Thursday

Date 23/5/19
Page

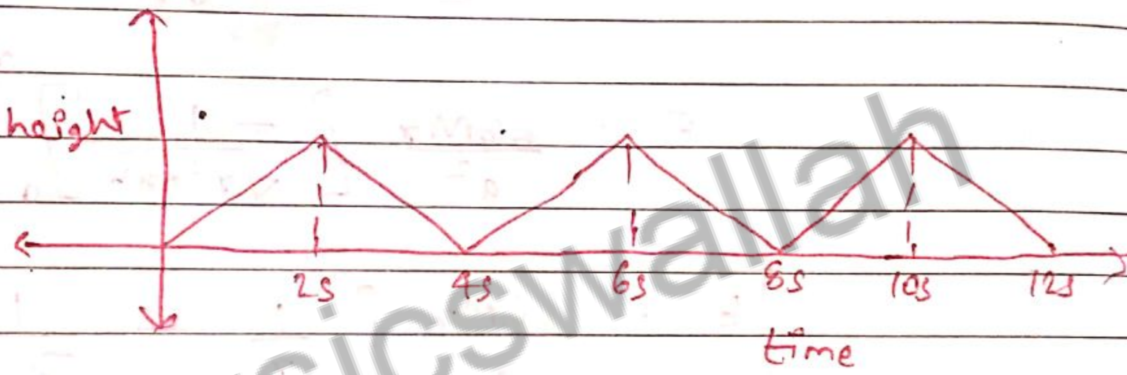
S. H. M

SHM is not a wave

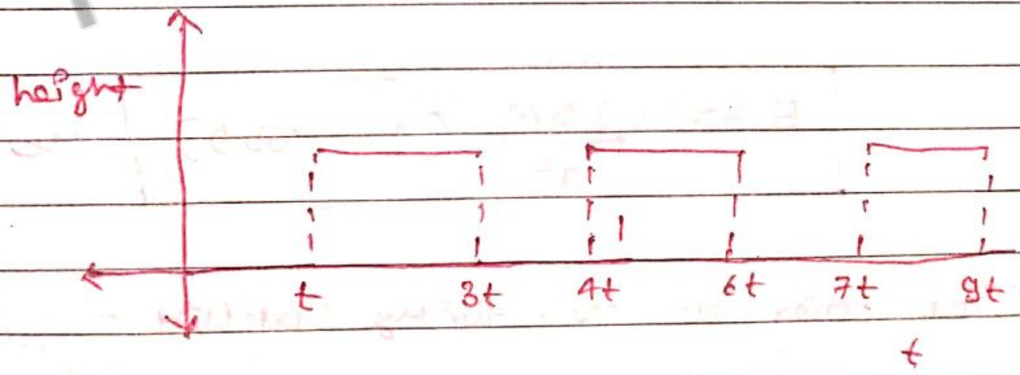
• Periodic Motion : (Fixed) Regular Interval
Repeat

$\boxed{\text{Harmonic}}$

(1) insect climbing a wall & falling down



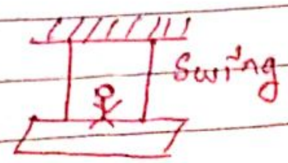
(2) Bacha → Steps → Climbs up & down



(3) U.C.M

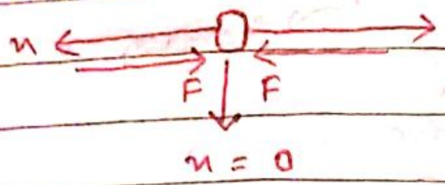
• Oscillatory : to & fro. periodic

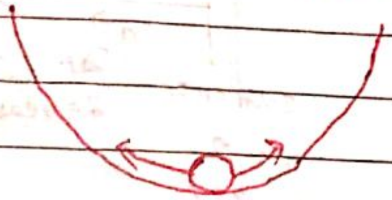
• All oscillatory motion are periodic (99%).



- Oscillatory ⟶ equation : $F = -kx^n$ $\{n = 1, 3, 5, 7\}$

x ↓ measured from mean position





Mean position (Stable Equilibrium position)

Force ↓ Mean position

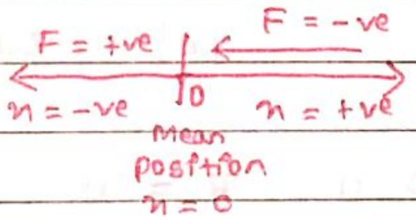
Pe जाना चाहती है।

"Yess, All oscillatory motions are periodic ~~tab~~ except in which Energy is lost."

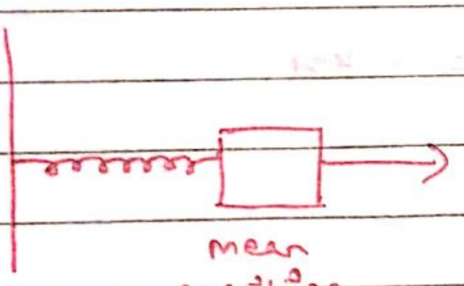
- SHM ⟶ Special case / simplest case of Oscillation

↓ St. line

$$\boxed{F = -kx^1}$$



$x \rightarrow$ measured from mean position



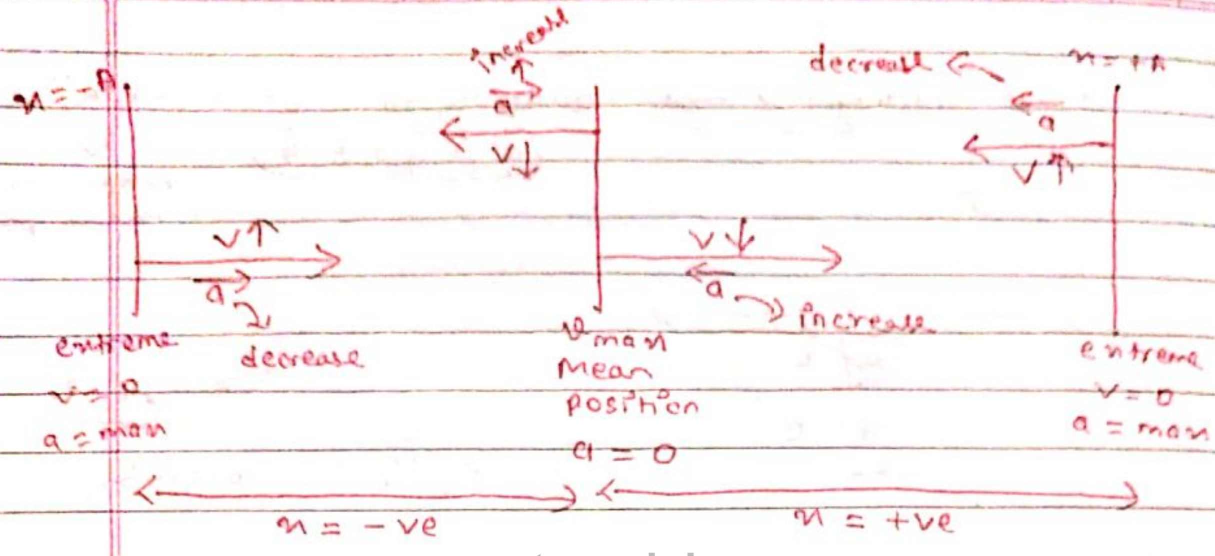
⟶ $a_{max} = \omega A$

extreme $x = \pm A$

⟶ $v = 0$ extreme

$V = \pm \omega\sqrt{A^2 - x^2}$ ⟶ $V_{max} = \pm A\omega$

mean



$$F = -kx$$

$$a = \frac{F}{m} = -\frac{kx}{m}$$

$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

$$\boxed{a = -\omega^2 x}$$

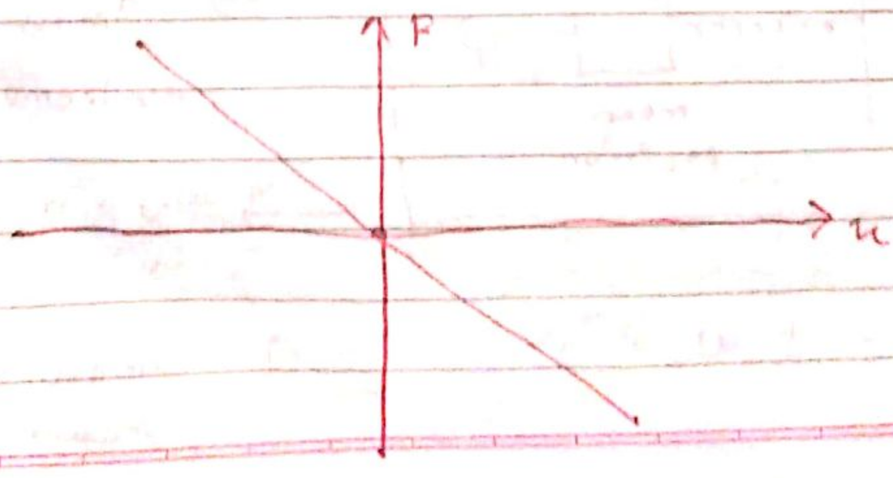
$a \rightarrow$ max, when $x = +A$ or $x = -A$

extreme position

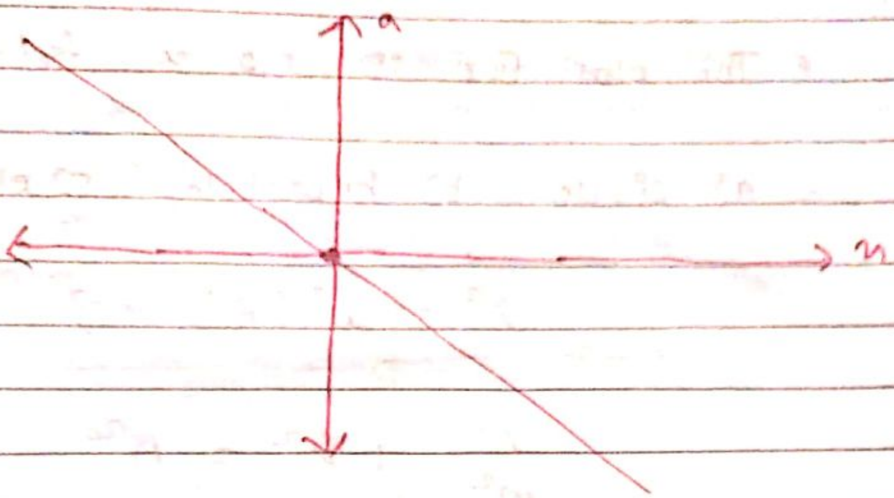
$$|a| = \omega^2 A$$

$a \rightarrow 0$, $x = 0$ (mean position)

- F v/s x $\quad F = -kx$



• a v/s x $\qquad a = -\omega^2 x$



• v v/s x

$$a = -\omega^2 x$$

$$\frac{dv}{dt} \times \frac{dx}{dx} = -\omega^2 x$$

$$\frac{v\,dv}{dx} = -\omega^2 x$$

$$\int_0^v v\,dv = -\int_{x=+A}^{x} \omega^2 x\,dx$$

$$\left[\frac{v^2}{2}\right]_0^v = -\omega^2\left[\frac{x^2}{2}\right]_A^x$$

$$\frac{v^2}{2} = -\frac{\omega^2}{2}(x^2 - A^2)$$

$$\boxed{v^2 = \omega^2(A^2 - x^2)}$$

- The plot for v v/s x is: (in SHM)

a) circle b) hyperbola c) ellipse d) parabola

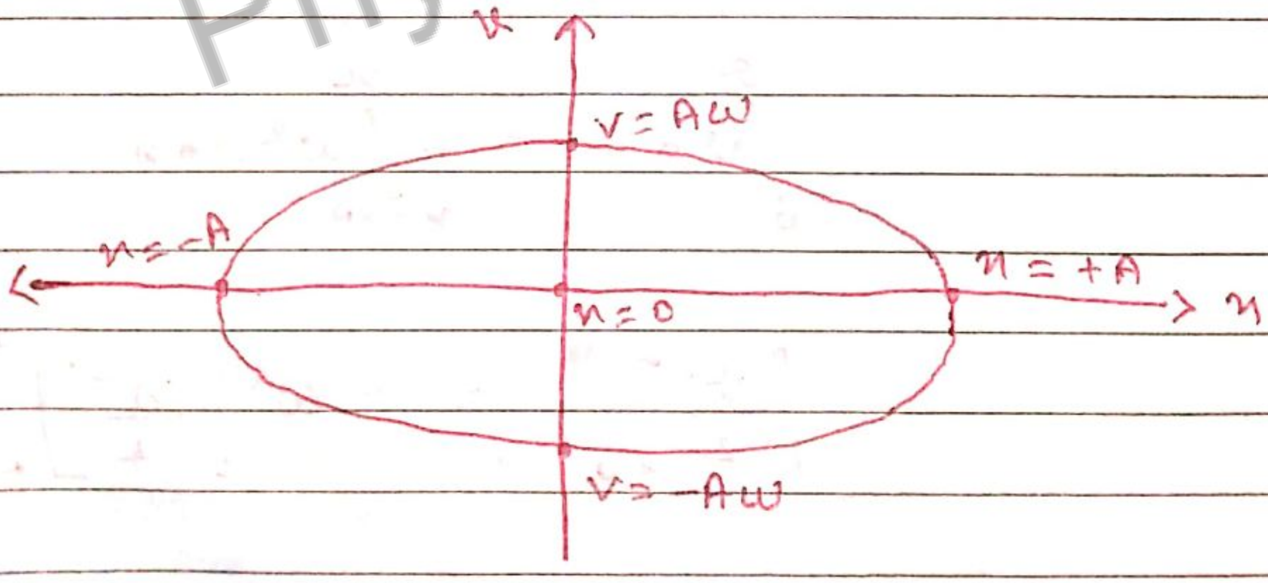
$$v^2 = \omega^2(A^2 - x^2)$$

$$\frac{v^2}{\omega^2} + x^2 = A^2$$

$$\boxed{\frac{v^2}{\omega^2 A^2} + \frac{x^2}{A^2} = 1}$$

Ellipse $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\right)$

$$\frac{x^2}{A^2} + \frac{v^2}{(\omega A)^2} = 1$$



$$\boxed{V_{max} = \pm A\omega}$$

Mean position

$$v = \pm \omega \sqrt{A^2 - x^2}$$

$$\frac{dx}{dt} = \pm \omega \sqrt{A^2 - x^2}$$

$$\int_0^x \frac{dx}{\sqrt{A^2 - x^2}} = \int_0^t \pm \omega \, dt$$

at $t = 0$, initially particle is at mean position $x = 0$

let $x = A\sin\theta$

$$\sqrt{A^2 - x^2} = \sqrt{A^2(1 - \sin^2\theta)} = A\cos\theta$$

$$dx = A\cos\theta \, d\theta$$

$$\int_0^x \frac{A\cos\theta \, d\theta}{A\cos\theta} = \int_0^t \pm \omega \, dt$$

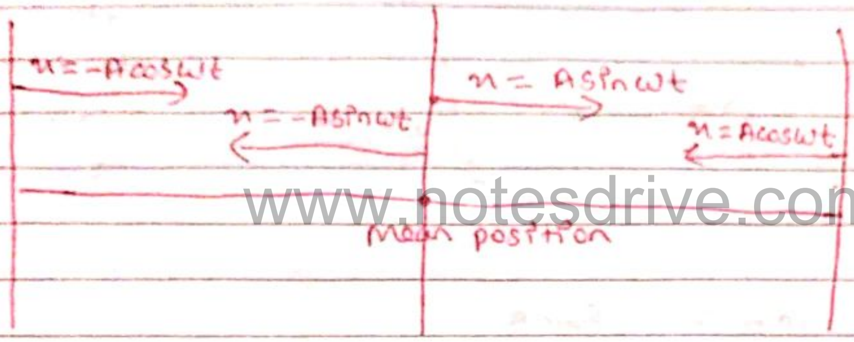
$$\left[\sin^{-1}\left(\frac{x}{A}\right)\right]_0^x = \pm \omega (t)_0^t$$

$$\sin^{-1}\frac{x}{A} = \pm \omega t$$

$$\frac{x}{A} = \sin(\pm \omega t)$$

$$\boxed{x = A\sin(\pm \omega t)}$$

$$\boxed{\begin{aligned} x &= A\sin\omega t \\ x &= -A\sin\omega t \end{aligned}}$$ ~~or x = A sin t~~



- At $t = 0$ particle is at $x = +A$ (extreme)

$$\int_{+A}^{x} \frac{dx}{\sqrt{A^2 - x^2}} = \int_0^t \pm\omega\, dt$$

$$\left[\sin^{-1}\left(\frac{x}{A}\right)\right]_A^x = \pm\omega(t)_0^t$$

$$\sin^{-1}\left(\frac{x}{A}\right) - \sin^{-1}\left(\frac{A}{A}\right) = \pm\omega t$$

$$\sin^{-1}\left(\frac{x}{A}\right) - \frac{\pi}{2} = \pm\omega t$$

$$\sin^{-1}\left(\frac{x}{A}\right) = \frac{\pi}{2} \pm\omega t$$

$\cancel{\sin^{-1}}(-1) = \sin\frac{3\pi}{2}$

Date
Page

$$x = A\sin\left(\frac{\pi}{2} \pm \omega t\right)$$

$$\boxed{x = A\cos\omega t}$$

- At $t = 0$ particle is at $x = -A$

$$\left[\sin^{-1}\left(\frac{x}{A}\right)\right]_{-A}^{x} = \pm\omega t$$

$$\sin^{-1}\left(\frac{x}{A}\right) - \sin^{-1}\left(-\frac{A}{A}\right) = \pm\omega t$$

$$\sin^{-1}\left(\frac{x}{A}\right) - \frac{3\pi}{2} = \pm\omega t$$

$$\sin^{-1}\left(\frac{x}{A}\right) = \frac{3\pi}{2} \pm \omega t$$

$$x = A\sin\left(\frac{3\pi}{2} \pm \omega t\right)$$

$$\boxed{x = -A\cos\omega t}$$

General eqn :-

$$\boxed{\begin{aligned} x &= A\sin(\omega t + \phi) \\ x &= A\cos(\omega t + \phi) \end{aligned}}$$

$x = A\sin(\omega t + \phi)$

$x = A\cos(\omega t + \phi)$

$\phi \rightarrow$ initial phase, $(\omega t + \phi) \rightarrow$ phase

- $x = A\sin(\omega t + \phi)$

$$\frac{dx}{dt} = A\omega\cos(\omega t + \phi)$$

$$\boxed{v = A\omega\cos(\omega t + \phi)}$$

$$a = \frac{dv}{dt} = A\omega^2(-\sin(\omega t + \phi))$$

$$\boxed{a = -A\omega^2\sin(\omega t + \phi)}$$

$\boxed{a = -\omega^2 x}$ differential equation of SHM

$$\frac{dv}{dt} = -\omega^2 x$$

$$\frac{d\left(\frac{dx}{dt}\right)}{dt} = -\omega^2 x$$

$$\boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0}$$

Date
Page

$$\boxed{\begin{aligned} v_{max} &= \pm A\omega \\ a_{max} &= \pm A\omega^2 \end{aligned}}$$

- Time period of 'SHM'

$$x_1 = x_2$$

$$A\sin(\omega t + \phi) = A\sin(\omega(t+T) + \phi)$$

$$\sin\theta = \sin\alpha$$

$$\theta = \alpha, \quad \theta = 2\pi + \alpha$$

$$\theta = \pi - \alpha$$

$$v = A\omega\cos(\omega t + \phi)$$

$$v_1 = v_2$$

$$\cancel{A\omega}\cos(\omega t + \phi) = \cancel{A\omega}\cos(\omega(t+T) + \phi)$$

$$\cos\theta = \cos\alpha,$$

$$\theta = \alpha, \quad \theta = 2\pi + \alpha$$

$$\theta = 2\pi - \alpha$$

$$\cancel{\omega t} + \cancel{\phi} + 2\pi = \cancel{\omega t} + \omega T + \cancel{\phi}$$

$$\omega T = 2\pi$$

$$\boxed{T = \frac{2\pi}{\omega}}$$

Q) Write the eq^n for SHM ($x - t$) if initially particle is at origin & moving in −ve direction w.r.t mean position.

(Mean position = origin) ($A = 2cm$, $T = 4s$)

$\Rightarrow$ $T = \frac{2\pi}{\omega}$

$\omega = \frac{\cancel{2}\pi}{\cancel{4}\,2} = \frac{\pi}{2}$

$x = -2\sin\left(\frac{\pi}{2}t\right)$

$$\boxed{x = -2\sin\left(\frac{\pi}{2}t\right)}$$

OR

$x = A\sin(\omega t + \phi)$ $\quad$ $v = A\omega\cos(\omega t + \phi)$

At $t = 0$
$x = 0$

$0 = A\sin(0 + \phi)$ $\quad$ $v < 0$
$t = 0$

$\sin\phi = 0$ $\quad$ $A\omega\cos(0 + \phi) < 0$

$\phi = 0$ or π $\quad$ $\cos\phi < 0$

$\cos 0 = 1$ $\quad$ $\cos 180° = -1$

$$\boxed{\phi = \pi}$$

$x = 2\sin(\omega t + \pi)$

$$\boxed{x = -2\sin\left(\frac{\pi}{2}t\right)}$$

$\boxed{0 \le \phi \le 2\pi}$

Date ______
Page ______

Q) Write the eqn for SHM $(x - t)$, $(v - t)$ & find 'v' at $t = 2sec$, $3sec$ if initially particle is at Right extreme of origin & moving in $-ve$ x-direction.
$(A = 2cm, T = 8sec)$

$\Rightarrow$

$x = A\sin(\omega t + \phi)$ $\qquad v = A\omega\cos(\omega t + \phi)$

At $t = 0$, $x = A$

$\cancel{x} = \cancel{A}\sin(0 + \phi)$ $\qquad v = A\omega\cos\left(\frac{\pi}{2} + \omega t\right)$

$\sin\phi = 1$

$\phi = \frac{\pi}{2}$ $\qquad v = -A\omega\sin(\omega t)$

$= -\frac{\cancel{2} \times \pi}{\cancel{4}\,2}\sin\left(\frac{\pi}{4}t\right)$

$x = A\sin\left(\frac{\pi}{2} + \omega t\right)$ $\qquad = -\frac{\pi}{2}\sin\left(\frac{\pi}{4}t\right)$

$= A\cos\omega t$

$T = \frac{2\pi}{\omega}$ $\qquad$ At $t = 2sec$,

$v = -\frac{\pi}{2}\sin\left(\frac{\pi}{2}\right)$

$\cancel{4}\ 8 = \frac{2\pi}{\omega}$ $\qquad \boxed{v = -\frac{\pi}{2}}$

$\omega = \frac{\pi}{4}$

At $t = 3sec$,

$v = -\frac{\pi}{2}\sin\left(\frac{3\pi}{4}\right)$

$x = 2 \times \cos\left(\frac{\pi}{4}t\right)$

$\boxed{v = -\frac{\pi}{2\sqrt{2}}}$ ..

$\boxed{x = 2\cos\left(\frac{\pi}{4}t\right)}$

At $t = 4sec$, $\boxed{v = 0}$ ..

$t_1 < t_2$

Q.3) Find $v-t$ for SHM, if initially particle is at $+\frac{A}{\sqrt{2}}$ of origin & moving initially in +ve direction.

$\Rightarrow$ $x = A\sin(\omega t + \phi)$ $\quad v = A\omega\cos(\omega t + \phi)$

At $t=0$

$x = \frac{A}{\sqrt{2}}$ $\quad \frac{A}{\sqrt{2}} = A\sin\phi$ $\quad A\omega\cos(\omega t + \phi) > 0$

$\cos(\phi) > 0$

$\sin\phi = \frac{1}{\sqrt{2}}$ $\quad \phi = \frac{\pi}{4}$

$\phi = \frac{\pi}{4}$

$$\boxed{v = A\omega\cos\left(\omega t + \frac{\pi}{4}\right)}$$

IIT 2001 :

If initially particle is at origin (SHM) & it ~~moves 'a' distance in~~ covers $0 \longrightarrow \frac{A}{2}$ in t_1 time & $\frac{A}{2} \longrightarrow A$ in t_2 time.

a) $t_2 = t_1$ $\quad$ b) $t_2 = t_1/2$

c) $t_2 = 2t_1$ $\quad$ d) $t_2 = t_1 + 2$

$\Rightarrow$ $\quad x = A\sin\omega t$

$\frac{A}{2} = A\sin\omega t_1$

$\sin \omega t_1 = \frac{1}{2}$

$\omega t_1 = \frac{\pi}{6}$

$\boxed{t_1 = \frac{\pi}{6\omega}}$

$\boxed{t_2 = \frac{\pi}{3\omega}}$

$x = A$

$t = (t_1 + t_2)$

$A = A \sin \omega (t_1 + t_2)$

$\omega (t_1 + t_2) = \frac{\pi}{2}$

$\omega t_2 = \frac{\pi}{2} - \frac{\pi}{6}$

$\omega t_2 = \frac{\pi}{3}$

JEE Mains 2014:

SHM $\longrightarrow$ A particle starts from rest & covers 'a' distance in 'τ' sec & another '2a' distance in next 'τ' sec in the same direction. Which of the following is correct?

a) $A = 3a$ b) $A = 4a$

c) $T = 6\tau$ d) $T = 8\tau$

$\Rightarrow$

$x = A\cos\omega t$

At $T = \tau$

$x = a$

$a = A\cos\omega\tau$

At $t = 2\tau$

$x = 3a$

$3a = A\cos 2\omega\tau$

$3 \times \cancel{A}\cos\omega\tau = \cancel{A}\cos 2\omega\tau$

$3\cos\omega\tau = \cos 2\omega\tau$

$a = \cancel{A\sin\omega t}\ A\cos\omega(t + t_0)$

$t = 0$

$x = 0$

extreme

$\Rightarrow$ At $t = T_1$

$x = A - a$

$A - a = A\cos\omega t$

Date
Page

At, $t = 2T$

$x = A - 3a$

$A - 3a = A\cos\omega 2T$

$\left[\cos\omega T = \frac{A-a}{A}\right]$

$= A(2\cos^2\omega t - 1)$

$A - 3a = A\left(2\left(\frac{A-a}{A}\right)^2 - 1\right)$

$A - 3a = A\left[\frac{(A^2 + a^2 - 2Aa)2}{A^2} - 1\right]$

$= A\left[\frac{2A^2 + 2a^2 - 4Aa - A^2}{A^2}\right]$

$\cancel{A^2} - 3aA = \cancel{A^2} + 2a^2 - 4Aa$

$Aa = 2a^2$

$\boxed{A = 2a}$

$A - a = A\cos\omega t$

$\cancel{a} = 2\cancel{a}\cos\omega t$

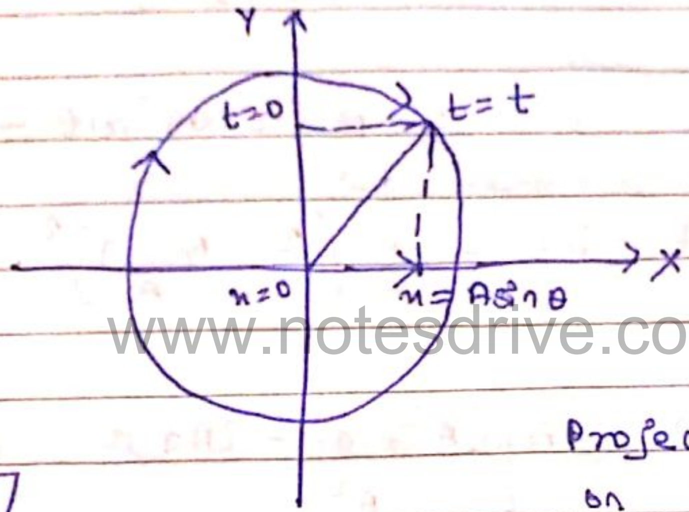
$\cos\omega t = \frac{1}{2}$

$\omega t = \frac{\pi}{3}$

$\frac{2\pi}{T} \times t = \frac{\pi}{3}$

$$\boxed{T = 6\tau}$$

- SHM & Circular Motion :



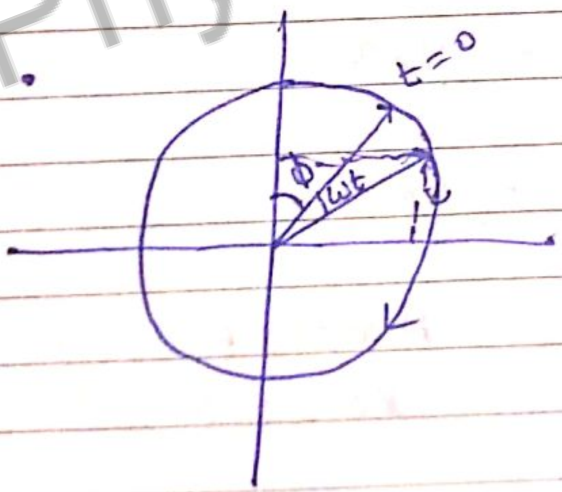
Projection is doing to & fro motion

Projection of particle on diameter

$$\boxed{\theta = \omega t}$$

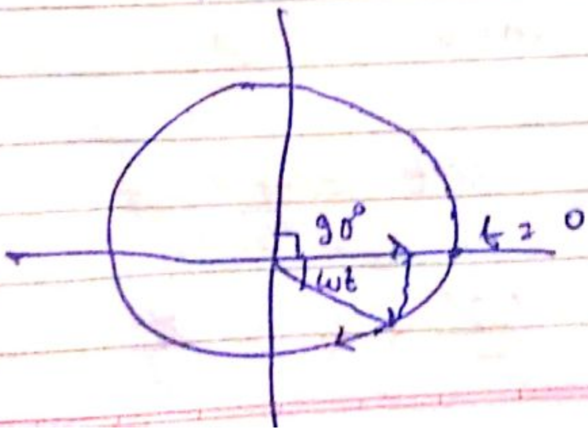
$$x = A\sin\theta$$

$$\boxed{x = A\sin\omega t}$$ ⟹ Yes SHM



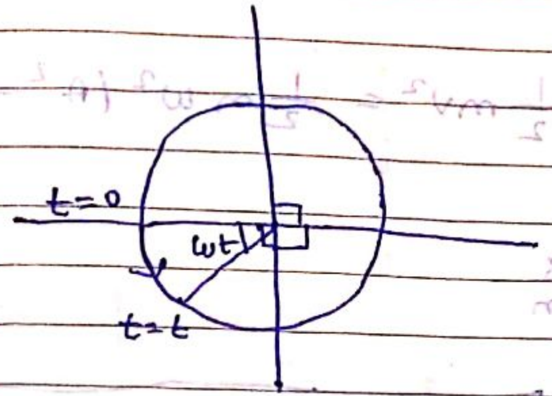
$$\boxed{x = A\sin(\omega t + \phi)}$$

→ Equation for SHM



$$\boxed{x = A\cos\omega t}$$

Scanned by CamScanner



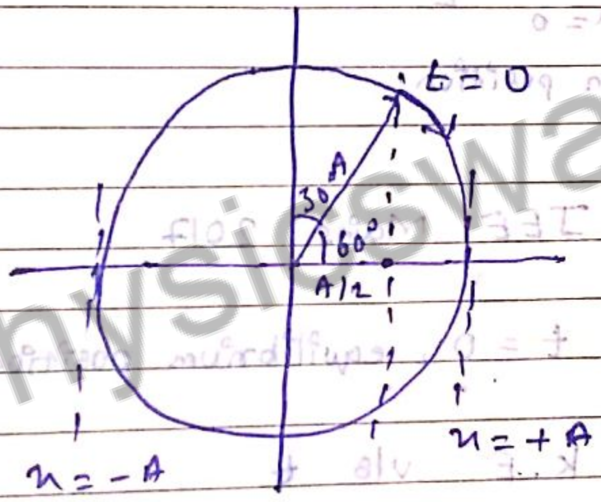
$$x = A\sin(\omega t + \phi)$$

$$x = A\sin(270 - \omega t)$$

$$\boxed{x = -A\cos\omega t}$$

Q) SHM : $t = 0$ initially $x = +\frac{A}{2}$ & initially it is moving in +ve x direction. find equation for x.

⟹



$$x = A\sin(\omega t + \phi)$$

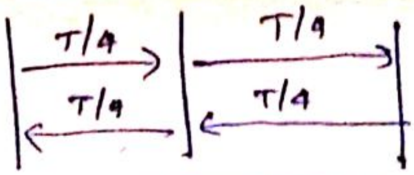
$$\boxed{x = A\sin\left(\omega t + \frac{\pi}{6}\right)}$$

SHM

$F = -Kx$

⤷ conservative

Kinetic Energy (Variable) + Potential Energy (Variable) — Total Mechanical Energy ⇓ Constant



- $K.E = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$

$\omega^2 = \frac{k}{m}$

$$\therefore \boxed{K.E = \frac{1}{2}k(A^2 - x^2)}$$

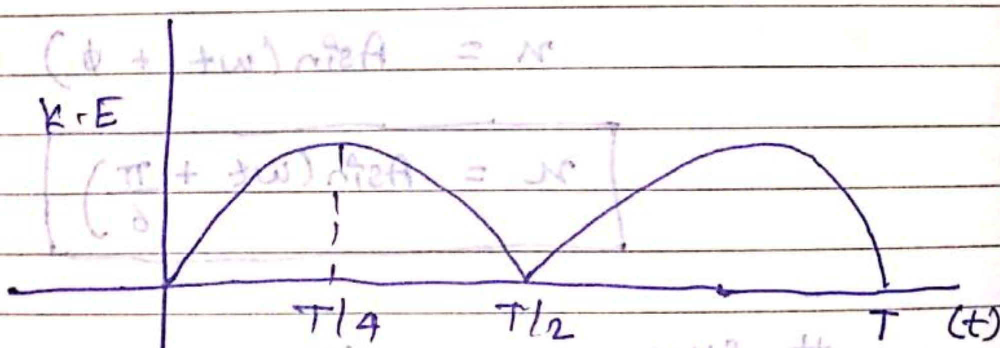
$Max = \frac{1}{2}kA^2$ | $Min = 0$
$x = 0$ | $x = \pm A$
Mean position | extreme position

JEE Mains 2017

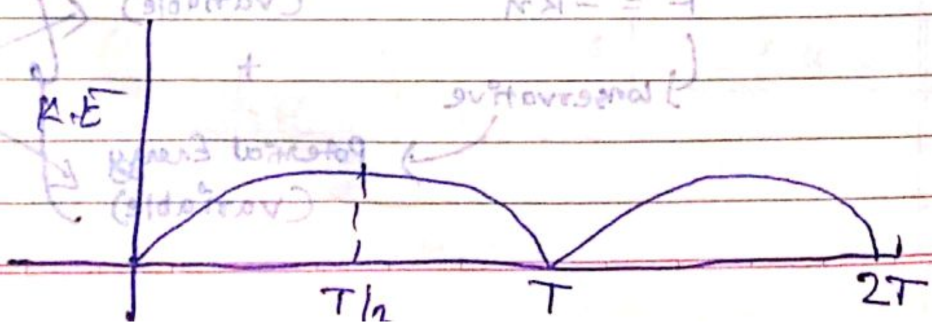
$t = 0$, equilibrium position (Mean position)

K.E v/s t

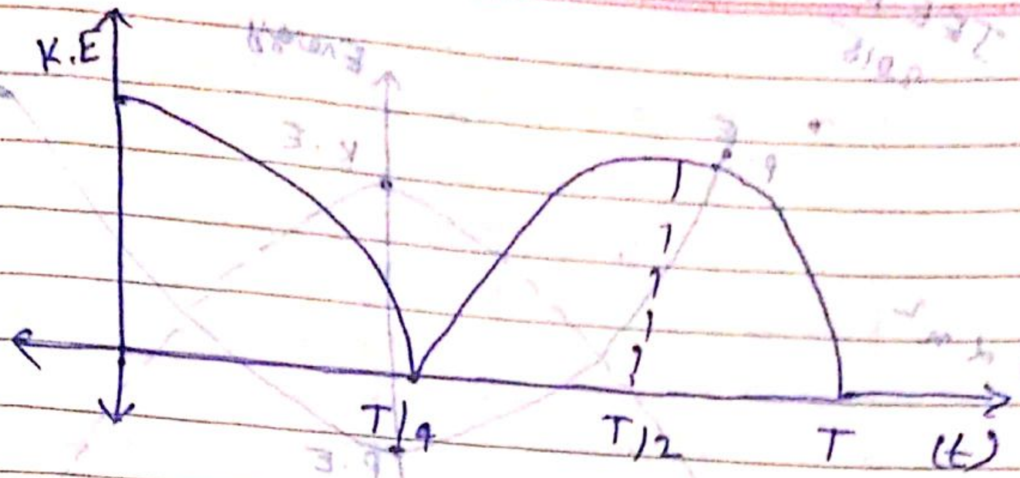
a) ✗



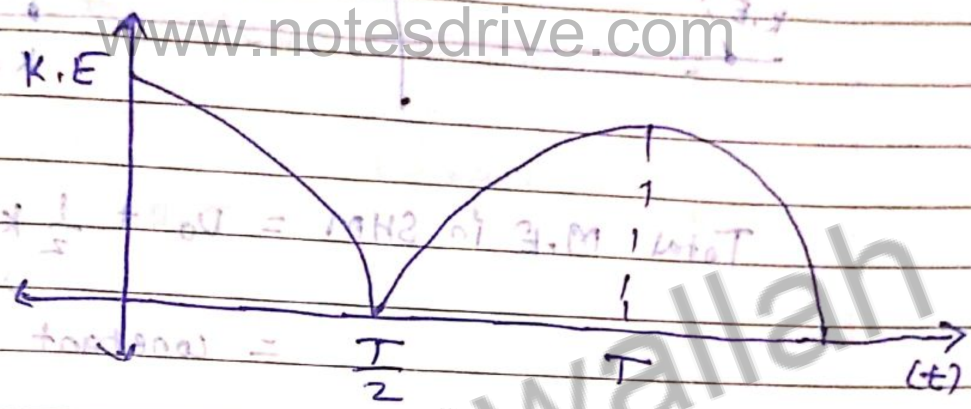
b) ✗



c)



d)



• P.E $\qquad \Delta U = -W_{\text{conservative force}}$

$F = -kx$

conservative force

$$\int dU = -\int dW_c$$

mean position ⇓ U_0

$$\int_{U_0}^{U} dU = + \int_{x=0}^{x=x} kx\,dx\cos 180^\circ$$

$$\boxed{U - U_0 = \frac{1}{2}kx^2}$$

$$\boxed{U = U_0 + \frac{1}{2}kx^2}$$

Min, $x = 0$, mean position $\qquad U_{min} = U_0$

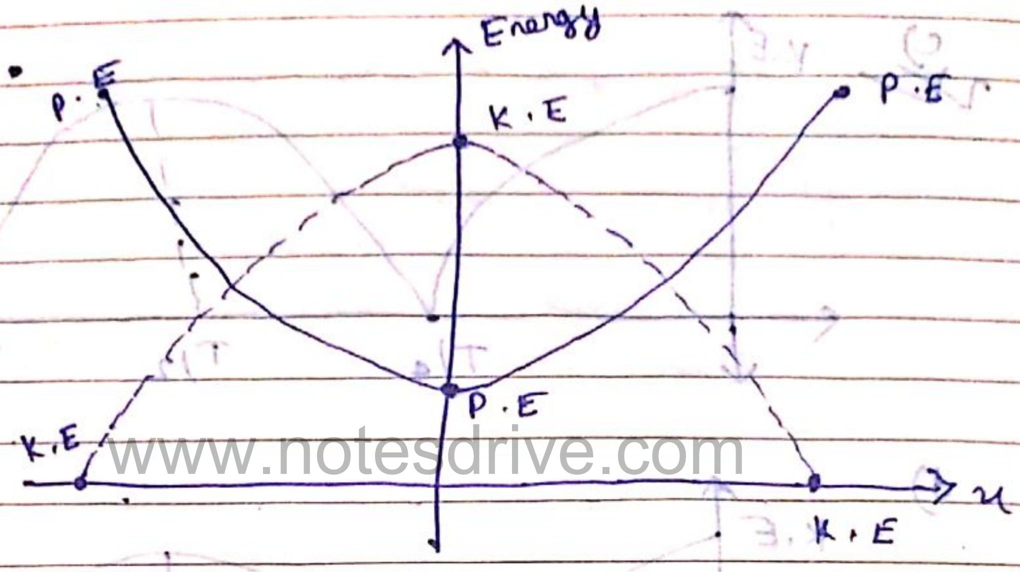
Max, $x = \pm A$, entreme $\qquad U_{max} = U_0 + \frac{1}{2}kA^2$

$F = -kx$ $\qquad -\frac{k}{m} = \omega^2$

$a = \frac{-kx}{m}$

JEE Mains 2015

$U \propto x^2$



Total M.E in SHM $= U_0 + \frac{1}{2}kA^2$

$= \text{constant}$

Q) Assuming $U = 0$ at mean position Find Total M.E of particle undergoing SHM of mass 2 kg,

$$x = 2\sin\left(\frac{\pi}{4}t + \frac{\pi}{4}\right)$$

$A = 2$

$\omega = \frac{\pi}{4}$

$m = 2\,kg$

$k = \omega^2 m$

$= \frac{\pi^2}{16} \times 2$

$= \frac{\pi^2}{8}$

$$T.E = \frac{1}{2}kA^2$$

$$= \frac{1}{2} \times \frac{\pi^2}{8^2} \times 4$$

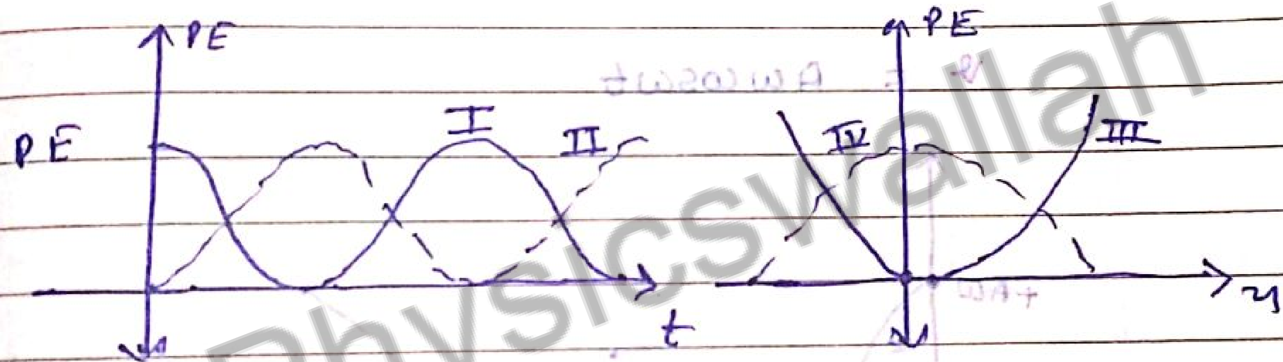
$$= \frac{\pi^2}{4}$$

IIT 2003:

SHM, $x = A\cos\omega t$

P.E v/s t

P.E v/s x



a) I & IV b) II & III c) I & III d) II & IV

• $PE = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\sin^2(\omega t + \phi)$

• $K.E = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2\cos^2(\omega t + \phi)$

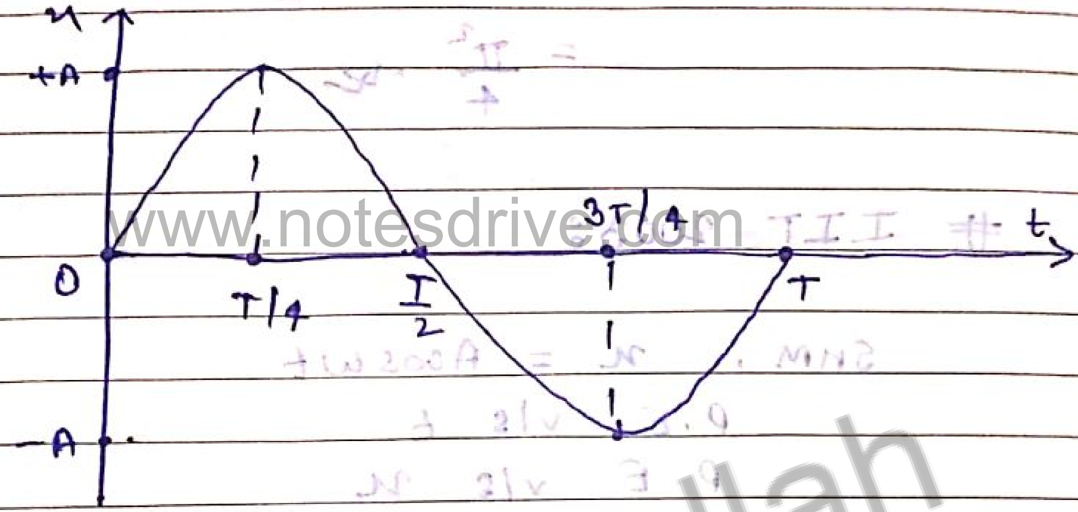
$$= \frac{1}{2}kA^2\cos^2(\omega t + \phi)$$

• $TE = \frac{1}{2}kA^2$

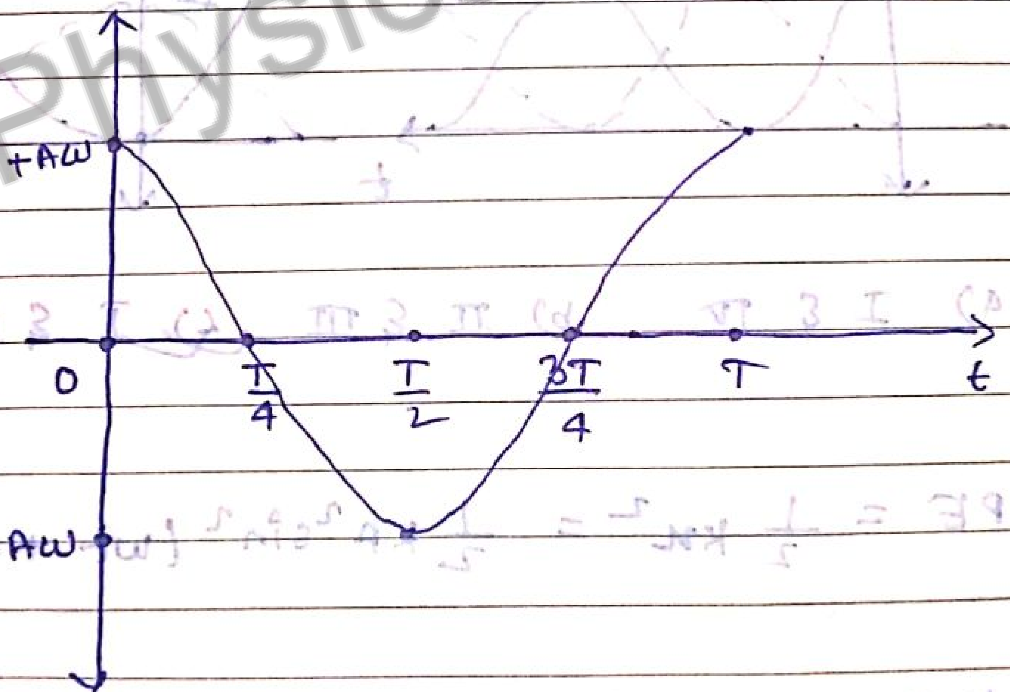
Date
Page

- $x = A\sin\omega t$
$x = A\sin\frac{2\pi}{T} \times t$

$t = 0$
$x = 0$
+ve direction motion

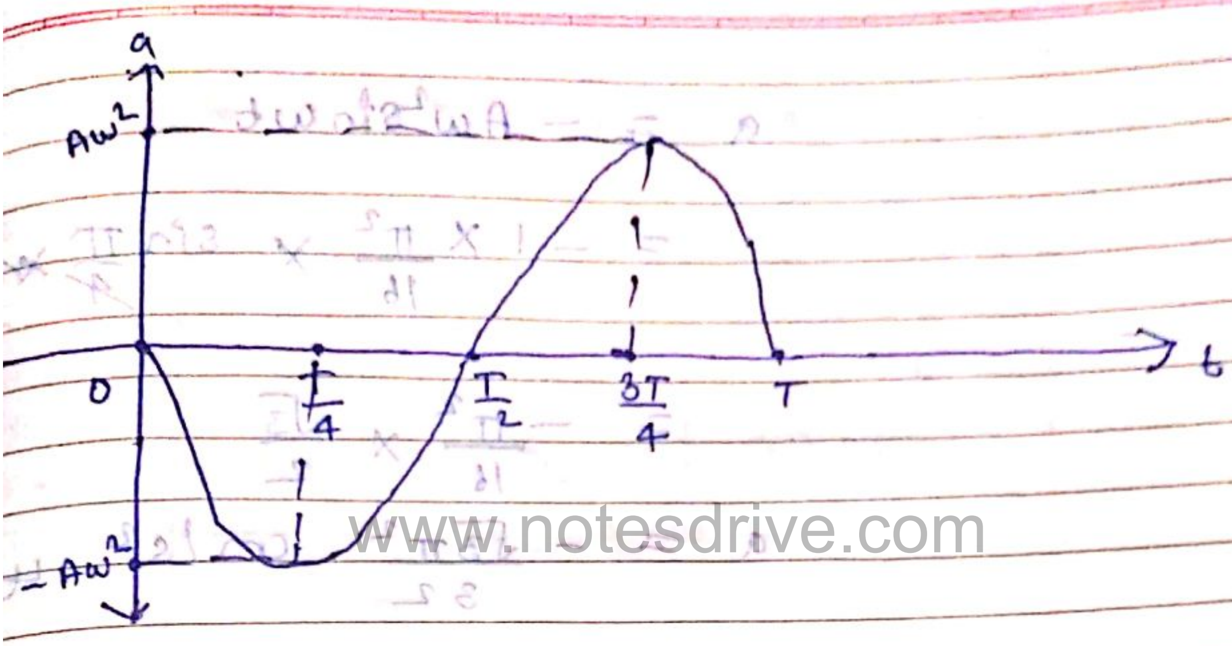


$v = A\omega\cos\omega t$

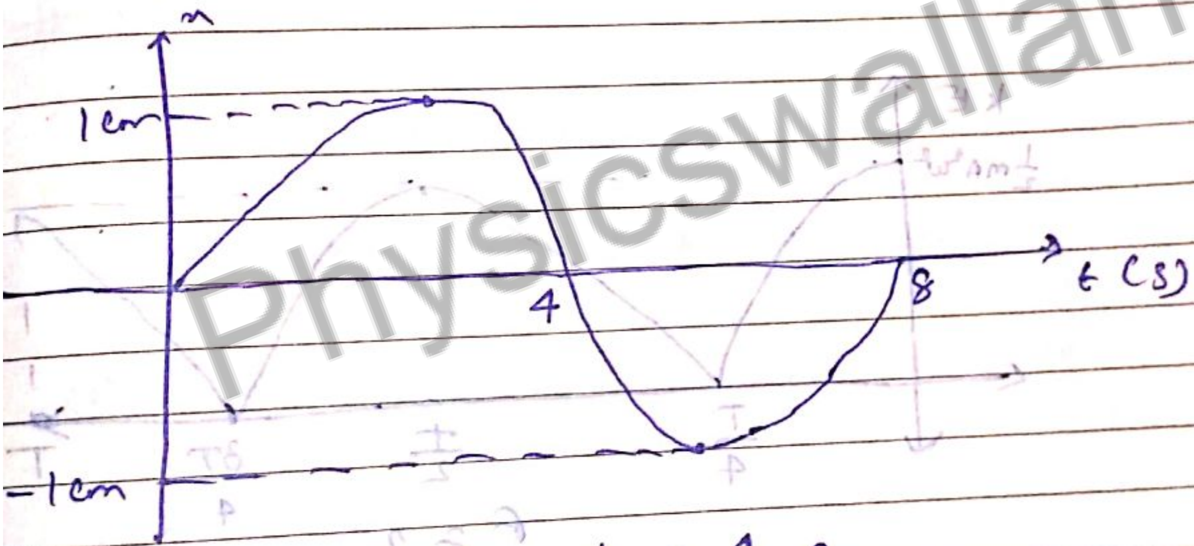


$a = -A\omega^2\sin\omega t = -A\omega^2\sin\frac{2\pi}{T}t$

$= -\omega^2 x$



IIT 2009



a when $t = \frac{4}{3}$ s

a) $\frac{\pi^2}{32}$ cm/s² b) $-\frac{\pi^2}{32}$ cm/s² c) $\frac{\sqrt{3}\pi^2}{32}$ cm/s²

d) $(-)\frac{\sqrt{3}}{32}\pi^2$ cm/s²

$\Rightarrow$ $T = 8$ sec $x = A\sin\omega t$

$A = 1$ cm

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}$$

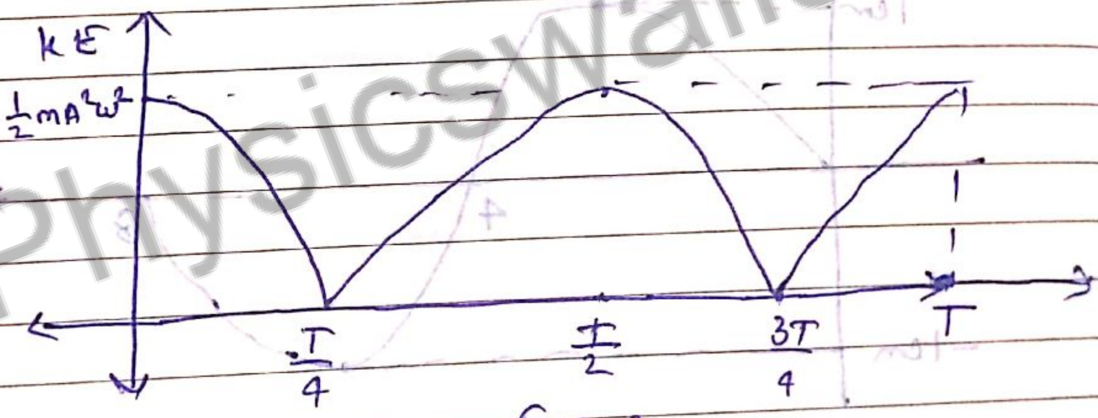
$$a = -A\omega^2 \sin \omega t$$

$$= -1 \times \frac{\pi^2}{16} \times \sin \frac{\pi}{\cancel{4}} \times \frac{\cancel{4}}{3}$$

$$= -\frac{\pi^2}{16} \times \frac{\sqrt{3}}{2}$$

$$a = -\frac{\sqrt{3}\pi^2}{32} \ cm/s^2$$

- $K.E = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2\cos^2\omega t$



$f = 2$

In SHM,

$x \rightarrow$ SHM $\quad A\sin\omega t \ (\omega)$ $\quad f = 1$

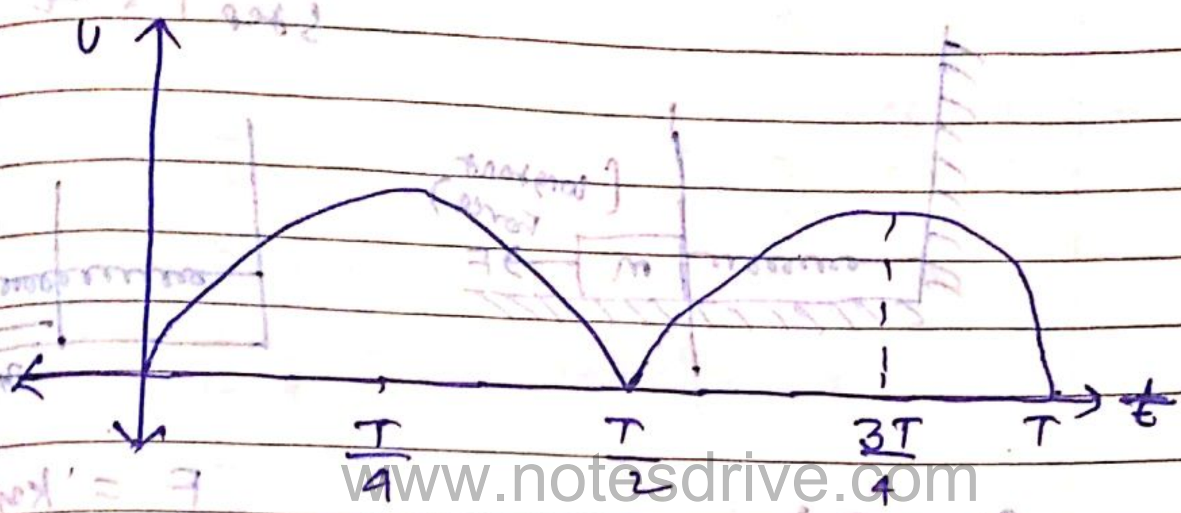
$v \rightarrow$ SHM $\quad A\omega\cos\omega t \ (\omega)$

$a \rightarrow$ SHM $\quad -A\omega^2\sin\omega t \ (\omega)$

K.E $\rightarrow$ ~~SHM~~ Periodic (2ω)

P.E $\rightarrow$ ~~SHM~~ Periodic (2ω)

$$U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 A^2 \sin^2 \omega t$$



Steps to Find Time Period of any S.H.M :-

① Find Equilibrium position $F_{ext} = 0$

Write Balanced force equation.

② Displace the object slightly (x) from mean position & find new F_{net}. & mathematically arrive at $F_{net} = -kx$

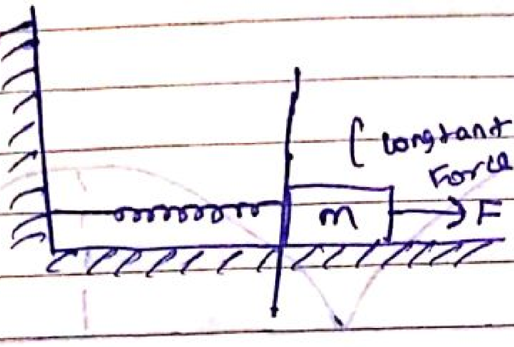
③ $K = m\omega^2 \Rightarrow \omega = \sqrt{\frac{k}{m}}$

$$\boxed{T = 2\pi\sqrt{\frac{m}{K}}}$$

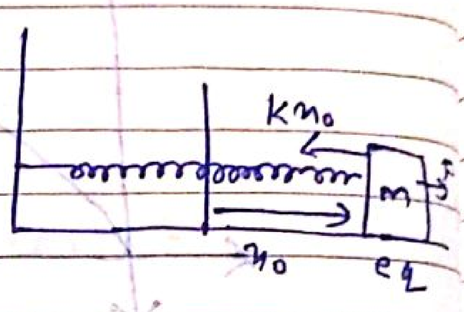
SHM constant

Spring constant only for spring block system.

- Time Period of Spring Block System :

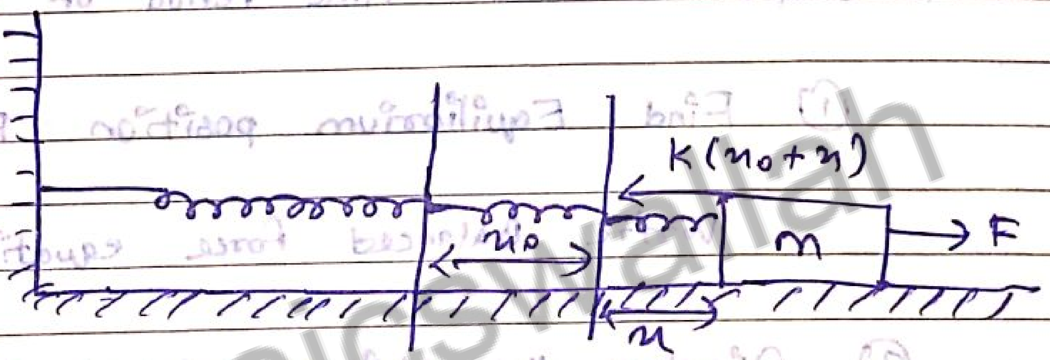


Step 1 : eq



$$F = kx_0$$

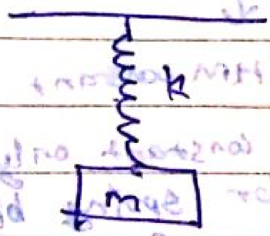
Step 2 :



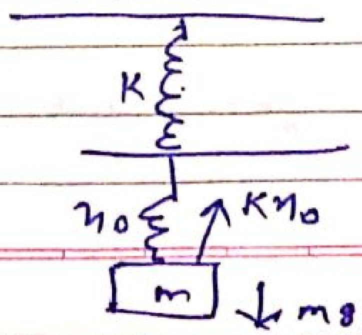
$$F_{net} = F - k(x_0 + x)$$
$$= F - kx_0 - kx$$

$$\boxed{F_{net} = -kx}$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

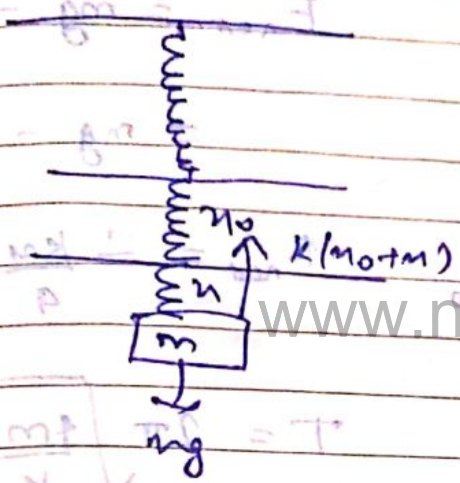
- 

Step 1 eq



$$mg = kx_0$$

SHO II:



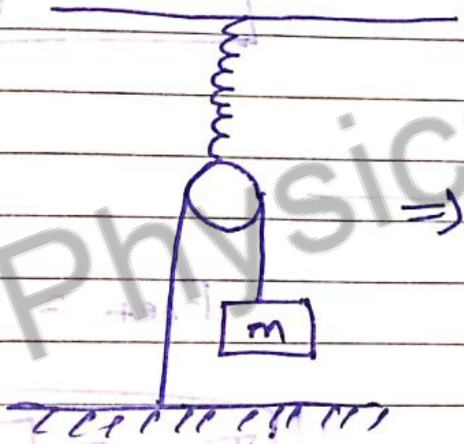
$$F_{net} = mg - kx_0 - kx$$

$$= kx_0 - kx_0 - kx$$

$$\boxed{F_{net} = -kx}$$

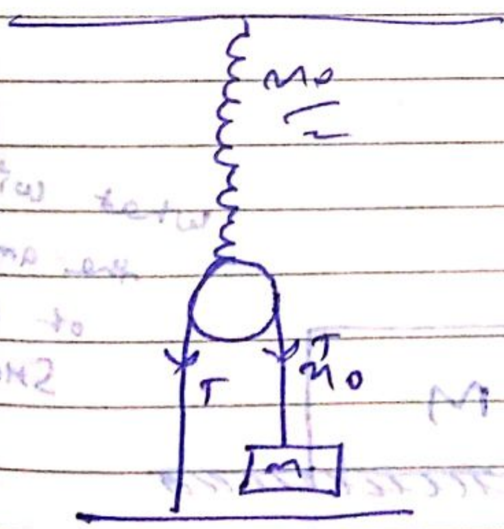
$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

3)



$$\cancel{F_{net} = -\frac{kx}{2}}$$

$$\cancel{T = 2\pi\sqrt{\frac{2m}{k}}}$$

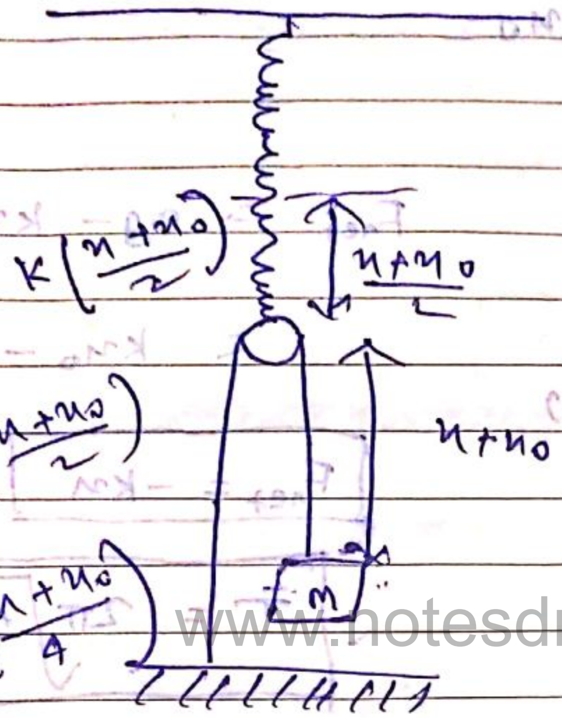


$$2T = \frac{kx_0}{2}$$

$$T = \frac{kx_0}{4}$$

$$mg = T$$

$$mg = \frac{kx_0}{4}$$



$2T' = k\left(\frac{x+x_0}{2}\right)$

$T' = k\left(\frac{x+x_0}{4}\right)$

$F_{net} = mg - T'$

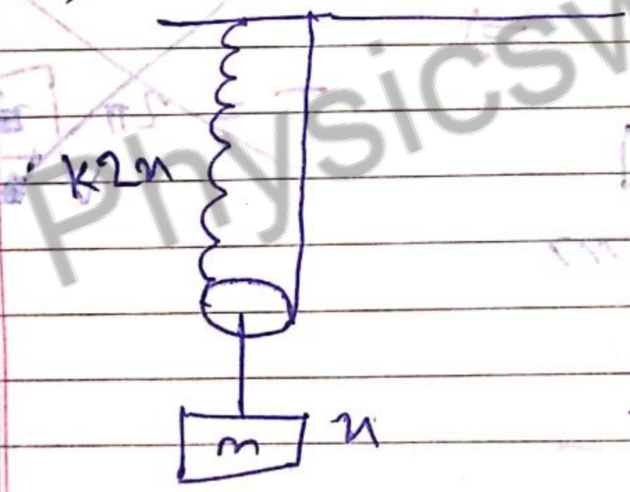
$= mg - \frac{kx}{4} - \frac{kx_0}{4}$

$F_{net} = -\frac{kx}{4}$

$T = 2\pi\sqrt{\frac{4m}{k}}$

$$\boxed{T = 4\pi\sqrt{\frac{m}{k}}}$$

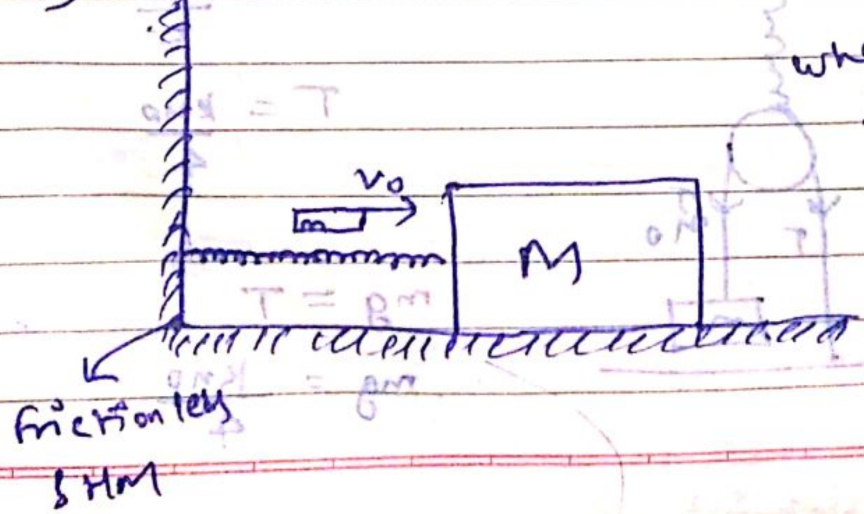
Q)



$F_{net} = 4kx$

$$\boxed{T = \pi\sqrt{\frac{m}{k}}}$$

Q)



what will be the amplitude of that SHM??

$$mv_0 = (m+M)v'$$

$$v' = \frac{mv_0}{m+M}$$

$$\frac{1}{2} \times \frac{m^2 v_0^2}{m+M} = \frac{1}{2} k x^2$$

$$x^2 = \frac{m^2 v_0^2}{(m+M)k}$$

$$x = \frac{m v_0}{\sqrt{(m+M)k}}$$

$$k = (M+m)\omega^2$$

$$\omega = \sqrt{\frac{K}{m+M}} \qquad V_{mean\ max} = \omega A$$

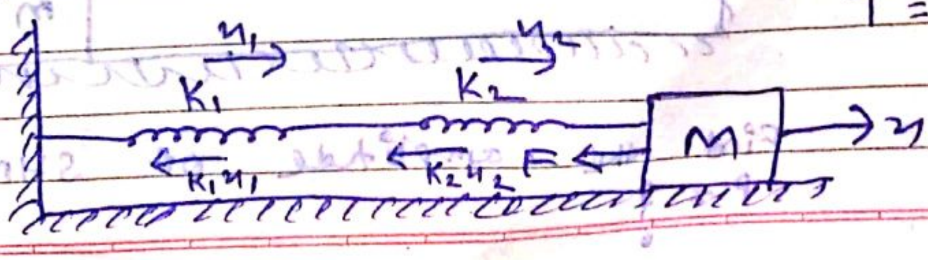
$$\frac{mv_0}{m+M} = \omega A$$

$$\frac{mv_0}{m+M} = \sqrt{\frac{k}{M+m}}\, A$$

$$\boxed{A = \frac{mv_0}{\sqrt{(M+m)k}}}$$

- Series

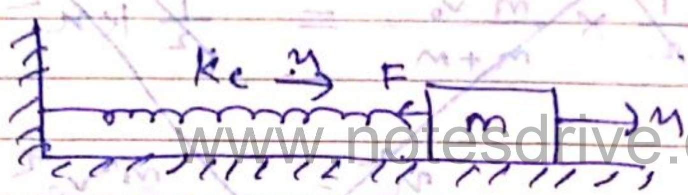
$$T = 2\pi\sqrt{\frac{m}{k_e}}$$



$K \uparrow\uparrow \Rightarrow$ string stiff

$$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$K_e = \frac{K_1 K_2}{K_1 + K_2}$$



$$F = K_e x = K_1 x_1 = K_2 x_2$$

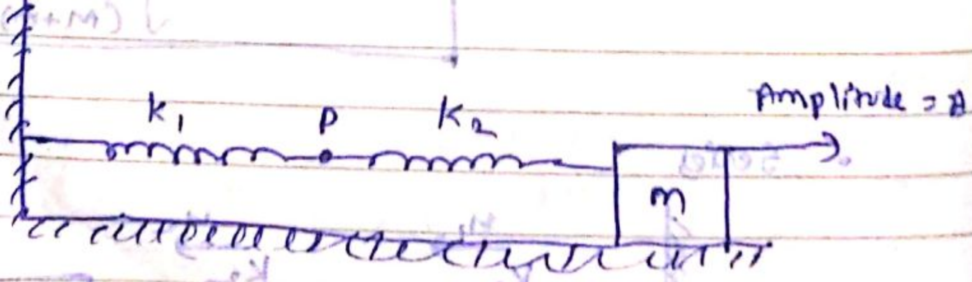
$$K_2 x_2 = F = K_1 x_1$$

$$x = x_1 + x_2$$

$$\frac{F}{K_e} = \frac{F}{K_1} + \frac{F}{K_2}$$

$$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2}$$

IIT 2009



Find the amplitude of SHM of P = ?

a) $\frac{k_1 A}{k_1 + k_2}$ b) $\frac{k_2 A}{k_1 + k_2}$ c) $\frac{k_1 A}{k_2}$ d) A

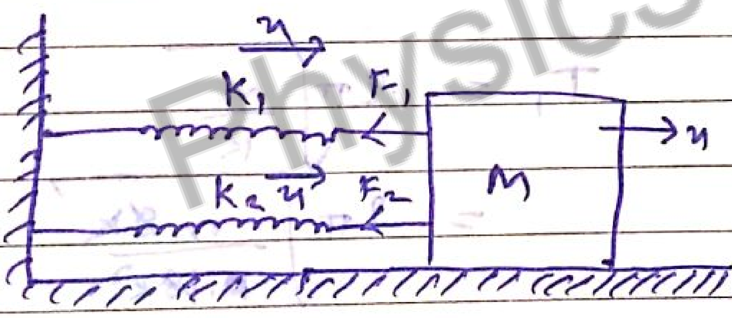
$$\Rightarrow \quad \boxed{F = k_1 x_1 = k_2 x_2} = k_E A$$

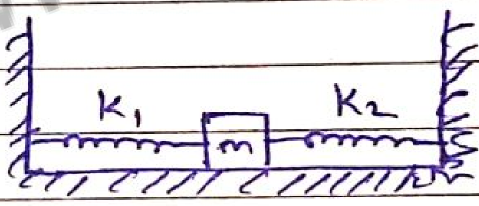
$$x_1 = \frac{k_E A}{k_1}$$

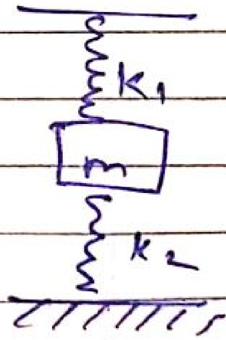
$$= \frac{k_1 k_2 A}{(k_1 + k_2) k_1}$$

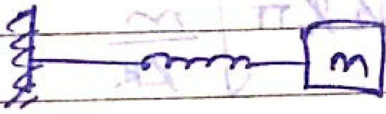
$$x_1 = \frac{k_2 A}{k_1 + k_2}$$

Parallel

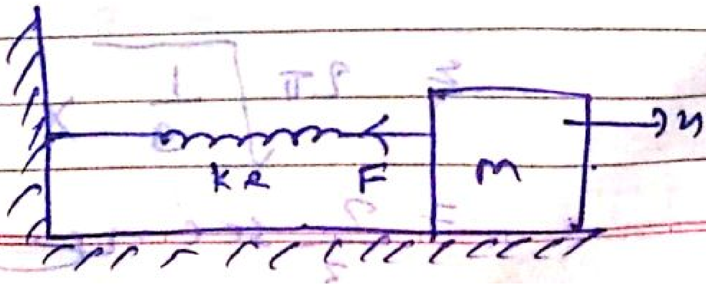








$$\boxed{K_e = k_1 + k_2}$$

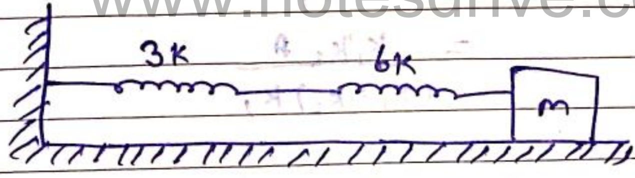


$$F = F_1 + F_2$$

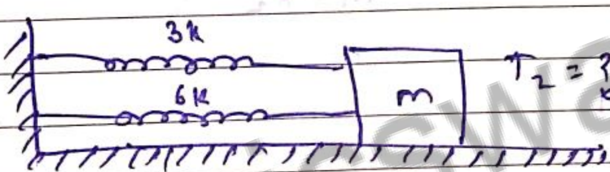
$$K_e x = k_1 x + k_2 x$$

$$\boxed{K_e = k_1 + k_2}$$

Q)



$T_1 = \sqrt{2}\,s$



$T_2 = ?$

$$T_1 = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{m\,9K}{\cancel{18}K^2}}$$

$$\cancel{\sqrt{2}} = \cancel{\sqrt{2}}\,2\pi\sqrt{\frac{m}{2K}}$$

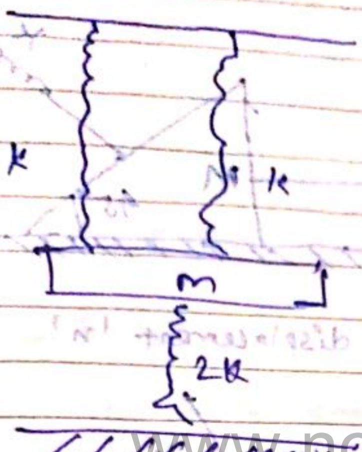
$$T_2 = 2\pi\sqrt{\frac{m}{9K}}$$

$$= 2\pi\sqrt{\frac{1}{9}} \times \frac{1}{\pi}$$

$$= \frac{2}{3}\ \text{sec}$$

Q)

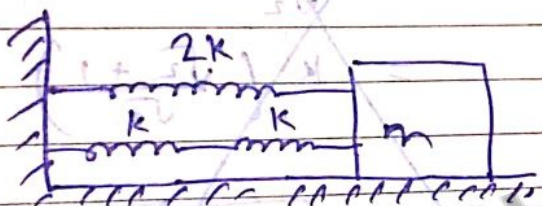
a)



$$T = 2\pi\sqrt{\frac{m}{4k}}$$

$$T_1 = \pi\sqrt{\frac{m}{k}}$$

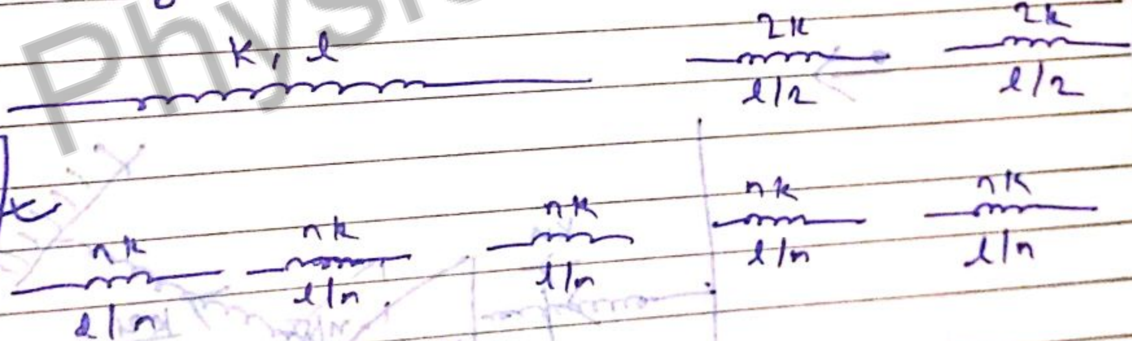
b)



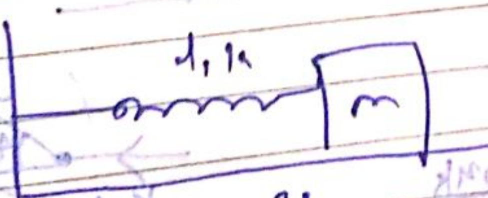
$$T_2 = 2\pi\sqrt{\frac{2m}{5k}}$$

Cutting of Springs

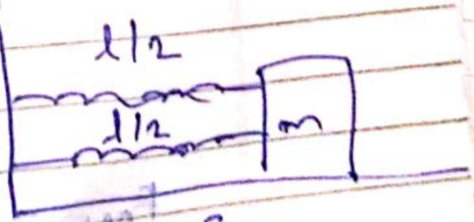
$$\boxed{k \propto \frac{1}{l}}$$



c)



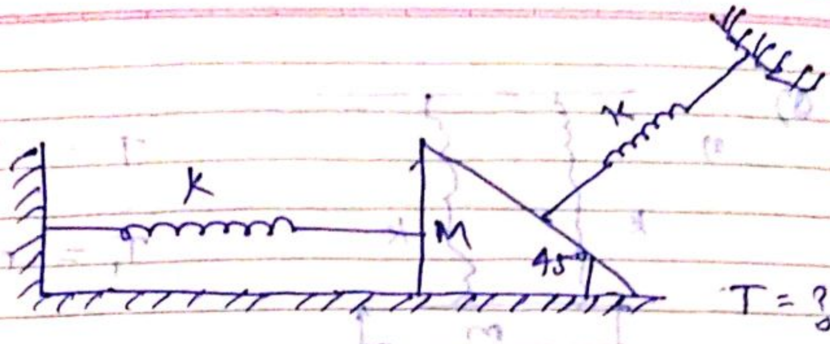
$T_1 = 2s$



$T_2 = ?$

$$\boxed{T_2 = \sqrt{2}\ \text{sec}}$$

Q)



$T = ?$

Small Horizontal displacement 'x'

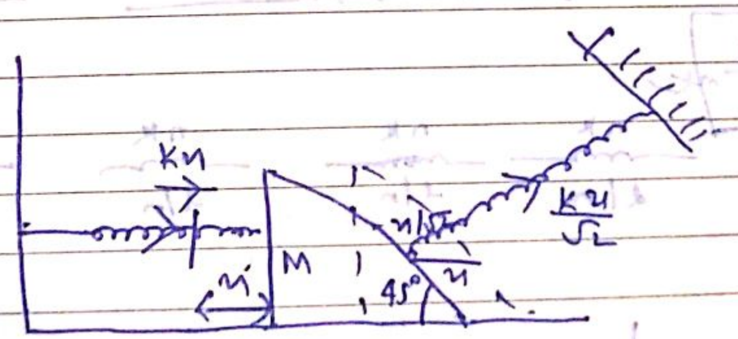
$$\cancel{K_e = K + \frac{K}{\sqrt{2}}}$$

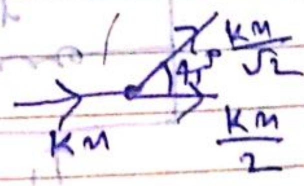
$$\cancel{= \frac{\sqrt{2}K + K}{\sqrt{2}}}$$

$$\cancel{= K\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)}$$

$$\cancel{T = 2\pi\sqrt{\frac{\sqrt{2}m}{(\sqrt{2}+1)K}}}$$

$\Rightarrow$

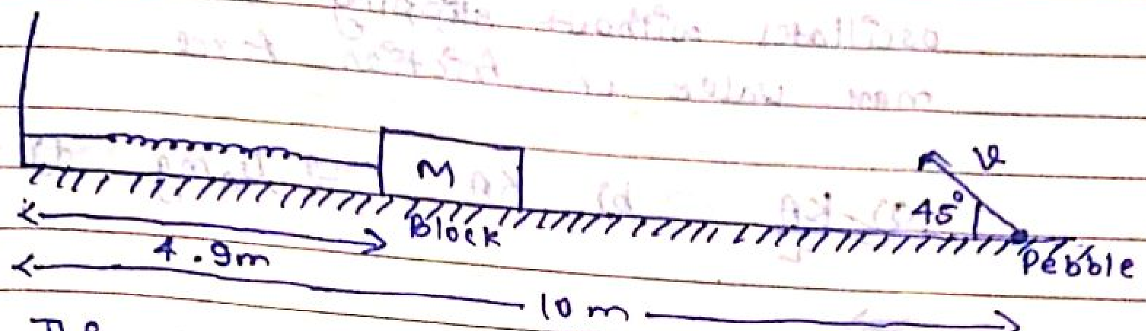




$$F_{net} = -\frac{3xk}{2}$$

$$\boxed{T = 2\pi\sqrt{\frac{m2}{3k}}}$$

IIT 2012 :



This block is stretched to $x = 0.2$ m & released from rest at $t = 0$. Block $\rightarrow$ SHM

$\rightarrow$ $\omega = \frac{\pi}{3}$. At $t = 0$ pebble is projected

If both meets at $t = 1$s, then find $v = ?$

a) $\sqrt{50}$ b) $\sqrt{51}$ c) $\sqrt{52}$ d) $\sqrt{53}$

$\Rightarrow$ $T = \frac{2\pi}{\pi} \times 3 = 6$ sec

At $t = 1$ sec,

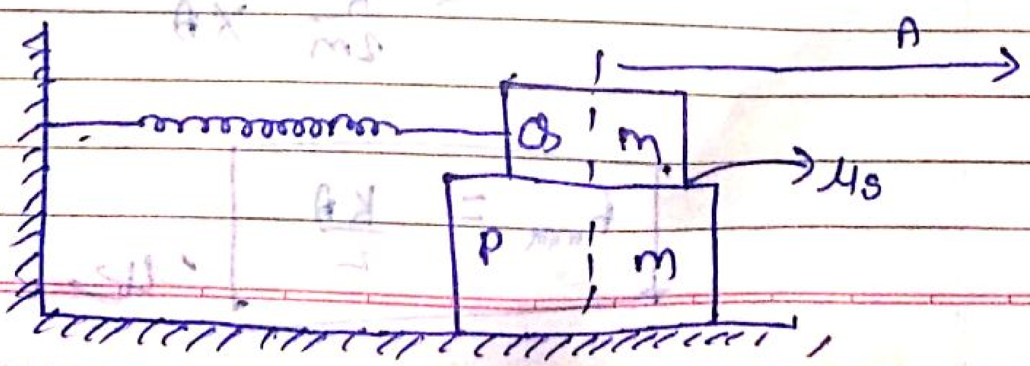
$R = 5$m

$= \frac{v^2}{g}$

$5 \times 10 = v^2$

$v = \sqrt{50}$ m/s.

IIT 2004

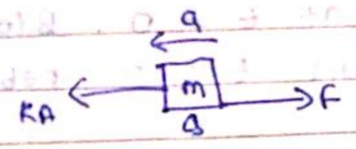


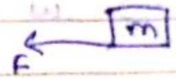
Two blocks pulled by distance A & oscillates without slipping what is the max value of friction force.

a) $\frac{KA}{2}$ b) KA c) $\mu_s mg$ d) zero

$\Rightarrow$

At extreme points



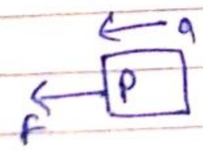


$-f + KA = ma$ $\qquad$ $f = ma$

$2f = KA$

$f = \frac{KA}{2}$. Ans

OR



$f = ma$

$f_{max} = m a_{max}$

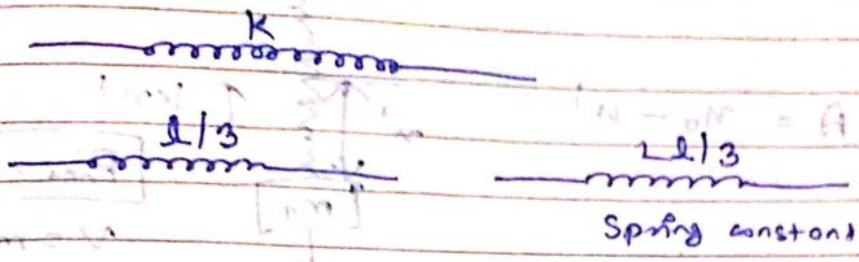
SHM,

$a_{max} = \omega^2 A$

$= \frac{K}{2m} \times A$

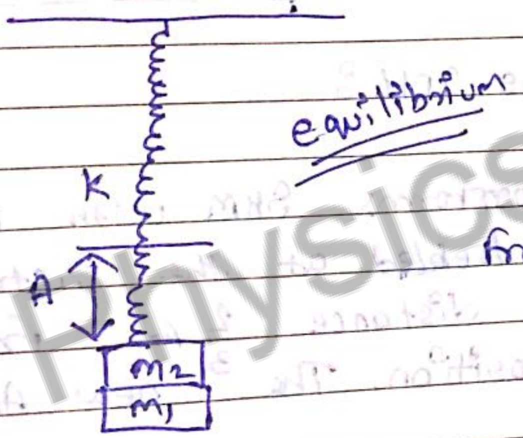
$\boxed{f_{max} = \frac{KA}{2}}$. Ans

IIT 1999



$\Rightarrow \quad k' = \frac{3k}{2}$

IIT 1981 :



m_1 is removed without any disturbance.
m_2 oscillates with freq ω & Amplitude A.
$\omega = ?$, $A = ?$

$\Rightarrow \quad Kx_0 = (m_2 + m_1)g$

$x_0 = \frac{(m_1 + m_2)g}{K}$

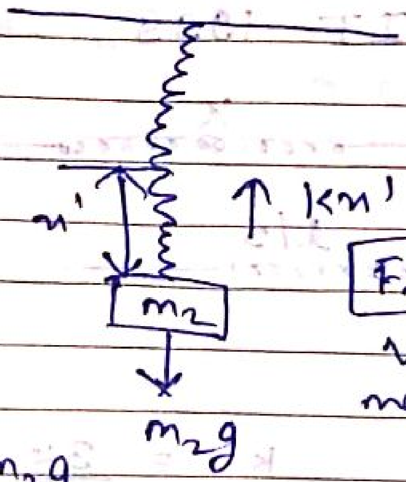
$\cancel{KA - m_2 g = m_2 a}$

$\cancel{m_1 g = m_2 a}$

$\cancel{a = \frac{m_1 g}{m_2}}$

~~ω =~~

$$A = x_0 - x'$$



$$\boxed{F_{net} = 0}$$

v = max
mean position

$$A = \frac{m_1 g + m_2 g}{K} - \frac{m_2 g}{K}$$

$$\boxed{A = \frac{m_1 g}{K}} \qquad \boxed{\omega = \sqrt{\frac{K}{m_2}}}$$

JEE Mains 2016 :

A particle performs SHM with Amp 'A'. Its speed is trebled at the instant when it is at a distance $\frac{2}{3}A$ from equilibrium position. The new Amplitude of SHM ⟶

a) $\frac{A\sqrt{41}}{3}$ b) $3A$ c) $A\sqrt{3}$ d) $\frac{7A}{3}$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{1}{2}Kx^2$$

$$\frac{1}{2} \times \cancel{m} \times \cancel{9} \times \omega^2 \left(\frac{5A^2}{9}\right) = \frac{1}{2}Kx^2$$

$$\frac{5A^2}{2} = x^2$$

$$v_f = 3v_p$$

$$\cancel{\omega}\sqrt{A_f^2 - \left(\frac{2}{3}A\right)^2} = 3\cancel{\omega}\sqrt{A^2 - \left(\frac{2}{3}\right)A^2}$$

$$A_f^2 - \frac{4A^2}{9} = \frac{\cancel{9} \times 5A^2}{\cancel{9}}$$

$$A_f^2 = 5A^2 + \frac{4}{9}A^2$$

$$\boxed{A_f = \frac{7A}{3}}$$

IIT 2010

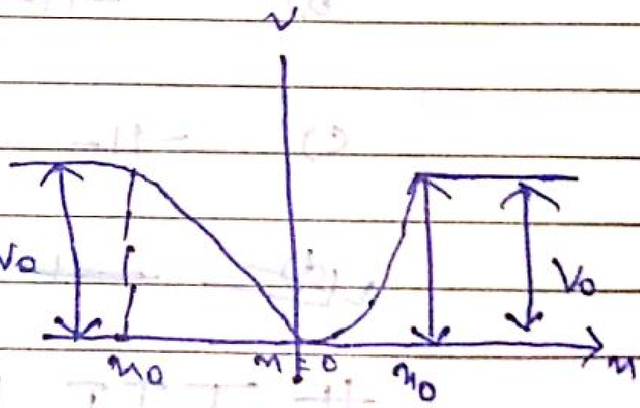
$V(x) = Kx^2$ SHM, time period $\propto \sqrt{\frac{m}{k}}$

$V(x) = \alpha x^4$ dimensional analysis

↓

increases on both sides of $x = 0$

Periodic motion



$V(x) = \alpha x^4 \quad |x| < X_0$

$V(x) = V_0$ (constant) $|x| > X_0$

Total Energy ⟶ infinity enscape X

1) If the motion is periodic & Total Energy of particle is E, then

Date
Page

a) $E < 0$; ~~b)~~ $E > 0$, ~~c)~~ $V_0 > E > 0$

d) $E > V_0$

② Periodic, time period proportional α

a) $A\sqrt{\frac{m}{\alpha}}$ ~~b)~~ $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$, c) $A\sqrt{\frac{\alpha}{m}}$

d) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

③ acceleration of this particle for

$|x| > X_0$

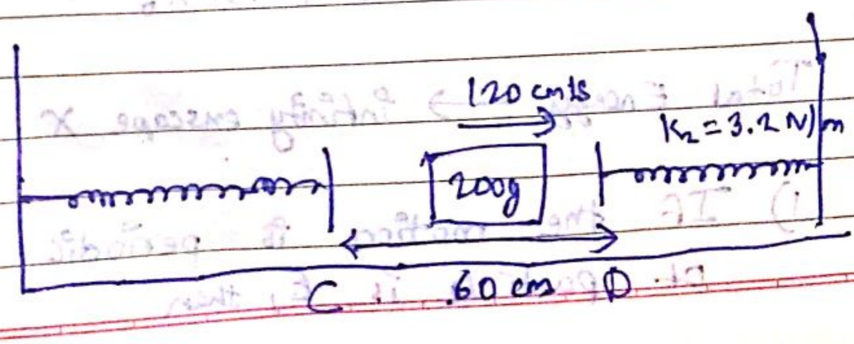
a) proportional to V_0

b) — 11 — $\frac{V_0}{mX_0}$

c) — 11 — $\sqrt{\frac{V_0}{mX_0}}$

~~d)~~ — 11 — zero

IIT 1985



$>E>0$

α

If mass 200g moves with 120 cm/sec between C & D. Find the time period of oscillation of 200 g mass.

a) 2.42 s b) 2.82 s c) 3.42 s d) 3.62 s

$\Rightarrow$

$0.2v^2 = 3.2 \times x^2$

$\cancel{0.2} \times 0.12 \times 0.12 = \overset{16}{\cancel{3.2}} \times x^2$

$t = \frac{1}{2} \times \frac{0.2 \times 0.12 \times 0.12}{0.03 \times 3.2}$

$\frac{0.12}{4} = x$ 0.03

$= \frac{\cancel{2} \times \cancel{12} \times 0.12}{\cancel{2} \times \cancel{3} \times 32}$ 0.03

$x = 0.03$ m

$= 30$ cm

$= 0.015$ sec

$\cancel{0.2} \times 0.12 \times 0.12 = \overset{9}{\cancel{1.8}} \times x^2$

2.03

+

$\frac{0.12}{3} = x$

$t = \frac{1}{\cancel{2}} \times \frac{\cancel{0.2} \times 0.12 \times 0.12}{0.04 \times 1.8}$

$x = 0.04$ m

$= 40$ cm

$= \frac{0.12 \times 12}{4 \times 18}$

$= 0.02$

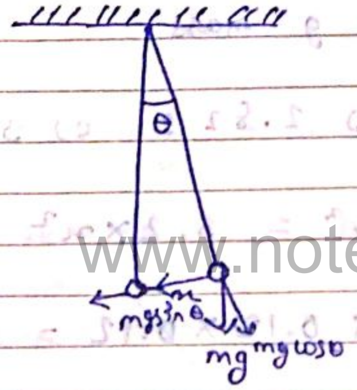
$$T = \frac{120}{120} + \pi\sqrt{\frac{0.2}{3.2}} + \pi\sqrt{\frac{0.2}{1.8}}$$

$= 1 + \pi/4 + \pi/3$

$= 1 + 0.78 + 1.04$

$= 2.82$ Sec.

Time Period of Simple Pendulum :



if θ is small, then path is a straight line → SHM

$$\theta = \frac{x}{l}$$

$$\sin\theta \approx \theta \approx \frac{x}{l}$$

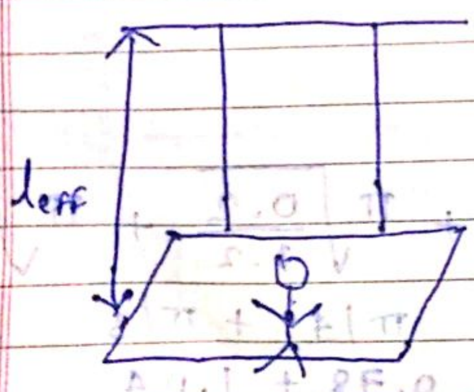
$$F = -mg\sin\theta$$
$$= -\frac{mgx}{l}$$
$$-kx = -\frac{mgx}{l}$$

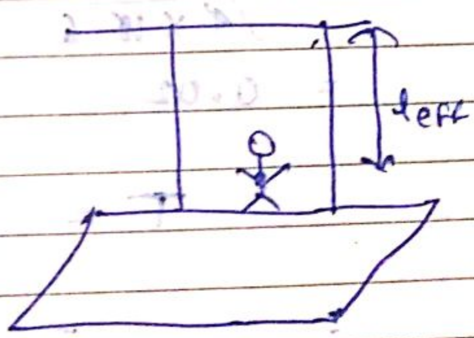
$$k = \frac{mg}{l}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$= 2\pi\sqrt{\frac{ml}{mg}}$$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

•



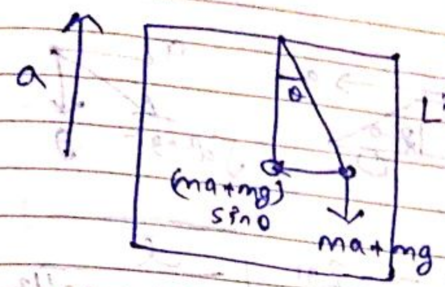


$$T = 2\pi\sqrt{\frac{l_{eff}}{g}}$$

$l_{eff} \downarrow \quad T \downarrow$

$\omega \uparrow$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g_{eff}}}}$$



$F = -\frac{(ma+mg)x}{l}$

$k = \frac{ma+mg}{l}$

$$\boxed{T = 2\pi\sqrt{\frac{l}{a+g}}}$$

IIT JEE 2005

Pendulum T_1

Point of suspension move upward

$y = kt^2$ ($k = 1 m/s^2$) ($t \to$ time)

new Time Period $\to T_2$

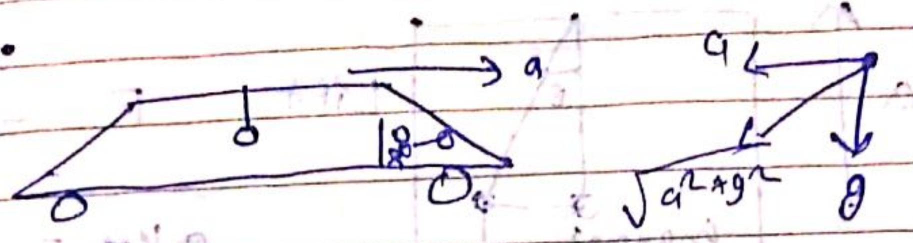
$\frac{T_1^2}{T_2^2} = ?$

a) $\frac{5}{6}$ b) $\frac{6}{5}$ ✓ c) 1 d) $\frac{1}{2}$

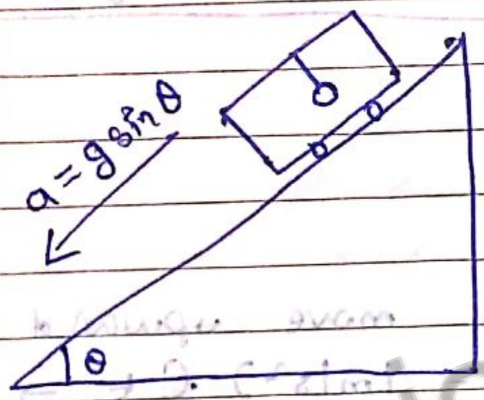
$\Rightarrow$ $y = kt^2$

$a = 2$ sec

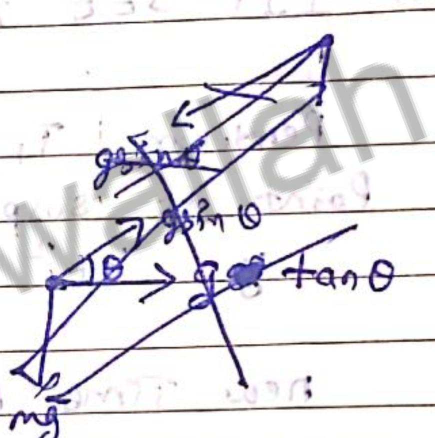
$\frac{T_1^2}{T_2^2} = \frac{12}{10} = \frac{6}{5}$

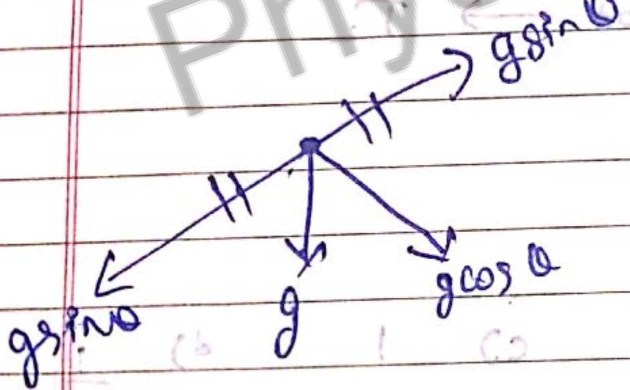


$$T = 2\pi\sqrt{\frac{l}{(a^2+g^2)^{1/2}}}$$



T = ?





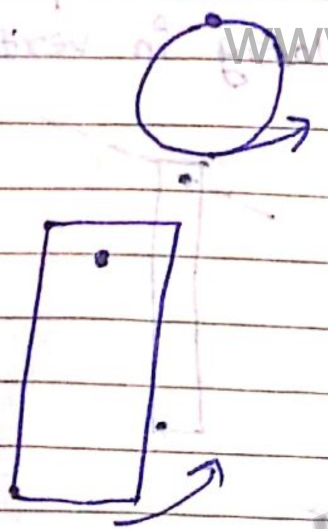
$$\boxed{T = 2\pi\sqrt{\frac{l}{g\cos\theta}}}$$

* Physical Pendulum :

Linear SHM	Angular SHM
① $F = -kx$	$\tau = -k\theta$
② $T = \frac{2\pi}{\omega}$	$T = \frac{2\pi}{\omega}$

③ $\omega = \sqrt{\frac{K}{m}}$ | $\omega = \sqrt{\frac{K}{I}}$

④ $\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$ | ④ $\boxed{T = 2\pi\sqrt{\frac{I}{k}}}$

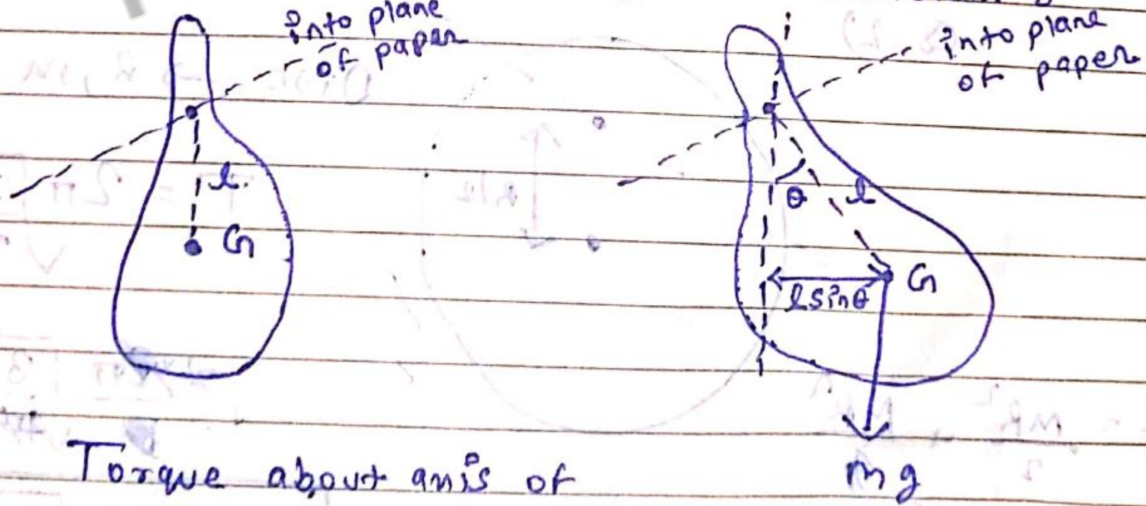


Rigid Body (pivoted) oscillate

of Any shape

oscillations should be very very small

- Time Period of Physical Pendulum :



Torque about axis of rotation :

$$\tau = mg(l\sin\theta)$$
$$= -mgl\theta$$

$\therefore \boxed{K = mgl}$

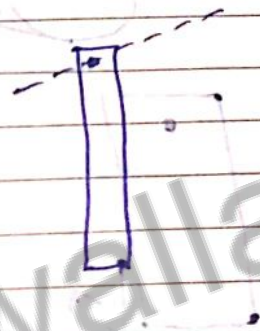
$$\tau = -K\theta$$

$$T = 2\pi\sqrt{\frac{I}{k}}$$

$$\boxed{T = 2\pi\sqrt{\frac{I}{mgl}}}$$ $I \rightarrow$ about axis of rotation

Q) Rod of Length L, Mass M, pivoted about one end & oscillating in vertical plane.

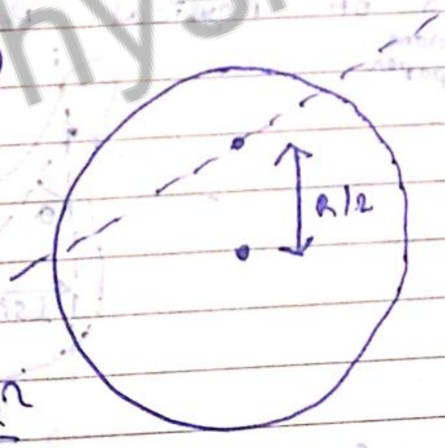
$T = ?$



$$T = 2\pi\sqrt{\frac{\cancel{m}l^{\cancel{2}}}{3\cancel{m}g\cancel{l}}}$$

$$\boxed{T = 2\pi\sqrt{\frac{l}{3g}}}$$

Q.2)



Disc $\rightarrow$ R, M

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

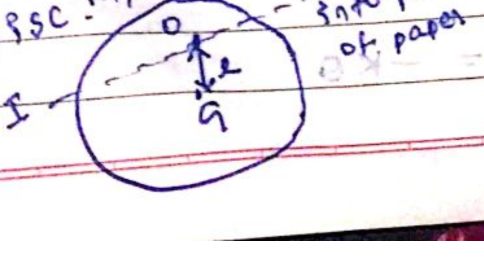
$$I = \frac{MR^2}{2} + \frac{MR^2}{4}$$

$$= \frac{3MR^2}{4}$$

$$= 2\pi\sqrt{\frac{3\cancel{m}l^{\cancel{2}}\times 2}{4\cancel{m}g\cancel{l}}}$$

$$\boxed{T = 2\pi\sqrt{\frac{3l}{2g}}}$$

Q) Disc M, R



For what distance OG,
① Time Period $\longrightarrow$ Max
② Time Period $\longrightarrow$ Min

Also find these Two Time Periods.

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{I}{mgl}}$$

$$T = 2\pi\sqrt{\frac{\frac{MR^2}{2} + ML^2}{MgL}}$$

$$y = \frac{R^2}{2gL} + \frac{L}{g}$$

$T \to$ min, $y \to$ min

$$\frac{dy}{dL} = 0$$

$$-\frac{R^2}{2gL^2} + \frac{1}{g} = 0$$

$$L^2 = \frac{R^2}{2}$$

$$L = \pm\frac{R}{\sqrt{2}}$$

$$T = 2\pi\sqrt{\frac{\left(\frac{R^2}{2} + \frac{R^2}{2}\right)\sqrt{2}}{Rg}}$$

$$= 2\pi\sqrt{\frac{\sqrt{2}R}{g}}$$

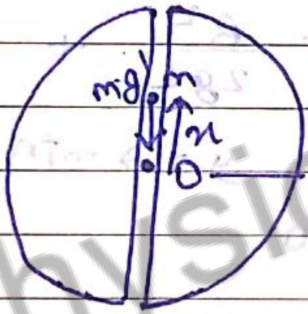
$$\boxed{T = 2\pi\sqrt{\frac{1.414R}{g}}}$$

$T_{max} \longrightarrow \infty$

$$\boxed{l = 0}$$

- Tunnel through Earth :

$$\boxed{T = 2\pi\sqrt{\frac{R}{g}}} \approx 84.6 \text{ min}$$



Mean position $\left\{ F_{net} = 0, \; g = 0 \right\}$

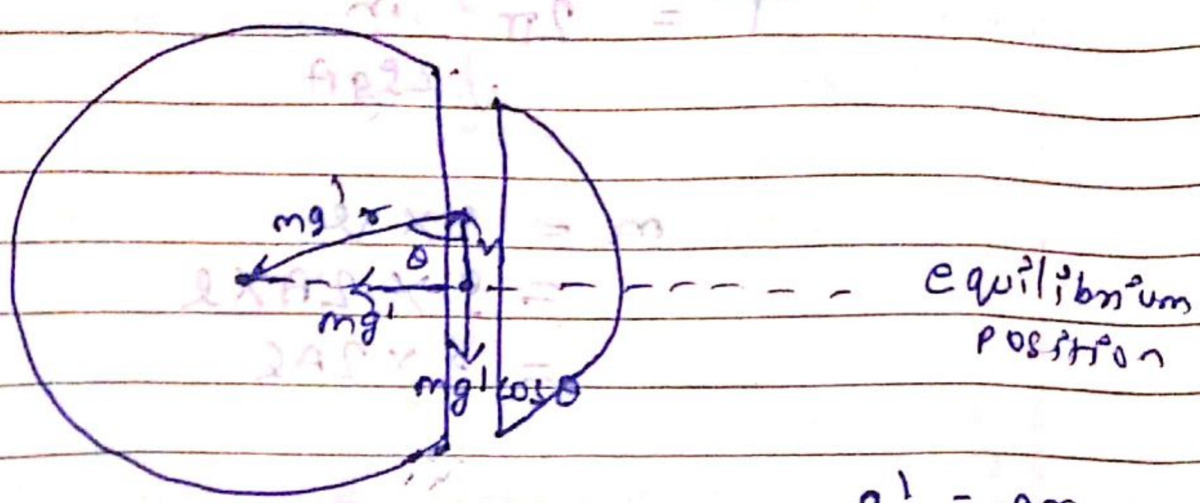
$$F_{net} = mg' = -\frac{mgx}{R}$$

$$k = \frac{mg}{R}$$

$$T = 2\pi\sqrt{\frac{mR}{mg}}$$

$$\boxed{T = 2\pi\sqrt{\frac{R}{g}}}$$

$T \approx 84.6$ min.



$$g' = \frac{gr}{R}$$

$$F_{net} = \frac{mgr}{R}\cos\theta$$

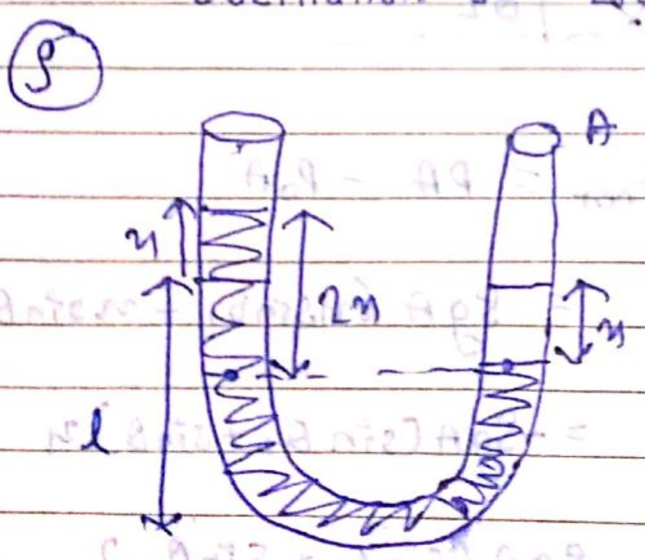
$$= \frac{mg\cancel{r}}{R} \times \frac{x}{\cancel{r}}$$

$$= -\frac{mgx}{R} \qquad k = \frac{mg}{R}$$

$$-k\cancel{x} = -\frac{mg\cancel{x}}{R}$$

$$\boxed{T = 2\pi\sqrt{\frac{R}{g}}}$$

- Oscillation of Liquid in a U-tube :



$$F_{net} = PA - P_0A$$

$$F_{net} = (P_0 + \rho g 2x)A - P_0A$$

$$= -\rho g 2xA$$

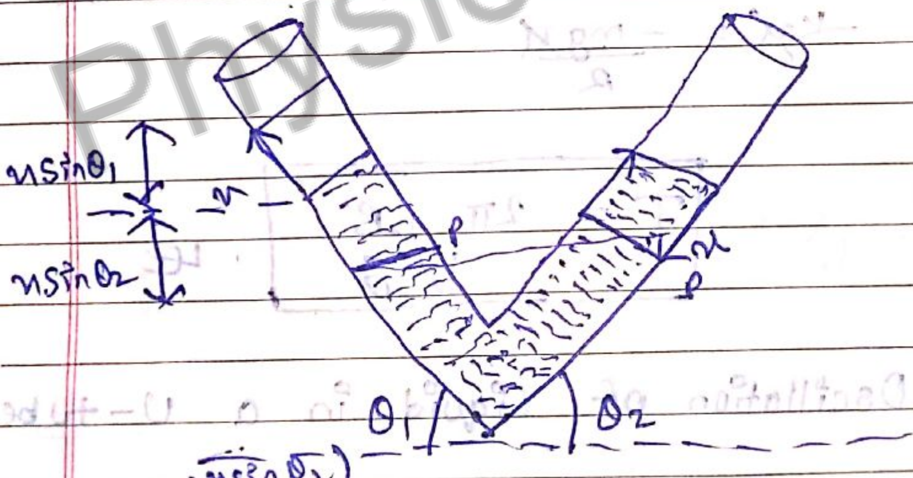
$$F_{net} = -2\rho gAx$$

$$T = 2\pi\sqrt{\frac{m}{2\rho g A}}$$

$$m = \rho \times V$$
$$= \rho \times 2A \times l$$
$$= \rho \times 2Al$$

$$T = 2\pi\sqrt{\frac{\cancel{2}\cancel{\rho}\cancel{A}l}{\cancel{2}\cancel{\rho}g\cancel{A}}}$$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$



m, A, ρ

$$P = P_0 + \rho g(x\sin\theta_1 + x\sin\theta_2)$$

$$F_{net} = PA - P_0A$$

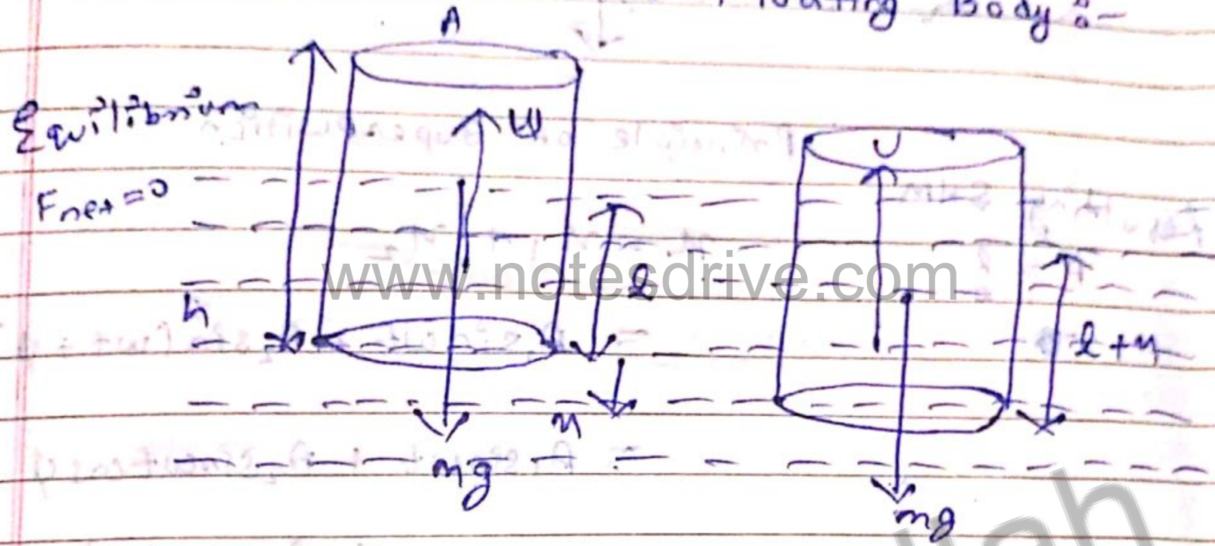
$$= \rho g A(x\sin\theta_1 + x\sin\theta_2)$$

$$= -\rho g A(\sin\theta_1 + \sin\theta)x$$

$$K = \rho g A(\sin\theta_1 + \sin\theta_2)$$

$$T = 2\pi\sqrt{\frac{m}{\rho g A(\sin\theta_1 + \sin\theta_2)}}$$

- Oscillation of a Floating Body :-



$U = mg$

$V\rho g = mg$

$Al\rho g = mg$ —(1)

$F_{net} = U - mg$

$= V'\rho g - mg$

$= A(l+x)\rho g - mg$

$= Al\rho g + Ax\rho g - mg$

$F_{net} = -A\rho g x$

$F_{net} = -kx$

$k = A\rho g$

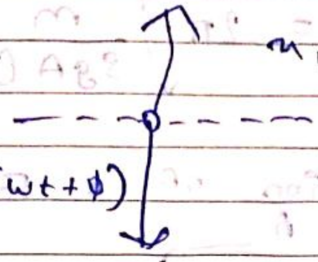
$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{Al\rho}{A\rho g}}$$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

Combination of SHM :-

$x_1 = A_1 \sin \omega t$



$x_2 = A_2 \sin(\omega t + \phi)$

Principle of Superposition

Resulting SHM

$A = ?$

$$x = x_1 + x_2$$

$$= A_1 \sin \omega t + A_2 \sin(\omega t + \phi)$$

$$= A_1 \sin \omega t + A_2 \sin \omega t \cos \phi + A_2 \cos \omega t \sin \phi$$

$$= \sin \omega t \underbrace{(A_1 + A_2 \cos \phi)}_{A \cos \theta} + \cos \omega t \underbrace{(A_2 \sin \theta)}_{A \sin \theta}$$

$$x = A \sin \omega t \cos \theta + A \cos \omega t \sin \theta$$

$$\boxed{x = A \sin(\omega t + \theta)}$$

$$A \sin \theta = A_2 \sin \phi \quad \text{—— (I)}$$

$$A \cos \theta = A_1 + A_2 \cos \phi \quad \text{—— (II)}$$

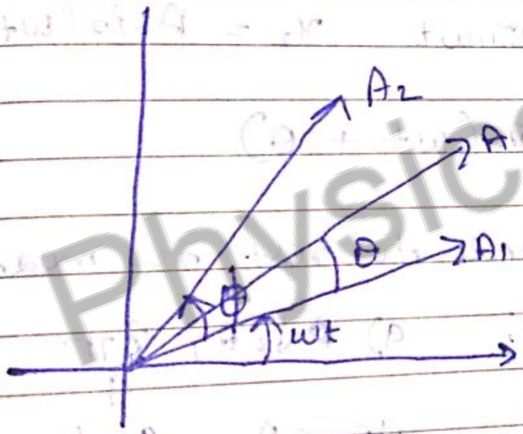
$$A^2 = A_2^2 \sin^2\phi + A_1^2 + A_2^2\cos^2\phi + 2A_1A_2\cos\phi$$

$$\boxed{A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

Divide (I) ÷ (II)

$$\boxed{\tan\theta = \frac{A_2\sin\phi}{A_1 + A_2\cos\phi}}$$

Shortcuts:-



$x_1 = A_1 \sin\omega t$

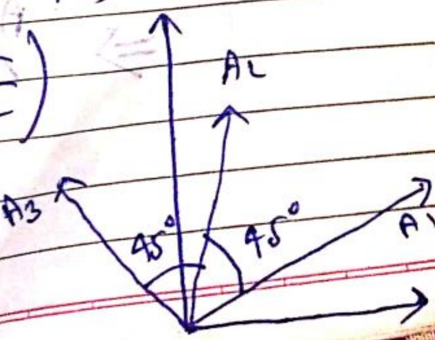
$x_2 = A_2 \sin(\omega t + \phi)$

$x = A\sin(\omega t + \theta)$

(Q) $x_1 = 2\sin\omega t$

$x_2 = 2\sqrt{2}\sin\left(\omega t + \frac{\pi}{4}\right)$

$x_3 = \sin\left(\omega t + \frac{\pi}{2}\right)$



$\Rightarrow A_x = 2 + 2 = 4$

$A_y = 1 + 2 = 3$

$A = \sqrt{A_x^2 + A_y^2}$

$= 5$

$\tan\theta = \frac{3}{4}$

$\boxed{\theta = 37^\circ}$

$\boxed{x = 5\sin(\omega t + 37^\circ)}$

Q) IIT 2011

$x_1 = A\sin\omega t \qquad x_2 = A\sin\left(\omega t + \frac{2\pi}{3}\right)$

$x_3 = B\sin(\omega t + \theta)$

particles comes to rest to mean position.

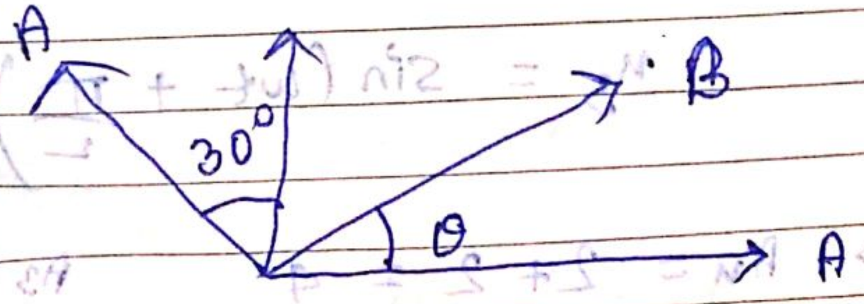
$B, \theta = ?$

a) $\sqrt{2}A, \frac{3\pi}{4}$

b) $A, 4\pi/3$

c) $\sqrt{3}A, \frac{5\pi}{6}$

d) $A, \pi/3$

$\Rightarrow$ 

Date
Page

$$A + B\cos\theta - \frac{A}{2} = 0$$

$$B\cos\theta = -\frac{A}{2}$$

$$\frac{\sqrt{3}A}{2} + B\sin\theta = 0$$

$$B\sin\theta = \frac{-\sqrt{3}A}{2}$$

$$\tan\theta = \sqrt{3}$$

$\boxed{\theta = 60^\circ}$ But, ∵ B is opp.

$\boxed{\theta = 240^\circ}$

$\boxed{B = A}$