

# # REAL NUMBERS

## Euclid's Division Lemma

### Statement

Given positive integers 'a' & 'b', there exist a unique integers 'q' and 'r' satisfying

$$a = b \times q + r$$

where  $0 \leq r < b$

## Euclid's Division Algorithm

$$c = d \times q + r$$

### Statement

→ An Application of Euclid's Division Lemma

→ Helps in finding HCF of numbers

### \* Algorithm

It is a series of well defined steps which gives a procedure for solving a type of problem.

### \* Lemma

It is a proven statement used for proving another statement

① Find the HCF of 60 & 36 using EDA.

By EDA,  $a = bq + r$

→  $60 = 36 \times 1 + 24 \quad (r \neq 0)$

$$36 = 24 \times 1 + 12 \quad (r \neq 0)$$

$$24 = 12 \times 2 + 0 \quad (r = 0)$$

$$\therefore \text{HCF} = 12$$

② Find the HCF of 24 & 15 using EDA.

→ By EDA,  
 $a = bq + r$

$$24 = 15 \times 1 + 9 \quad (r \neq 0)$$

$$15 = 9 \times 1 + 6 \quad (r \neq 0)$$

$$9 = 6 \times 1 + 3 \quad (r \neq 0)$$

$$6 = 3 \times 2 + 0 \quad (r = 0)$$

$$\therefore \text{HCF} = 3$$

③ Find the HCF of 8 & 6.

→ By EDA,

$$8 = 6 \times 1 + 2 \quad (r \neq 0)$$

$$6 = 2 \times 3 + 0 \quad (r = 0)$$

$$\therefore \text{HCF} = 2$$

④ Find the HCF of 135 & 225.

→ By EDA,

$$225 = 135 \times 1 + 90 \quad (\mu \neq 0)$$

$$135 = 90 \times 1 + 45 \quad (\mu \neq 0)$$

$$90 = 45 \times 2 + 0 \quad (\mu = 0)$$

$$\therefore \text{HCF} = 45$$

⑤ Find the HCF of 867 & 255.

→ By EDA,

$$~~255~~ 867 = 255 \times 3 + 102 \quad (\mu \neq 0)$$

$$255 = 102 \times 2 + 51 \quad (\mu \neq 0)$$

$$102 = 51 \times 2 + 0 \quad (\mu = 0)$$

$$\therefore \text{HCF} = 51$$

⑥ Find the HCF of 616 & 32.

$$\rightarrow 616 = 32 \times 19 + 8 \quad (\mu \neq 0)$$

$$32 = 8 \times 4 + 0 \quad (\mu = 0)$$

$$\therefore \text{HCF} = 8$$

(7) Prove that every positive even integer is of the form  $2q$  & every positive odd integer is of the form  $2q+1$ , where  $q$  is some integer.

→ Let  $a$  be any +ve integer &  $b=2$   
( $a = bq + r$ )

Apply EDA to ' $a$ ' & ' $b$ '

$$a = 2q_1 + r, \quad 0 \leq r < 2$$

$$\therefore r = 0, 1$$

$$a = 2q_1 \rightarrow \text{Even}$$

$$a = 2q_1 + 1 \rightarrow \text{Odd}$$

# Every composite no. can be factorised as a product of prime nos in a unique way (apart from order)

(8) Can  $4^n$  end with a zero? (where  $n$  is a natural no.)

$$\rightarrow 4^n = 2^{2n}$$

It cannot end with zero as it has only factor 2.

⑨ Find LCM & HCF of 6 & 20 using method of prime factorisation.

$$\rightarrow 6 = \underline{2} \times 3$$

$$20 = \underline{2} \times 2 \times 5$$

\* HCF is the product of smallest powers of each common prime factor of the numbers.

$$\therefore \text{HCF} = 2$$

\* LCM is the product of the greatest power of each prime factor involved of the numbers.

$$\therefore \text{LCM} = 2 \times 2 \times 3 \times 5 \\ = 60$$

#  $\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$  (Product of two nos.)

#  $\text{HCF} \times \text{LCM} = \text{Product of two numbers}$

⑩ Find the HCF of 96 and 404 using prime factorisation and use it to find their LCM.

$$\rightarrow 96 = 32 \times 3 \\ = 2^5 \times 3$$

$$404 = 2^2 \times 101$$

$$\therefore \text{HCF} = 2^2 = 4$$

HCF  $\times$  LCM = Product of two numbers

$$2^2 \times \text{LCM} = 96 \times 404$$

$$\text{LCM} = \frac{96 \times 404}{4}$$

4

$$\text{LCM} = 9696$$

(11) Find the LCM & HCF of 12, 15, 21 by prime factorisation.

$$\rightarrow 12 = 2^2 \times 3$$

$$15 = 3 \times 5$$

$$21 = 3 \times 7$$

$$\text{HCF} = 3$$

$$\text{LCM} = 2^2 \times 3 \times 5 \times 7$$

$$= 420$$

(12) Find LCM & HCF of 92 & 510 by prime factorisation

$$\rightarrow 92 = 2^2 \times 23$$

$$510 = 2 \times 3 \times 5 \times 17$$

$$\therefore \text{HCF} = 2$$

$$\text{LCM} = 2^2 \times 3 \times 5 \times 17 \times 23$$

$$= 23460$$

## \* Irrational Number

A number is called irrational, if it cannot be expressed as  $p/q$ , where 'p' & 'q' are integers and  $q \neq 0$

# Theorem 8- If 'p' be a prime number

If 'p' divides  $a^2$ , then 'p' will divide 'a' also, 'a' is a positive integer.

(13) Prove that  $\sqrt{2}$  is irrational.

→ By using Method of Contradiction

Assume  $\sqrt{2}$  is a rational number.

$$\sqrt{2} = a/b \quad (b \neq 0)$$

$$b\sqrt{2} = a \quad (a \text{ \& } b \text{ are co-primes})$$

(H.C.F. as 1)

Squaring both the sides

$$(b\sqrt{2})^2 = a^2$$

$$2b^2 = a^2 \quad (2 \text{ divides } a)$$

$$\text{Let } a = 2k \quad (k \text{ is an integer})$$

$$2b^2 = 4k^2$$

$$b^2 = 2k^2 \quad (2 \text{ divides } b)$$

∴ a & b have 2 as a common factor

∴ Our assumption is wrong.

Hence,  $\sqrt{2}$  is an irrational number.

\* Any real number which has a decimal expansion that terminates can be expressed as a rational number whose denominator is a power of 10.

\* Rational number =  $p/q$

where,  $q = 2^n \times 5^m$  and 'n', 'm' are non-negative integers

\* If 'x' is a rational number whose decimal expansion terminates, then 'x' can be written in the form  $p/q$  ( $p, q$  are co-primes) and the prime factorisation of 'q' will be of the form  $2^n \times 5^m$ , where 'n' & 'm' are non-negative integers.

\* If the rational no.  $x = p/q$  ( $q \neq 0$ ;  $p, q$  are co-primes) such that the prime factorisation of 'q' is of the form of  $2^n \times 5^m$  or either only  $2^n$  or  $5^m$  where 'n' & 'm' are non-negative integers, then 'x' will have decimal expansion which terminates.

\* The square of natural number can never end on 2, 3, 7, or 8.



If  $q = 2^n \times 5^m$  the decimal expansion will terminate.

$$\text{For e.g. } = 3/8 = 3/2^3 = 0.375$$

$$1/8 = 1/2^3 = 0.125$$

$$875/10000 = 875/2^4 \times 5^4 = 0.0875$$

If  $q \neq 2^n \times 5^m$  the decimal expansion will not terminate repeating.

$$\text{For e.g. } = 1/7 = 0.\overline{142857}$$

(14) Without using long division, check if the given rational numbers are terminating or recurring.

$$(a) 13/3125 = p/q$$

$$\text{To verify } = 3125 = 2^n \times 5^m$$

$$q = 3125 = 5^5 \times 2^0$$

∴ Its terminating

we have,

$$23 \overline{) 1056(45}$$

$$\underline{92}$$

$$136$$

$$\underline{115}$$

$$21$$

$$\text{Require no.} = 23 - 21$$

$$= 2$$

(15) The least number must be added to 1056, so that the sum is completely divisible by 23 & 2.