Chapter - 1 Electric Charges and Fields

Electric Charge - The physical property of matter that causes it to experiance force when placed in an electromagnetic field is called charge.

* Electric charge is a scalar quantity.
* There are 2 kinds of charges
  i) Positive charge ii) Negative charge
* An object can attend a positive charge by loosing electron. while another attend negative charge by gaining electron.
* Like charges repel eachother while unlike charges attrack eachother.
* An object can be charged by different method like; friction, conduction and Induction etc.

Conductor and Insulators - Those substances which can be used to carry electric charge from one point to another and allow electricity to pass through them is called conductor.
for ex- Fe, Cu, Ag, Al etc.

Those substances which can not conduct electricity are called insulators or diectric.
for ex- Glass, rubber, wood etc.

Basic Properties of Electric Charges

① **Additive Nature of Charges** - Electric charge is additive in nature it means if a system consists

of n charges $q_1, q_2, q_3 \ldots\ldots q_n$ then the total charge of the system will be $q_1+q_2+q_3 \ldots\ldots q_n$
In order to calculate the net charge on a system we have to just add algebraically all the charges present in the system this is known as the principle of Superposition of charges.

② Conservation of electric charge- During any process the net electric charge of an isolated system remain constant it means charge can neither be created nor be distroyed but it may get transfered from one part of system to another this is called conservation of electric charge.

③ Quantisation of electric charge- The charge on any body can be express as an integral multiple of basic unit charge e this is called quantisation of electric charge. It can be written as—

$$[q = \pm ne]$$

where $n = 1, 2, 3, 4$

The SI unit of charge is coloumb and It is denoted by C.
The value of unit charge $[e = 1.6 \times 10^{-19} C]$

Coulomb's Law - According to coulomb's law the electric force between two charges $q_1 \overset{r}{\longleftrightarrow} q_2$ is directly proportional to the product of magnitude of charges and inversely proportional to the square of distance between the two charges.

Let two charges q_1 & q_2 are separated by a distance r in vaccum then According ~~cot~~ coulomb law electric force between these two charges.

$$① \quad F \propto q_1 q_2$$

$$② \quad F \propto \frac{1}{r^2}$$

$$\Rightarrow F \propto \frac{q_1 q_2}{r^2}$$

$$\Rightarrow F = K \frac{q_1 q_2}{r^2} \quad -①$$

where K is constant.

The value of K depends upon the unit of charge, distance & force.

If force is in a Newton, charge in coloumb & distance in metre. The value of $K = 9 \times 10^9 \, N\text{-}m^2/C^2$

The constant k is usually equal to $\frac{1}{4\pi\epsilon_0}$

therefore from - (1)

$$\left[F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \right]$$

ϵ_0 is called permitivity of space.

[The value of $\epsilon_0 = 8.85 \times 10^{-12} C^2/N\text{-}m^2$]

If the charges of placed in an dielectric medium like wood, paper, oil etc. Then the electric force b/w these two charges —

$$\left[F = \frac{1}{4\pi\epsilon_0 K} \frac{q_1 q_2}{r^2} \right]$$

where K is dielectric constant of medium

$\epsilon_0 K = \epsilon$

where ϵ is permitivity of medium.

The value of $K > 1$ (for medium)

For air or vacuum $K = 1$

Comperision of electrostatic force and gravitational force.

$e = -1.6 \times 10^{-19} C$

$P = +1.6 \times 10^{-19} C$

$m_e = 9.1 \times 10^{-31} kg$

$m_p = 1.67 \times 10^{-27} kg$

$$\overset{e^-}{q_{1E}} \text{ —— } r \text{ —— } \overset{+p}{q_2}$$

According to Coulomb's law

$$F_E = \left(\frac{1}{4\pi\epsilon_0}\right) \frac{q_1 q_2}{r^2} = \frac{9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{r^2}$$

$$= \frac{23.04 \times 10^{-29}}{r^2}$$

According to Newton's law of gravitational force between these two charges.

$$F_m = \frac{G m_1 m_2}{r^2} = \frac{6.67\times10^{-11}\times9.1\times10^{-31}\times1.6\times10^{-27}}{r^2}$$

$$= \frac{97.11\times10^{-69}}{r^2}$$

$$\therefore \frac{F_e}{F_m} = \frac{\frac{23.04\times10^{-29}}{\cancel{r^2}}}{\frac{97.11\times10^{-69}}{\cancel{r^2}}}$$

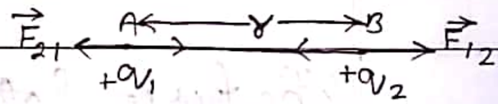
$$\frac{F_e}{F_m} = 10^{40}$$

$$\boxed{F_e \simeq 10^{40} F_m}$$

Therefore, the electrostatic force between two charge ~~par~~ particle will be more greater than gravitational force between these particles.

Coulomb's Law in Vector form

By coulomb's Law



$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{21}$$

so

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

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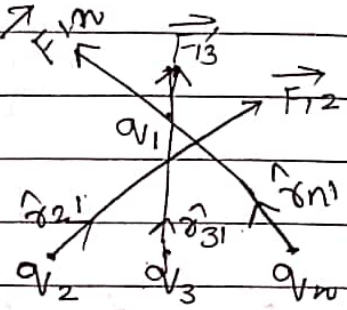
Force between multiple charges - (Super position Theory)

According to this principle forces on any charge due to number of other charges is the vector sum of all the force on that charge due to other charges.
The individual forces can not effected due to the presence of other charges

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21} + \frac{q_1 q_2}{r_{31}^2}\hat{r}_{31} + \cdots + \frac{q_1 q_n}{r_{n1}^2}\hat{r}_{n1}\right]$$



Total force on charge q_1 -

$$\vec{F} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \cdots + \vec{F}_{1n}$$

Electric field

The force that a unit positive charge would experiance if placed at that point is called the electric field due to a charge.

The intensity of Electric charge-

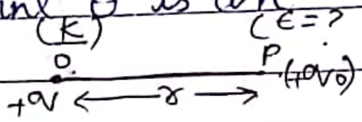
The force experiance per unit positive charge placed in an electric field is called intensity of electric field.
It is denoted by E

$$\left[\vec{E} = \frac{\vec{F}}{q_0}\right]$$

The unit of intensity of E.F is N/C.
dimensional formula $[MLT^{-3}A^{-1}]$

The intensity of Electric field due to a charge.

Let a +q charge is placed at point O is an dielectric media whose dielectric constant is K. There is a point P at distance r from O whe have to find the intensity of electric field at point P.



Let us place a testing positive charge $+q_0$ on point P. the force b/w charges

$$F = \frac{1}{4\pi\epsilon_0 K}\frac{q q_0}{r^2}$$

There fore the intensity of e.f. at point P.

$$E = \frac{F}{q_0} = \frac{\frac{1}{4\pi\epsilon_0 K}\,\cancel{q_0} q / r^2}{\cancel{q_0}}$$

$$\left[E = \frac{1}{4\pi\epsilon_0 K}\frac{q}{r^2}\right]$$

for Air $K = 1$

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$\left[E \propto \frac{1}{r^2}\right]$$

The direction of E.F due to positive charge will be away from the charge and due to negative charge that will be towards the charge.

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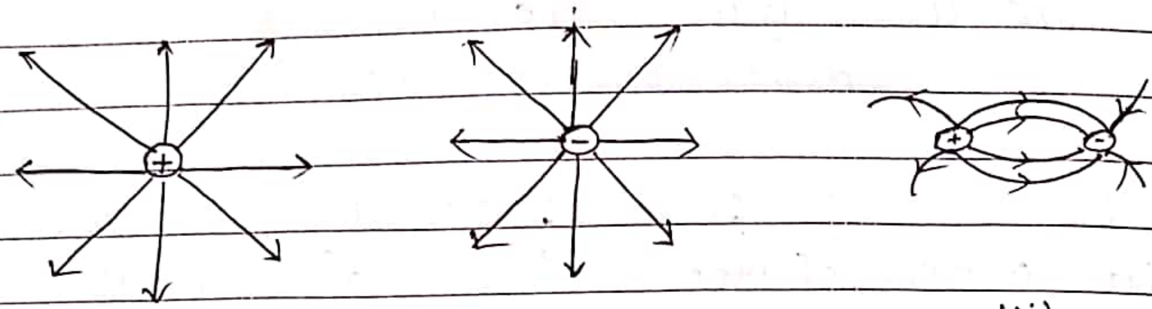
Electric Field Lines



i) ii) iii)

i) Electric field lines is a curve drawn in such a way that tangent to it at each point represent the direction of electric field at that point. Basically E.F.L are used to determine the electric field around the charge or configuration of charges.
The E.F.L due to an isolated positive charge will be away from the charge.
The electric field lines due to a negative charge will be towards the charge.

Properties of Electric Field Lines

1- E.F.L. Start from positive charge and end at negative charge.
2- Tangent to any point on E.F.L. shows the direction of electric field at that point.
3- Two field lines Can never bisect eachother because if they bisect eachother there will be two tangent at bisecting point this shows that there will be two direction of E.F. which is not possible.

4. The E.F.L donot form close loops.
5. E.F.L. are $\perp$ to the surface of a charge conductor.

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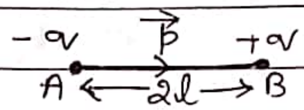
Dipole (electric)

A device in which two equal and opposite charges are separated by a small distance is called electric dipole



Electric Dipole Moment

The product of charge and the distance b/w the charge is called electric dipole moment.
It is denoted by p

$$[\vec{p} = q \times 2l]$$

Electric dipole moment is a vector quantity.
The direction of p is from ~~-q~~ negative to positive charge.
The SI unit of dipole moment is C-m
And dimensional formula dipole moment is [LTA]

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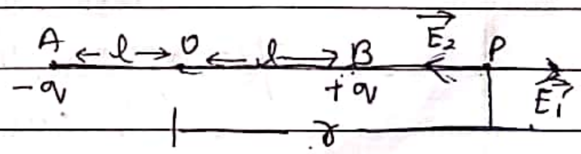
Electric field Intensity Due to an Electric dipole



The intensity of E.F at P due to q charge.

$$E_1 = \frac{1}{4\pi\epsilon_0 K} \frac{q}{(r-l)^2} \quad \text{(towards BP)}$$

The intensity of E.F at P due to $-q$ charge

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$E_2 = \frac{1}{4\pi\epsilon_0 k} \frac{q}{(r+l)^2}$ (towards PB)

The resultant intensity of Electric Field at P

$E = E_1 + E_2$

$= \frac{q}{4\pi\epsilon_0 K}\left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right]$

$= \frac{q}{4\pi\epsilon_0 K}\left[\frac{\cancel{r^2} + \cancel{l^2} + 2rl - \cancel{r^2} - \cancel{l^2} + 2rl}{(r^2 - l^2)^2}\right]$

$E = \frac{1}{4\pi\epsilon_0 K} \frac{2(2ql)r}{\cancel{r^4}\, r^3}$ $\because l << r$, $l^2 =$ Negligible

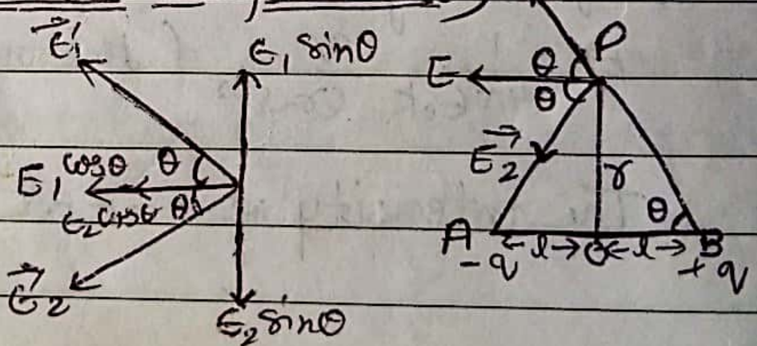
$$\boxed{E = \frac{1}{4\pi\epsilon_0 K} \frac{2p}{r^3}} \quad (\because p = 2ql)$$

For Air $K = 1$

$$\boxed{E = \frac{1}{4\pi\epsilon_0} \frac{2p}{r^3}}$$

The direction of resultant E.F will be from negative charge ~~to~~ positive charge.

The intensity of Electric field due to a dipole at a point on the equitorial line (Broadside on position)



$BP = AP \sqrt{r^2 + l^2}$

The intensity of E.F at P due to +q charge

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$$E_1 = \frac{1}{4\pi\epsilon_0 K}\frac{q}{PB^2} = \frac{1}{4\pi\epsilon_0 K}\frac{q}{(r^2+l^2)}$$

Similarly the intensity of electric field at P due to $-q$ charge

$$E_2 = \frac{1}{4\pi\epsilon_0 K}\frac{q}{(r^2+l^2)}$$

The resultant intensity of electric field due to dipole at P.

$$E = E_1\cos\theta + E_2\cos\theta$$

$$= 2\times\frac{1}{4\pi\epsilon_0 K}\frac{q}{(r^2+l^2)}\cos\theta$$

In ΔOPB

$$\cos\theta = \frac{l}{(r^2+l^2)^{1/2}}$$

Therefore

$$E = \frac{2}{4\pi\epsilon_0 K}\frac{q}{(r^2+l^2)}\times\frac{l}{(r^2+l^2)^{1/2}}$$

$$E = \frac{1}{4\pi\epsilon_0 K}\frac{(2ql)}{(r^2+l^2)^{3/2}} \quad \because l \ll r \quad \because l^2 = \text{Negligible}$$

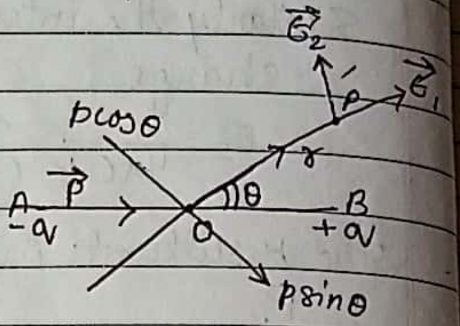
$$\boxed{E = \frac{1}{4\pi\epsilon_0 K}\frac{p}{r^3}} \quad \because 2ql = p$$

For Air $K=1$

$$\left[E = \frac{1}{4\pi\epsilon_0} = \frac{p}{r^3}\right]$$

"The direction of E is parallel to the axis dipole from positive charge to negative charge"

Electric field intensity at any point due to an electric dipole



The intensity of E.F at P due to $p\cos\theta$ component

$$E_1 = \frac{1}{4\pi\epsilon_0 K}\frac{2p\cos\theta}{r^3}$$

The intensity of E.F at P due to $P\sin\theta$ component

$$E_2 = \frac{1}{4\pi\epsilon_0 K}\frac{P\sin\theta}{r^3}$$

The resultant intensity of Electric Field at P

$$E = \sqrt{E_1^2 + E_2^2} = \frac{1}{4\pi\epsilon_0 k}\frac{P}{r^3}\sqrt{4\cos^2\theta + \sin^2\theta} \quad \because \sin^2\theta = 1-\cos^2\theta$$

$$E = \frac{1}{4\pi\epsilon_0 K}\frac{p}{r^3}\sqrt{4\cos^2\theta + 1 - \cos^2\theta}$$

$$\boxed{E = \frac{1}{4\pi\epsilon_0 K}\frac{p}{r^3}\sqrt{3\cos^2\theta + 1}}$$

i) For axial position

$\cancel{E = \frac{}{4}}$ $\theta = 0$

$$E = \frac{1}{4\pi\epsilon_0 K}\frac{p}{r^3}\sqrt{3\cos^2 0 + 1}$$

$$\boxed{E = \frac{1}{4\pi\epsilon_0 K}\frac{2p}{r^3}}$$

ii) For equitorial position

$\theta = 90^\circ$

$$E = \frac{1}{4\pi\epsilon_0 K}\frac{P}{r^3}\sqrt{3\cos^2 90 + 1}$$

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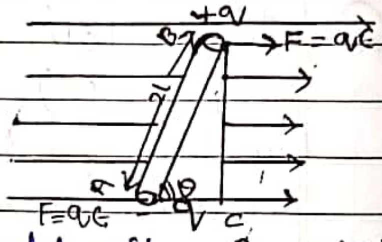
$$\boxed{E = \frac{1}{4\pi \epsilon_0 K} \frac{p}{r^3}}$$

Electric dipole in an uniform external fied.

Torque on Electric Dipole in uniform Electric field.

Let an electric dipole AB of length 2l is situated in an electric field E at ∠θ two forces act upon the charges will be equal, parallel and opposite therefore, they consitute a couple



BC = 2l sin θ

$$F = qE$$

The Torque — τ = force × Perpendicular Distance b/w the force

$$\tau = qE \times 2l \sin\theta$$

$$\tau = (2ql) E \sin\theta$$

$$[\tau = pE\sin\theta]$$

The vector notation is

$$[\vec{\tau} = \vec{p} \times \vec{E}]$$

The unit of Torque is N-m

The dimensional formula of Torque $[ML^2T^{-2}]$

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① If $\theta = 0$
Then $[\tau = 0]$
In this position the dipole is in <u>stable equiliberium.</u>

② If $\theta = 90^\circ$
then τ will be maximum.
$[\tau_{max} = p \times E]$

③ If $\theta = 180^\circ$
$[\tau = 0]$
In this position the dipole is in <u>unstable equiliberium.</u>

Work done on a dipole in an Electric Field

When an electric dipole is placed in an uniform e.F. it experiences ~~torque~~ and tends ~~to altpe~~ alting it in such a way to attend stable equiliberium.

Small amount of work done in rotating the dipole through a small angle $d\theta$ against the torque is give by.

$$\tau = pE \sin\theta.$$

Word done in rotating dipole by small angle θ,

$$dw = \tau \times d\theta$$

If the dipole is rotated from θ_1 to θ_2

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the work done -

$$W = \int_{\theta_1}^{\theta_2} dW = \int_{\theta_1}^{\theta_2} \tau\, d\theta = \int_{\theta_1}^{\theta_2} p \sin\theta\, d\theta$$

$$W = pE \int_{\theta_1}^{\theta_2} \sin\theta\, d\theta$$

$$\therefore \int \sin\theta\, d\theta = \cos\theta$$

$$W = pE\,[-\cos\theta]_{\theta_1}^{\theta_2}$$

$$W = pE\,[-\cos\theta_2 + \cos\theta_1]$$

$$\boxed{W = pE\,[\cos\theta_1 - \cos\theta_2]}$$

This work done is stored in the form of energy which is called the Potential Energy of dipole

$$\therefore \boxed{U = pE\,[\cos\theta_1 - \cos\theta_2]}$$

If $\theta_1 = 90°$

$\theta_2 = \theta$.

Then Potential Energy

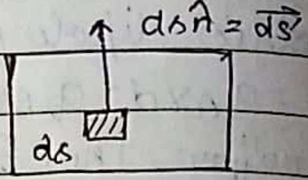
$$U = pE\,[\cos 90° - \cos\theta]$$

$$\boxed{U_0 = -pE\cos\theta}$$

Gauss's Theorem

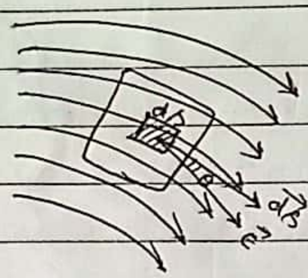
Area Vector – The vector associated with every area element of a closed surface is taken to ~~the~~ be in the direction of normal to the area is called area vector.

for ex.



$\hat{n}$ = unit vector

Electric flux –



Total number of E.F.L passes through the Surface is called electric flux linked with that Surface.

If $d\phi$ is a small electric flux through a small area ds due to an electric field, E at an angle θ then –

$$\left[d\phi = \vec{E}\cdot\vec{ds} = E\,ds\cos\theta \right]$$

The total electric flux link with the whole surface s.

$$\left[\phi = \oint_s \vec{E}\cdot\vec{ds} = \oint_s E\,ds\cos\theta \right]$$

$\oint_s$ = Surface Integration.

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① If $0 < \theta < 90^\circ$
then $\phi = +ve$.

② If $\theta = 90^\circ$
Then $\phi = 0$

③ If $90 < \theta < 180^\circ$
Then $\phi = -ve$.

The unit of Electric flux $= [Nm^2 - C^{-1}]$

The dimensional formula of $\phi = [ML^3T^{-3}A^{-1}]$

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Gauss's Theorem

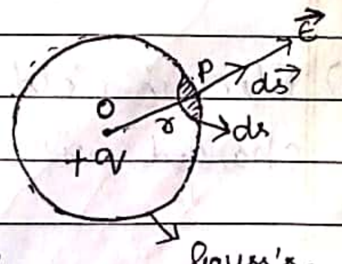
According to this theorm the total electric flux linked with a surface $= \frac{1}{\epsilon_0}$ time of charge bounded by that surface i.e.

$$\left[\phi = \frac{q}{\epsilon_0}\right]$$

$$\because \phi = \oint_S E ds \cos\theta$$

so, $$\left[\oint_S E ds \cos\theta = \frac{q}{\epsilon_0}\right]$$

Proof



The electric flux linked with small area ds

$$d\phi = \vec{E}.\vec{ds} = E\,ds\cos\theta = E\,ds$$

The total electric flux linked with Gauss's Surface

$$\phi = \oint_s E\,ds = E \oint_s ds$$

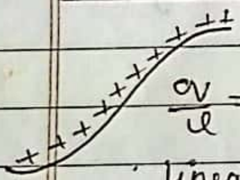
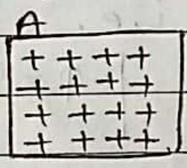
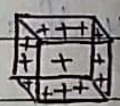
$$\because \quad E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \quad \text{and} \quad \oint_s ds = 4\pi r^2$$

$$\phi = \frac{1}{\cancel{4\pi}\epsilon_0}\frac{q}{\cancel{r^2}} \times \cancel{4\pi r^2}$$

$$\left[\phi = \frac{q}{\epsilon_0}\right] \text{ Proved}$$

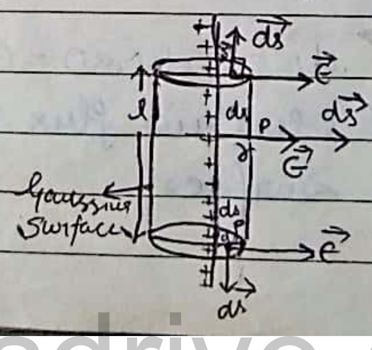
This is Gauss's Theorem.

Distribution of Charge —

Linear distribution	Surface distribution	Volume distribution
		
$\frac{q}{l} = \lambda$ — linear charge density	$\frac{q}{A} = \sigma$ — Surface Charge density	$\frac{q}{V} = \rho$ — Volume charge density
unit = C/m	unit = C/m²	unit = C/m²

Application of Gauss's Theorem

① Electric Field due to an infinately long thin straight charged wire.



Length = l
Radius = r

Total flux linked with Gauss's ~~F~~ Surface

$$\phi_E = \oint_S \vec{E}\cdot\vec{ds}$$

$$\phi_E = \underbrace{\oint_{S_{upper}} \vec{E}\cdot\vec{ds}}_{0} + \underbrace{\oint_{S_{lower}} \vec{E}\cdot\vec{ds}}_{0} + \oint_{S_{side}} \vec{E}\cdot\vec{ds}$$

$$\phi_E = \oint_{side} \vec{E}\cdot\vec{ds}$$

$$\phi_E = \oint_S E\,ds\cos\theta$$

$$\phi_E = E\oint_S ds \qquad \because \oint_S ds = 2\pi r l$$

$$\phi_E = E\times 2\pi r l \quad \text{--- (1)}$$

According to Gaussius theorem

$$\phi_E = \frac{q}{\epsilon_0} \quad \text{--- (2)}$$

from (1) & (II)

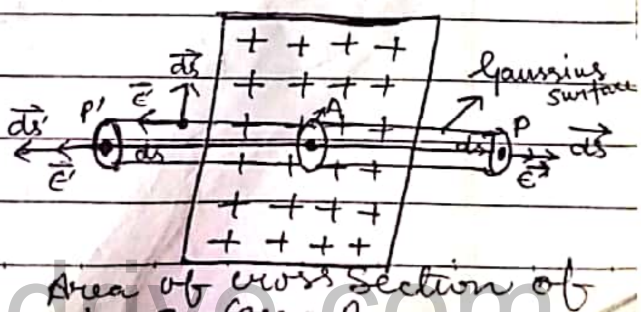
$$E\times 2\pi r l = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{2\pi\epsilon_0 r l} \qquad \therefore \frac{q}{l} = \lambda \text{ (linear charge density)}$$

$$\boxed{E = \frac{\lambda}{2\pi\epsilon_0 r}}$$

(2) ~~Elect~~ Electric field due to a thin infinite plane Sheet of charge.

Let us the charge density of plate σ



Area of cross Section of Gaussius surface = A

The total electric flux linked with Gaussius Surface.

$$\phi_E = \oint_S \vec{E}\cdot\vec{ds}$$

$$\phi_E = \underbrace{\oint_S \vec{E}\cdot\vec{ds}}_{\text{Left Surface}} + \underbrace{\oint_S \vec{E}\cdot\vec{ds}}_{\text{Right Surface}} + \underbrace{\oint_S \vec{E}\cdot\vec{ds}}_{\text{Curved Surface}}$$

$$\phi_E = \oint_S E\, ds \cos 0^\circ + \oint_S E\, ds \cos 0^\circ + \oint_S E\, ds \underbrace{\cos 90^\circ}_{=0}$$

$$\phi_E = E\oint_S ds + E\oint_S ds$$

$$\phi_E = EA + EA \qquad \because \oint_S ds = A$$

$$\phi_E = 2EA \quad \text{—} (1)$$

According to Gaussius Theorem.

$$\phi_E = \frac{q}{\epsilon_0} \quad \text{—} (2)$$

from eqn (1) & (2)

$$2EA = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{2\epsilon_0 A} \rightarrow \left[E = \frac{\sigma}{2\epsilon_0}\right] \quad \because \frac{q}{A} = \sigma$$

$$\cancel{\because \frac{q}{A} = \sigma}$$

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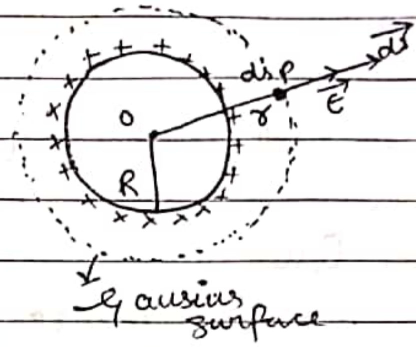
③ Electric field due to a uniformaly charged thick spherical ~~solid~~ shell

(i) At point P outside the charged spherical shell $(r > R)$



$$\phi_E = \oint_S \vec{E} \cdot \vec{ds} = \oint_S E \, ds \cos 0$$

$$\phi_E = E \oint_S ds$$

$$\phi_E = E \times 4\pi r^2 \quad \text{—} ① \quad \because \oint_S ds = 4\pi r^2$$

According to Gausius theorem

$$\phi_E = \frac{q}{\epsilon_0} \quad \text{—} ②$$

from eq ① & ②

$$E \times 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\boxed{E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}}$$

(ii) At ~~point~~ Surface of charged shell $[r = R]$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2}$$

$$E = \frac{1}{\epsilon_0} \times \frac{q}{(4\pi R^2)}$$

$$\boxed{E = \frac{\sigma}{\epsilon_0}} \quad \because \sigma = \frac{q}{4\pi R^2}$$

(vi) Inside the charged shell -

$r < R$

$$\begin{bmatrix} V = 0 \\ E = 0 \end{bmatrix}$$