$a_5 = 34$

Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

$a_2 = 14$

$a_3 = 18$

$d = a_3 - a_2 = 18 - 14 = 4$

$a_2 = a + d$

$14 = a + 4$

$a = 10$

$S_n = \frac{n}{2} [2a + (n-1)d]$

$S_{51} = \frac{51}{2} [2 \times 10 + (51-1) \times 4]$

$= \frac{51}{2} [2 + (20) \times 4]$

$= \frac{51 \times 220}{2}$

$= 51 \times 110$

$= 5610$

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If the sum of first n terms of an AP is $4n - n^2$, what is the first term (i.e. S_1)? What is the sum of first two terms? What is the second term? Similarly, find the third, the 10th and the n^{th} terms?

Given

$S_n = 4n - n^2$

$a = S_1 = 4(1) - (1)^2 = 4 - 1 = 3$

DATE :

$[a_n = S_n - S_{n-1}]$

sum of first two terms $= S_2$

$= 4(2) - (2)^2 = 8 - 4 = 4$

& $a_2 = S_2 - S_1 = 4 - 3 = 1$

$d = a_2 - a_1 = 1 - 3 = -2$

$a_n = a + (n-1)d$

$= 3 + (n-1)(-2)$

$= 3 - 2n + 2$

$= 5 - 2n$

$\therefore a_3 = 5 - 2 \times 3 = 5 - 6 = -1$

$a_{10} = 5 - 2 \times 10 = 5 - 20 = -15$

Hence, sum of first two terms is 4.
Second term is 1. 3rd, 10th, and nth terms are $-1, -15$ & $5 - 2n$ respectively.

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