Chapter - 10 Wave Optics

Wave-Optics

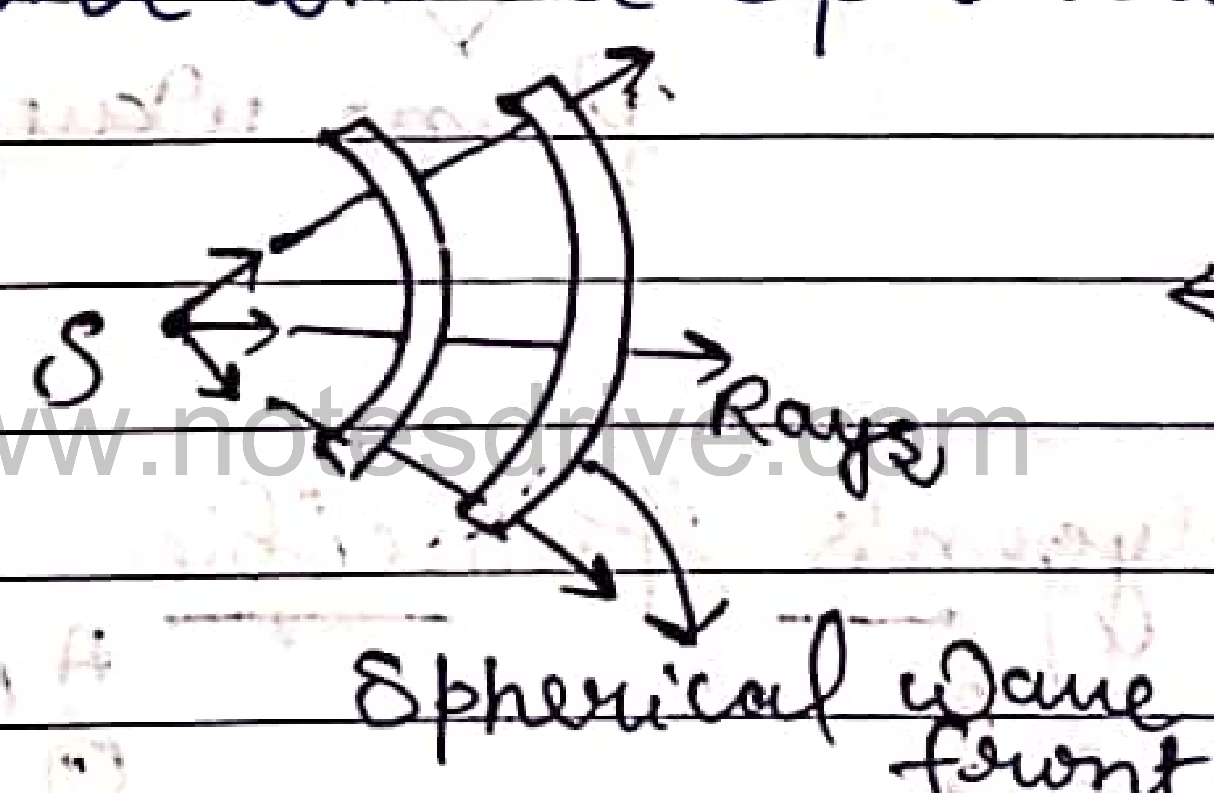
* Hygen's Principle

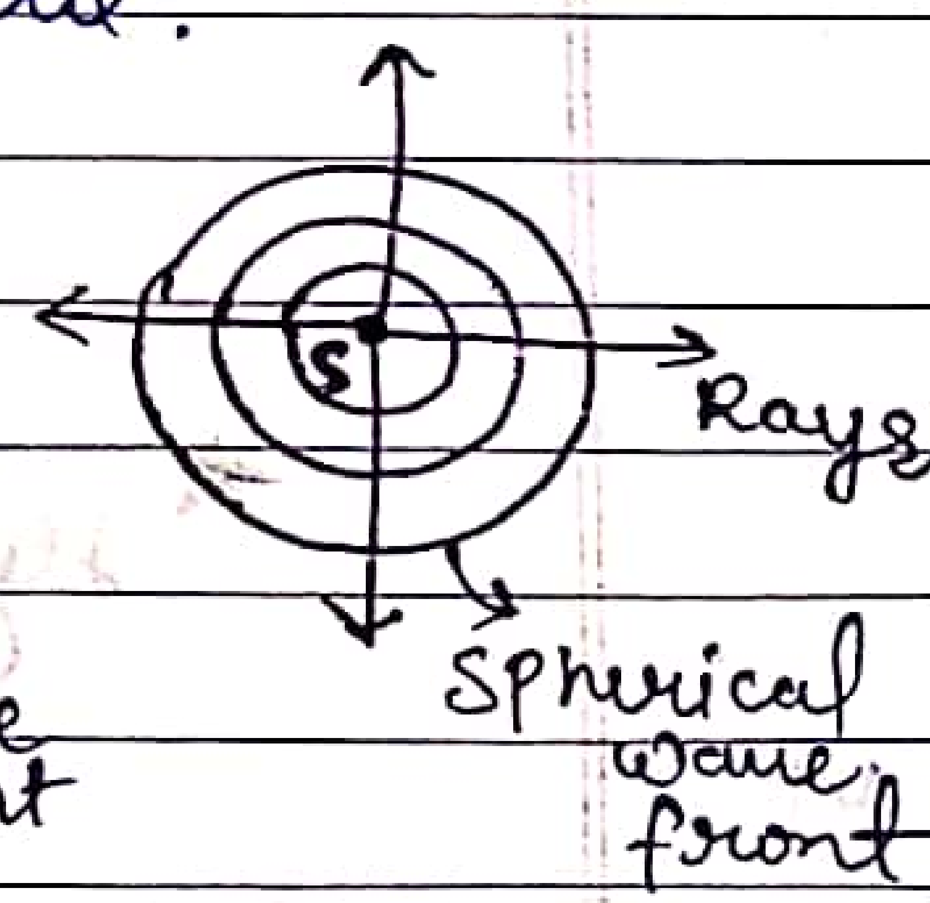
Wave front → A surface in any medium in which all the particles are same phase of oscillation is called wave front.

★ A line ⊥ to a wave front is called Ray.

★ Depending on the shape of source of light wave front can be classified in three form.

① Spherical wave front - When the source of light is point source the wave front will be spherical.

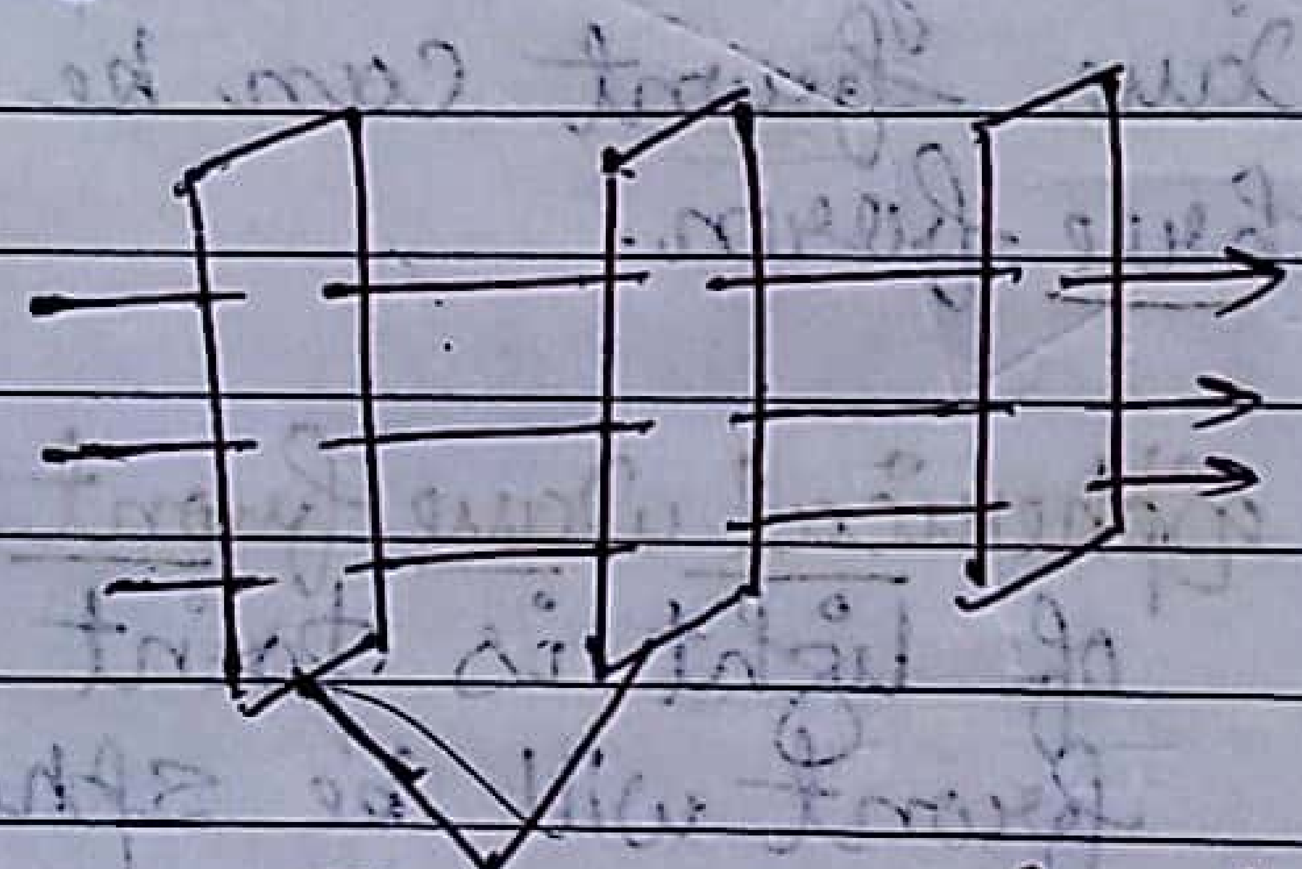




② Cylindrical wave front -

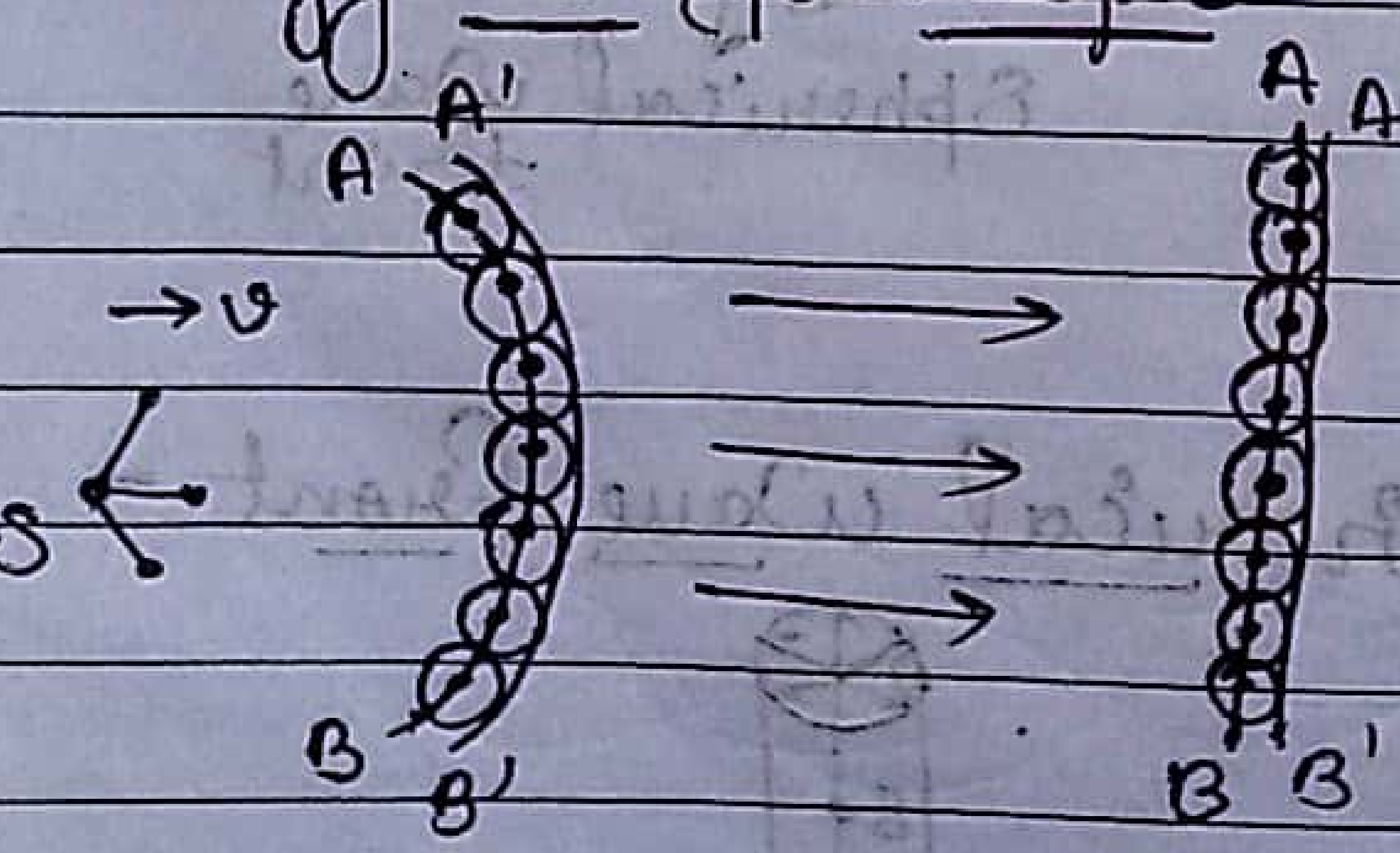
when the source of light is linear i.e. ~~is a~~ straight line ~~with be~~ source then all the point will be at same distance from the source and lies on a cylinder, therefore, the shape of wave front will be cylindrical

(3) Plane wave front – When the point source or linear source of light is at very large distance then the spherical or cylindrical wave front appears like a plane and it is called plane wave front.



Plane wave front

★ Hygen's Principle



* According to Hygen's Principle

1) Each point on a Given wavefront is the source of Secondary Wavelets.
And the Wavelets emminating from these point spread out in all directions with the speed of wave.

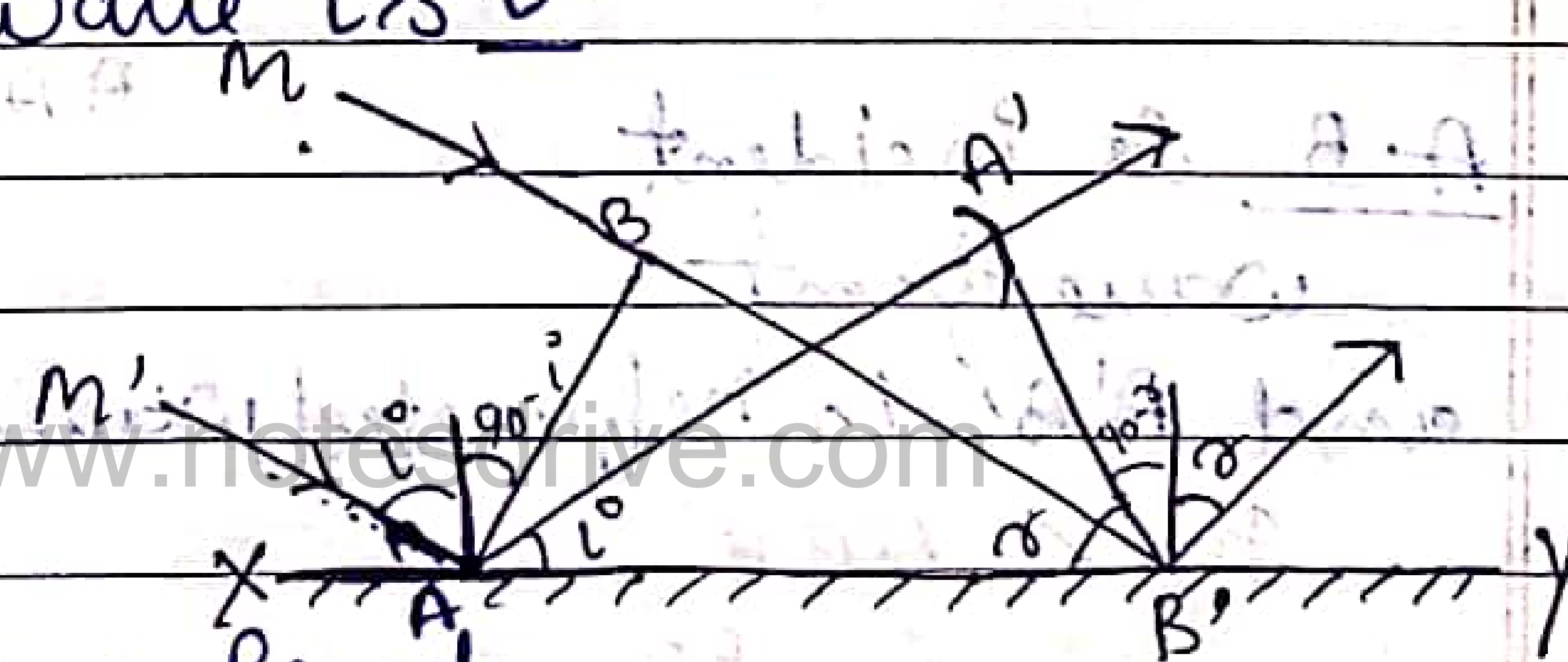
2) A Surface touching these Secondary wave tangitially in the forward direction at any time gives the new wave front i.e. called Secondary wave front.

Proving Law of Reflection by Hygen's principle

Let the velocity of wave is v

$BB' = vt$

$AA' = vt$



AB is incident wave front
and A'B' is Reflection of wave front

In Δ ABB' and AA'B'

$BB' = AA' - (vt)$

$\angle ABB' = \angle AA'B'$

AB' is common

$\Delta ABB' \cong AA'B'$

$\Rightarrow [\angle i = \angle r]$ This is the law of Reflection.

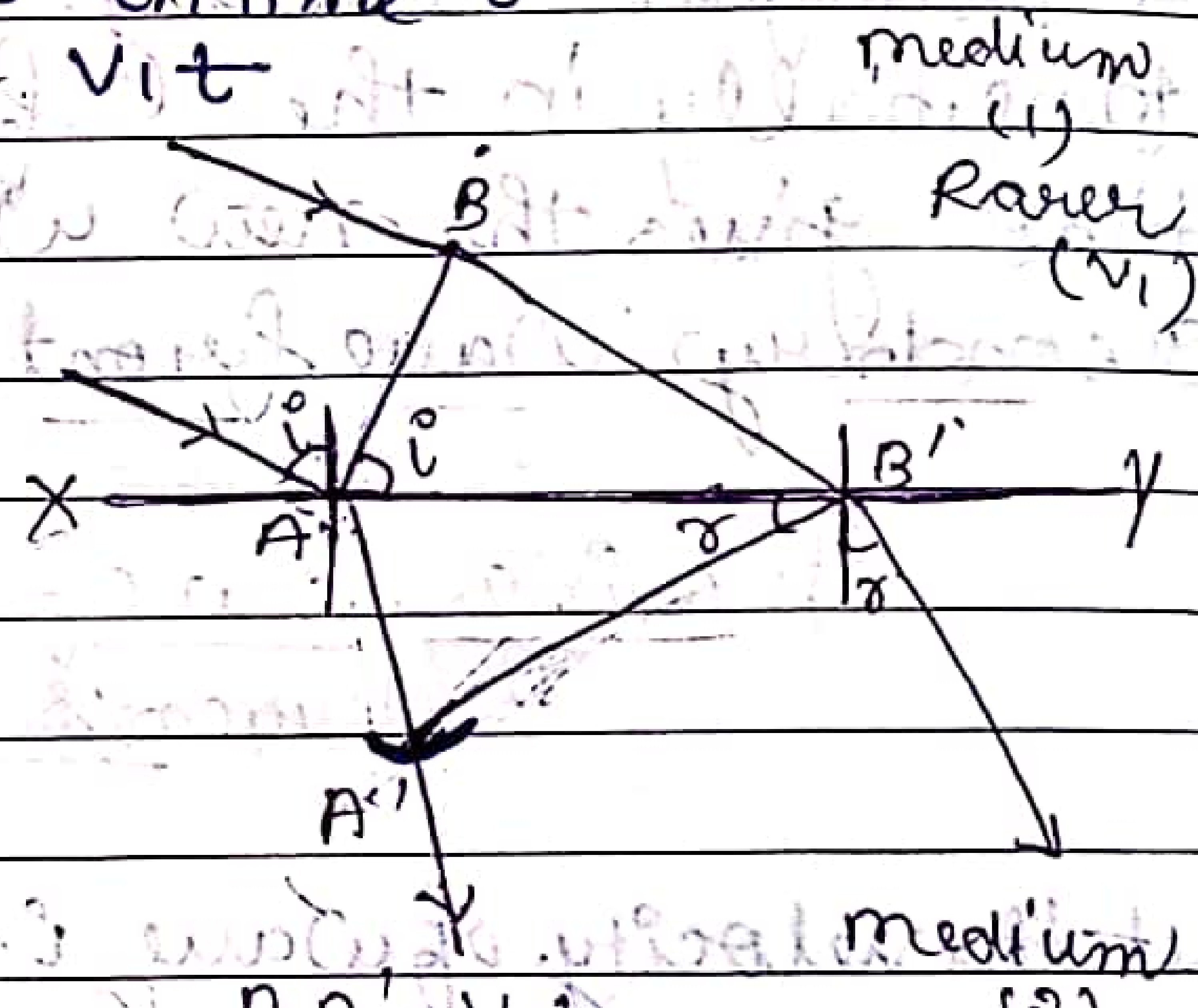
ii Since, the incident wavefront AB reflecting surface XY and reflected wavefront A'B' are all $\perp$ to the plane therefore incident rays, reflected rays and normal all lie in the same plane this is second law of reflection.

Proving the law of refraction by Hygen's Principle

Let B reached at B' in time t

$BB' = v_1 t$

Let velocity of wave in rarer medium ① is v_1 and in ② is v_2



$AA' = v_2 t$

☆ AB is incident wavefront and A'B' is refracted wavefront

In $\triangle ABB'$

$$\sin i = \frac{BB'}{AB'}$$

In $\triangle AA'B'$

$$\sin r = \frac{AA'}{AB'}$$

Therefore, $\dfrac{\sin i}{\sin r} = \dfrac{BB'/\cancel{AB'}}{AA'/\cancel{AB'}}$

$$\frac{\sin i}{\sin r} = \frac{BB'}{AA'} = \frac{v_1 \cancel{t}}{v_2 \cancel{t}} = \frac{v_1}{v_2} = \text{Constant}$$

$$\boxed{\frac{\sin i}{\sin r} = {}_1\mu_2}$$

This is the first law of refraction.

Interferance of light

Superposition Principle - According to this principle at a particular point in a medium the resultant displacement produce by number of waves is the vector sum of the displacement produced by each of wave.

For e.g.

Let two waves of displacement y_1 and y_2 travelling in a medium in same direction. Therefore at any point the resultant displacement

$$\boxed{y = y_1 + y_2}$$

Interferance of light waves

When two light waves of about equal frequency travel on same direction then the intensity of resultant wave does not remain uniform.

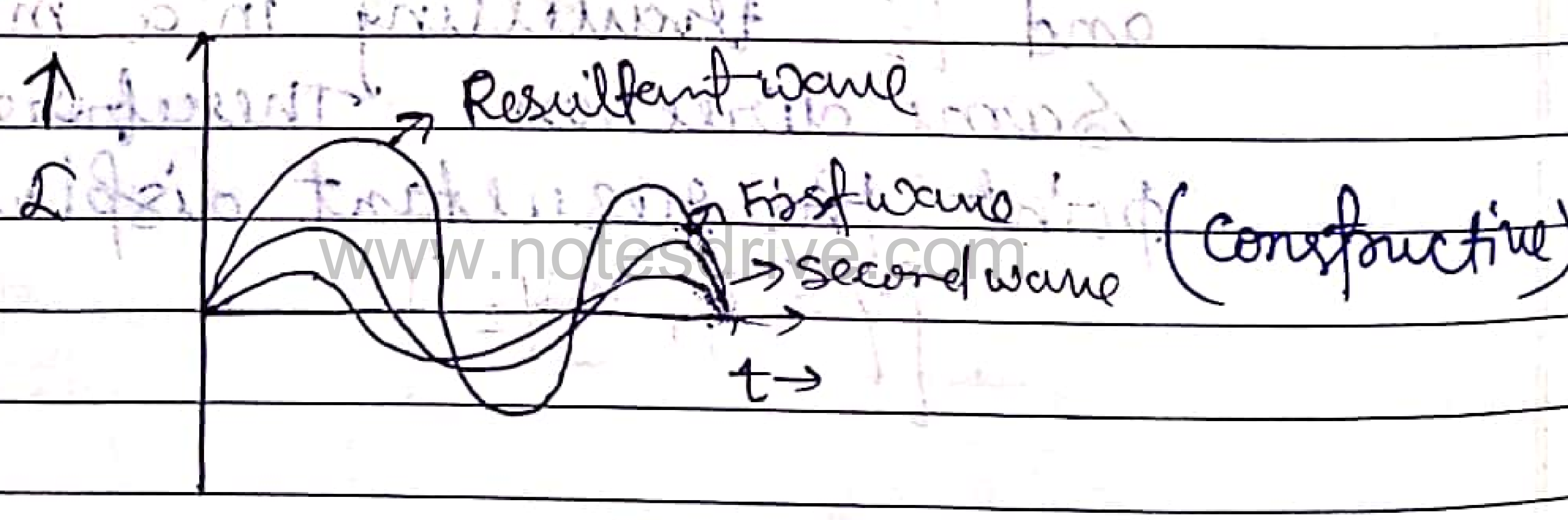
This phenomenon of formation of maximum intensity at some point and minimum intensity at some point is called the interferance of light wave

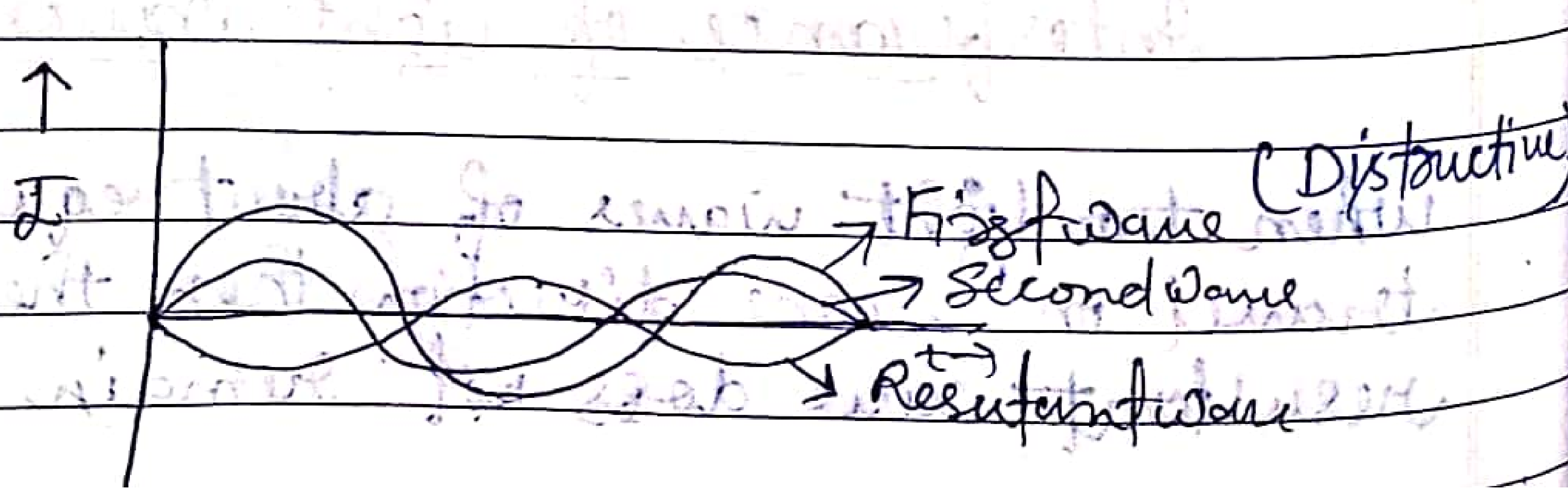
"The distribution of intensity of waves is called interferance."

☆ There are 2 type of interferance

① Constructive – At the points where the resultant intensity of light is max interferance is called constructive.

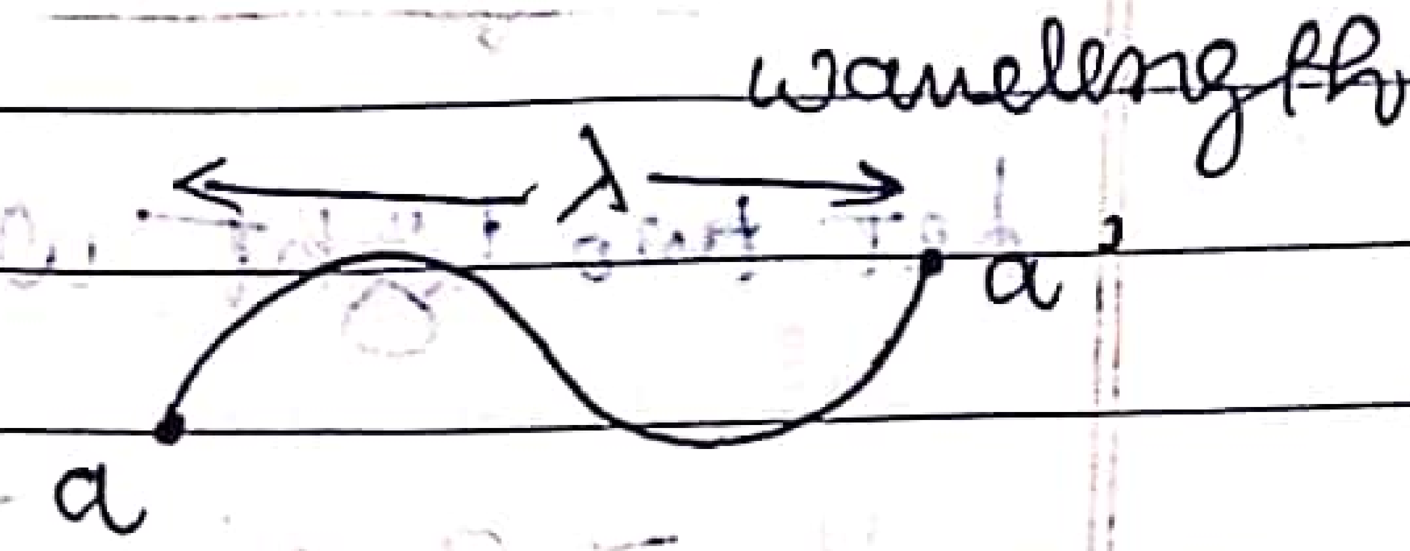
② Destructive – At the point where the resultant intensity of light is minimum the interferance is called destructive interferance.



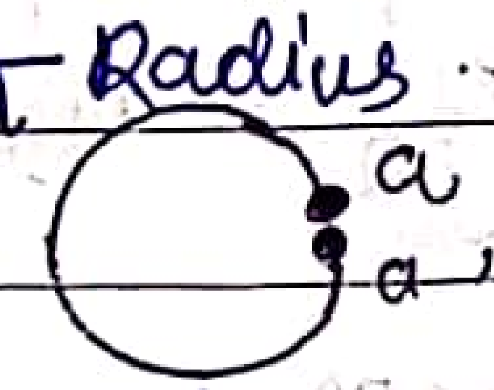


Phase difference

Path difference



Phase difference b/w a and a' = 2π



Path diff = λ

$$\phi = \frac{2\pi}{\lambda} x$$

$$\boxed{\text{Phase diff} = \frac{2\pi}{\lambda} \text{ path diff}}$$

eg

(1) If the phase diff b/w two vibrating particle is π

$$\cancel{\pi} = \frac{2\cancel{\pi}}{\lambda} \text{ path diff}$$

$$\boxed{\text{path diff} = \frac{\lambda}{2}}$$

(11) If phase diff = 2π

$$\cancel{2\pi} = \frac{\cancel{2\pi}}{\lambda} \text{ path diff}$$

$$\boxed{\text{path diff} = \lambda}$$

Interference b/w Two light Waves

Let two light wave are

$$y_1 = a_1 \sin(\omega t)$$
$$y_2 = a_2 \sin(\omega t + \phi)$$
Phase diff $= \phi$

According to superposition theory, the displacement of Resultant wave,

$$y = y_1 + y_2$$

$$y = a_1 \sin\omega t + a_2 \sin(\omega t + \phi)$$

$$y = a_1 \sin\omega t + a_2 [\sin\omega t \cos\phi + \cos\omega t \sin\phi]$$

$$y = a_1 \sin\omega t + a_2 \sin\omega t \cos\phi + a_2 \cos\omega t \sin\phi$$

$$y = (a_1 + a_2 \cos\phi) \sin\omega t + a_2 \sin\phi \cos\omega\phi$$

Let $a_1 + a_2 \cos\phi = A \cos\theta$ — (1)
$a_2 \sin\phi = A \sin\theta$ — (2)

$$y = A \cos \sin\omega t + A \sin\theta \cos\omega t$$

$$\boxed{y = A \sin(\omega t + \theta)}$$

This is the displacement equation of Resultant wave. Where A is the amplitude of Resultant wave.

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① ^{2} + ② ^{2}

$$A^2\sin^2\theta + A^2\cos^2\theta = a_1^2 + a_2^2\cos^2\theta + 2a_1a_2\cos\phi + a_2^2\sin^2\phi$$

$$A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$$

$$\left[A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}\right]$$

$$\left[I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi\right]$$

① <u>For Constructive Interference</u>

$$\cos\phi = +1$$
$$\Rightarrow [\phi = 2m\pi]$$

Where $m = 0, 1, 2, 3, 4$ ---

it means there ~~is~~ will be constructive interference when the phase difference b/w the waves will be ~~zero~~ $0, 2\pi, 4\pi, 6\pi$ ----

There will be Constructive interference where the path difference b/w Two waves will be $0, \lambda, 2\lambda, 3\lambda$ ----

$$[\text{path diff} = m\lambda]$$

Where $m = 0, 1, 2, 3$ ----

Therefore $I_{max} = I_1 + I_1 + 2\sqrt{I_1 I_2}$

$\cancel{I_{max} = (\sqrt{I_1} + \sqrt{\ })}$

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

② **For Destructive Interference**

$\cos\phi = -1$

$$\boxed{\phi = (2m-1)\pi}$$

Where $m = 1, 2, 3 - - -$.

$\phi = \pi, 3\pi, 5\pi, 7\pi - - - -$

path diff $= \frac{\lambda}{2\pi} \times$ phase diff

i) for π
path diff $= \lambda/2$

ii) for 3π
path diff $= 3\lambda/2$

iii) for $\phi = 5\pi$
path diff $= 5\lambda/2$

$$\boxed{\text{Path diff} = (2m-1)\lambda/2}$$

Where $m = 1, 2, 3, 4, - - -$.

Interference Pattern in light waves

Coherent and Incoherent Sources

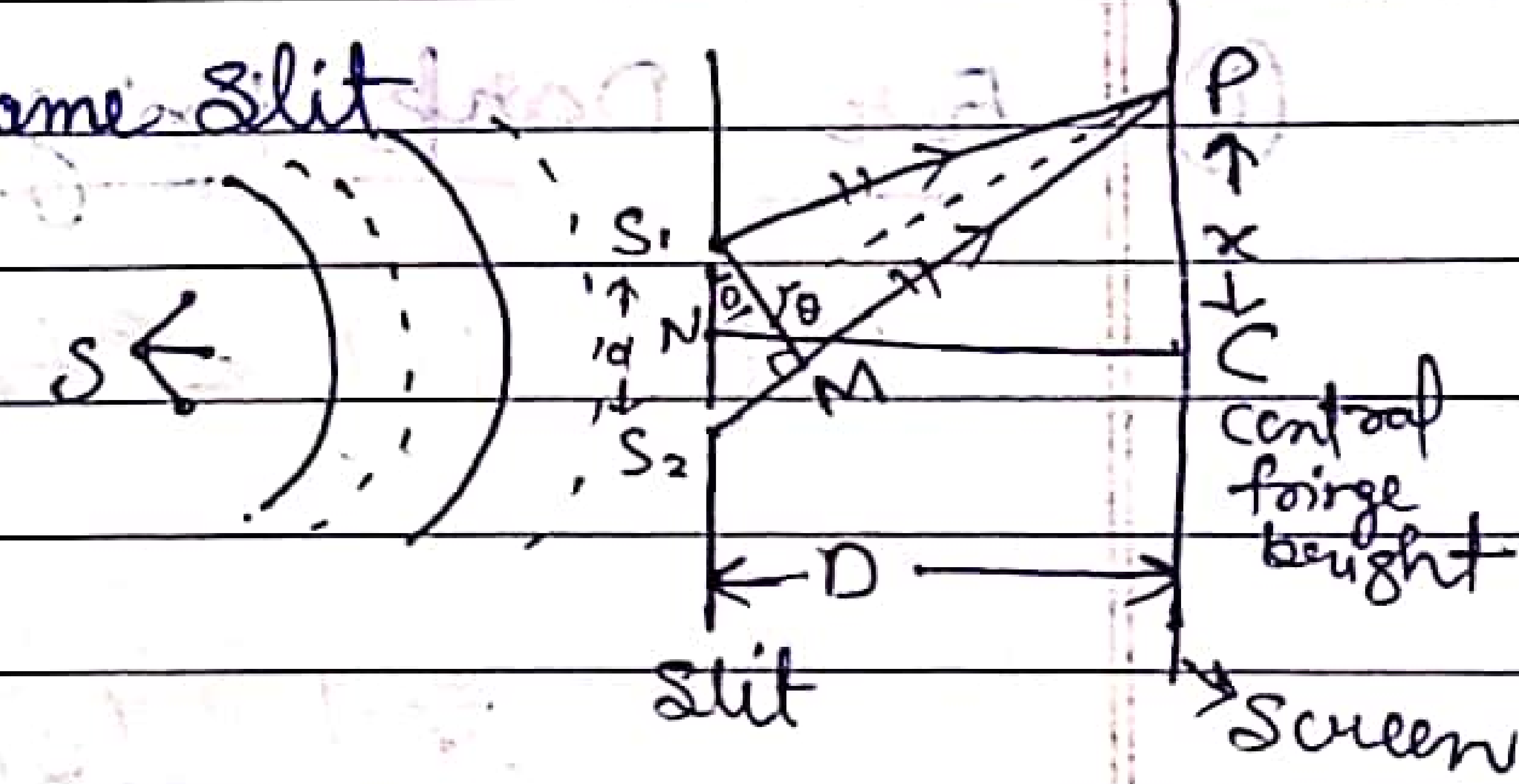
The Sources of light which emit light waves of same wavelength, same frequency and having constant phase difference are known as ~~cho~~ coherent sources.

Two such sources of light which do not emit light waves with constant phase diff. are called Incoherent sources.

Young's Double Slit Experiment

d = Width of Slit

D = Distance of screen frame slit



The ~~path~~ difference b/w two ~~path~~ Rays arriving at P

In $\Delta\ S_1 S_2 M$

$$\sin\theta = \frac{S_2 M}{S_1 S_2}$$

$$\sin\theta = \frac{S_2 M}{d} \quad -(I)$$

In Δ PNC

$$\tan\theta = \frac{PC}{NC} = \frac{x}{D} \quad -(II)$$

$\because$ θ is very very small.

$$\sin\theta \simeq \tan\theta$$

$$\frac{S_2 M}{d} = \frac{x}{D}$$

$$\text{Path diff} = S_2 M = \frac{xd}{D} \quad \text{--- (iii)}$$

(i) For Bright Fringes

$$\text{Path diff} = m\lambda$$

where $m = 0, 1, 2, 3 \ldots$

$$\frac{xd}{D} = m\lambda$$

$$\left[x = m\frac{D\lambda}{d} \right]$$

(ii) For Dark Fringes

$$\text{Path diff} = (2m-1)\lambda/2$$

where $m = 1, 2, 3 \ldots$

$$\frac{xd}{D} = (2m-1)\lambda/2$$

$$\left[x = (2m-1)\frac{D\lambda}{2d} \right]$$

The distance b/w two Successive Bright or dark fringes is called fringe bridth w

For Bright Fringe

$$x_n = \frac{nD\lambda}{d}$$

$$x_{n+1} = (n+1)\frac{D\lambda}{d}$$

$$W = x_{n+1} - x_n$$

$$= (n+1)\frac{D\lambda}{d} - \frac{nD\lambda}{d}$$

$$\cancel{\frac{nD\lambda}{d}} + \frac{D\lambda}{d} - \cancel{\frac{nD\lambda}{d}}$$

$$\boxed{W = \frac{D\lambda}{d}}$$

For Dark Fringe

$$x_n = (2n-1)\frac{D\lambda}{2d}$$

$$x_{n+1} = (2(n+1)-1)\frac{D\lambda}{2d}$$

$$= (2n+1)\frac{D\lambda}{2d}$$

$$W = (2n+1)\frac{D\lambda}{2d} - (2n-1)\frac{D\lambda}{2d}$$

$$\cancel{\frac{2nD\lambda}{2d}} + \frac{D\lambda}{2d} - \cancel{\frac{2nD\lambda}{2d}}$$

$$\boxed{W = \frac{D\lambda}{2d}}$$

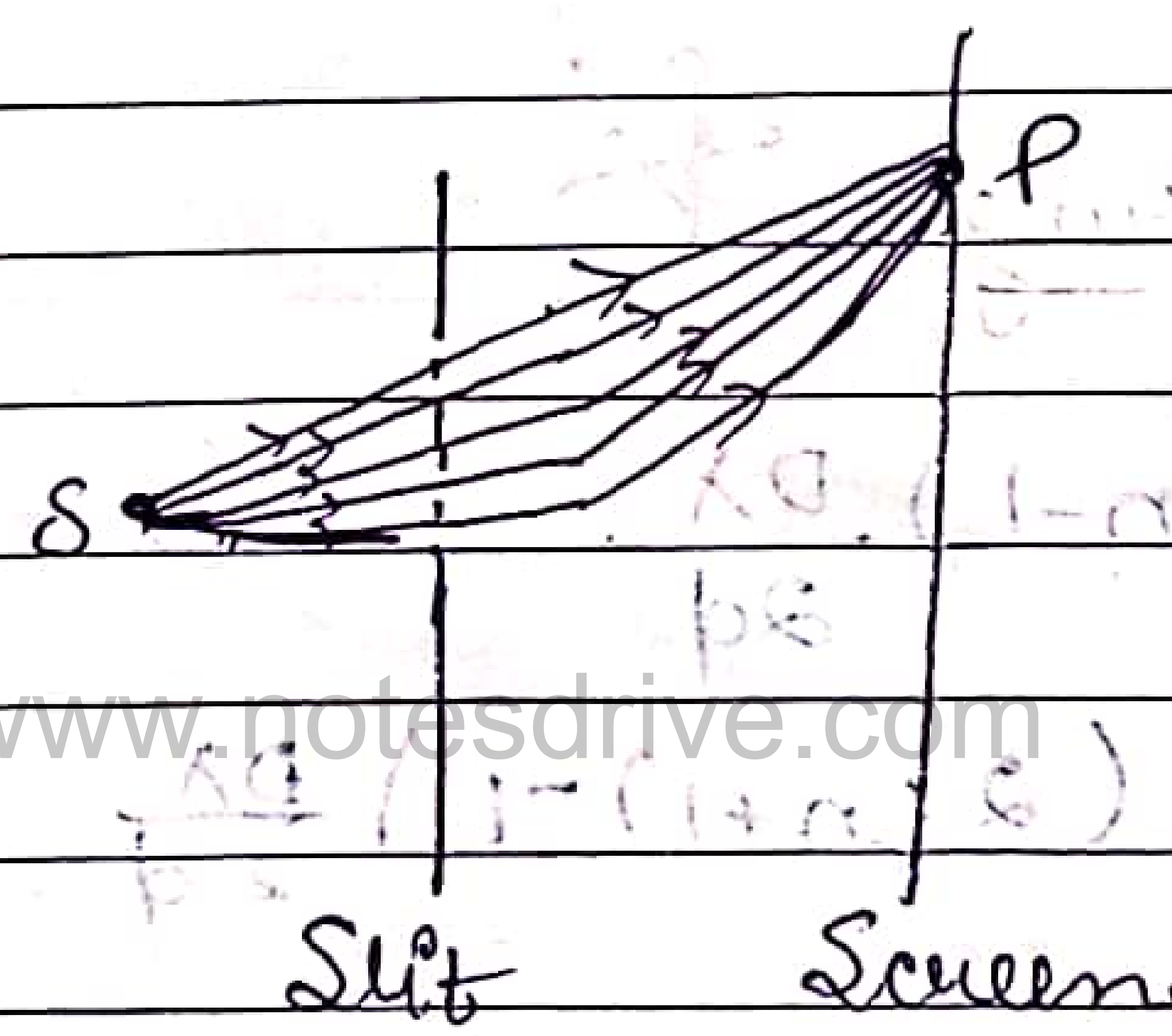
Diffraction of light

The phenomenon of bending of light around the sharp corners and the spreading of light within the geometrical shadow of the opaque obstacles is called diffraction of light.

For diffraction the aparture of the obstacle are comparable to the wavelength of light.

☆ There are 2 type of diffraction

① Freanel diffraction



If the distance b/w source and obstacle is short than the diffraction of light is called Freanel diffraction.

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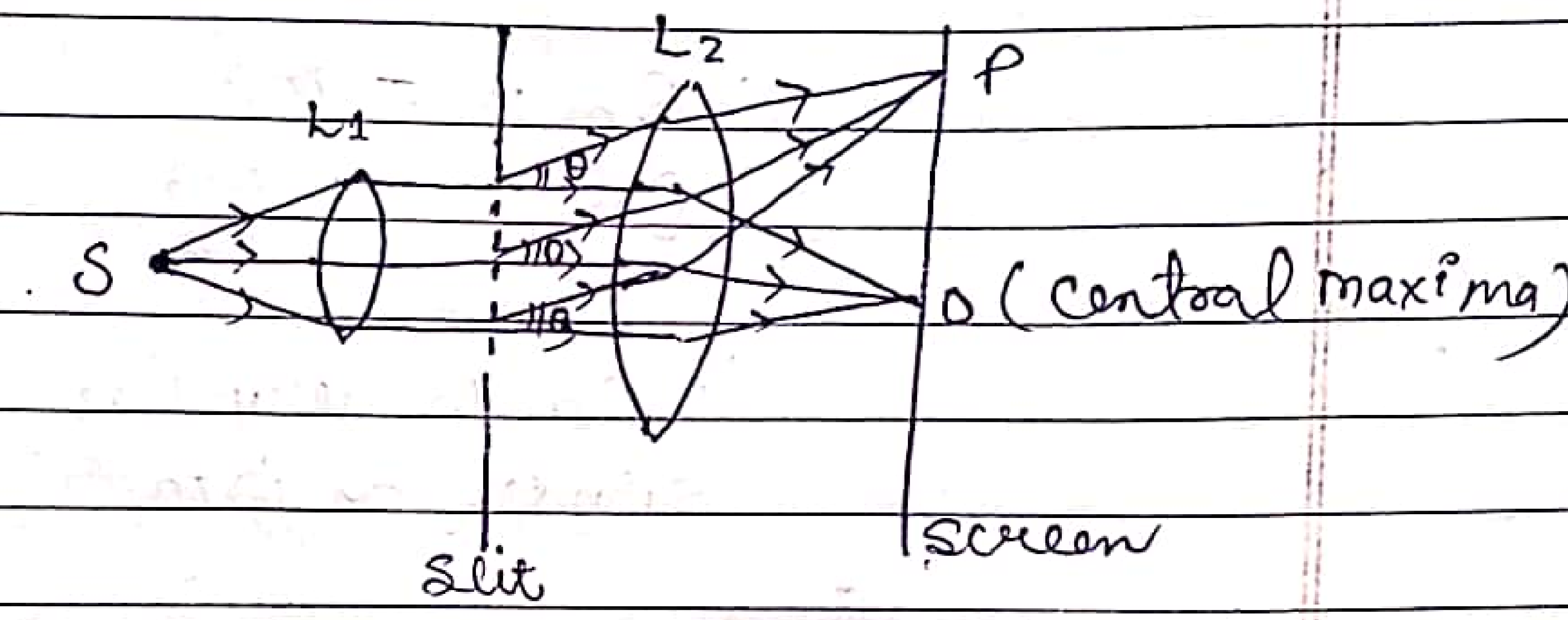
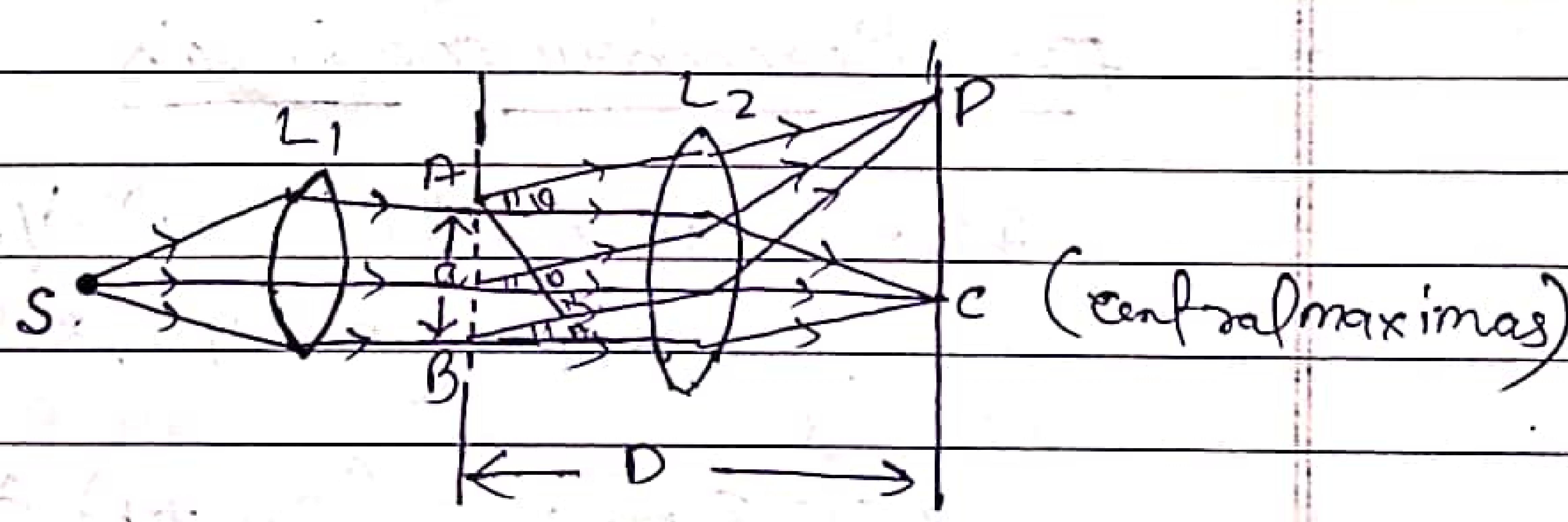
2) Fraunhalfer diffraction

θ = diffraction angle



When the diff distance b/w source and obstacle is large then the diffraction of light is called Fraunhalfer diffraction.

Diffraction of light at the single slit.



i) Secondary minimas
ii) Secondary maximas

For Secondary Minima's

Path diff = $m\lambda$ where $m = 1, 2, 3, 4 \ldots$

$BN = \pm m\lambda$

~~for~~ In $\triangle ABN$

$$\sin\theta = \frac{BN}{AB} = \frac{BN}{a}$$

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$BN = a \sin\theta$

for ~~se~~ mth Secondary minima's

$$a \sin\theta_m = \pm m\lambda$$
$$\sin\theta_m = \pm \frac{m\lambda}{a}$$

$\because \theta_m$ is very less

$\sin\theta_m \approx \theta_m$

$$\left[\theta_m = \pm \frac{m\lambda}{a}\right]$$

$m = 1, 2, 3, ------$

$\therefore \theta_1 = \pm \frac{\lambda}{a}, \ \theta_2 = \pm \frac{2\lambda}{a}, \ \theta_3 = \pm \frac{3\lambda}{a}$

For Secondary maxima's

Path diff $= \pm (2m+1)\lambda/2$

where $m = 1, 2, 3, 4 ---$

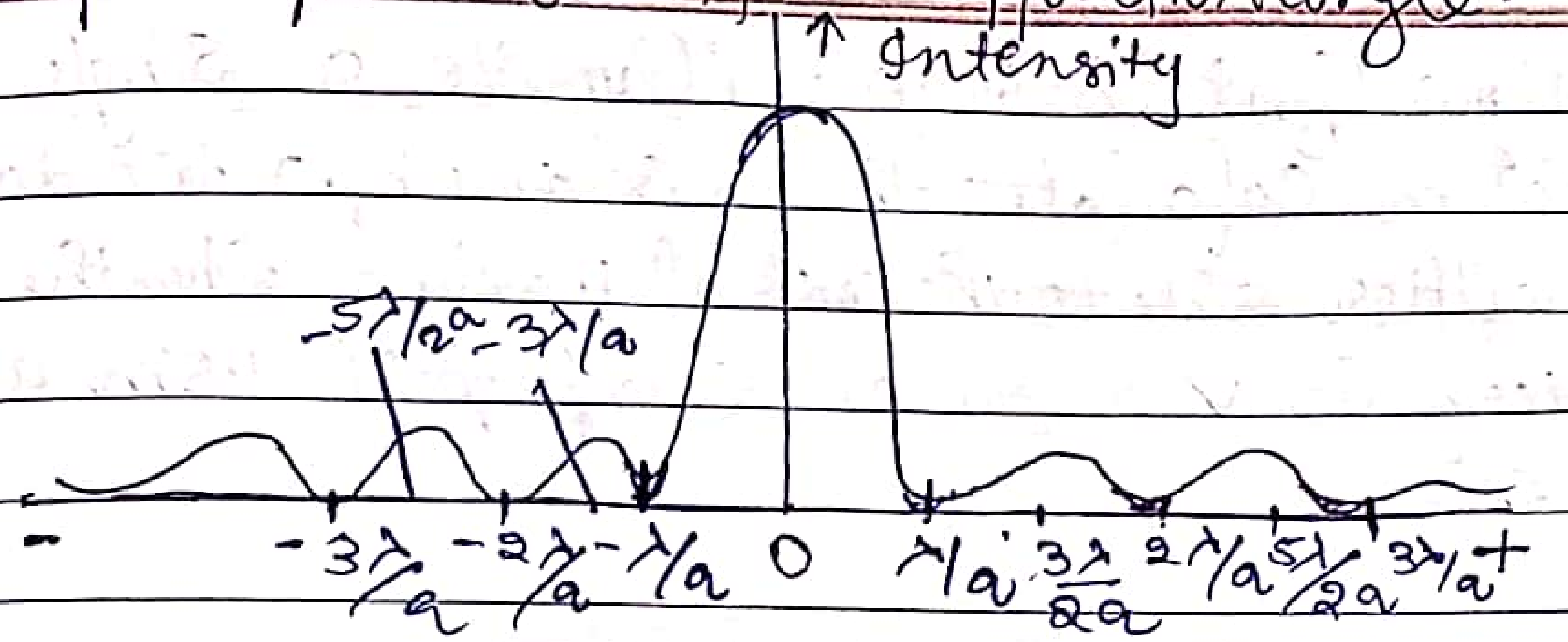
$$a \sin\theta_m = \pm (2m+1)\lambda/2$$
$$\sin\theta_m = \pm (2m+1)\lambda/2a$$
$$\left[\theta_m = \pm (2m+1)\lambda/2a\right]$$

$m = 1, 2, 3 --$

$\theta_1 = \pm \frac{3\lambda}{2a}$

$\theta_2 = \pm \frac{5\lambda}{2a}$

★ Graph b/w Intensity & diffraction angle.

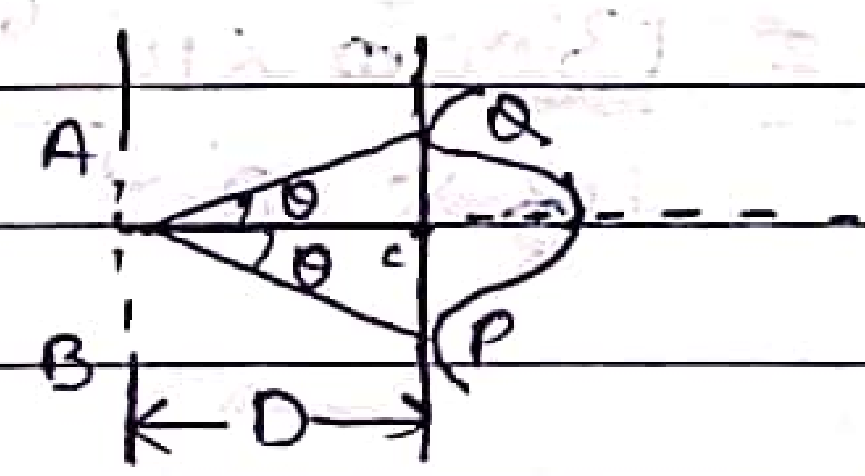


Angular width of central maxima

It is the distance b/w first secondary minima of both side of central maxima.

Angular width of central max (2θ) = $\lambda/a + \lambda/a$

$$\left[2\theta = \frac{2\lambda}{a}\right]$$



Linear width of central maxima

$$\text{Angle} = \frac{\text{Arc}}{\text{radius}} \qquad \because \theta = \lambda/a$$

$$\theta = \frac{PC}{D}$$

$$PC = D\theta$$

$$PC = \frac{D\lambda}{a}$$

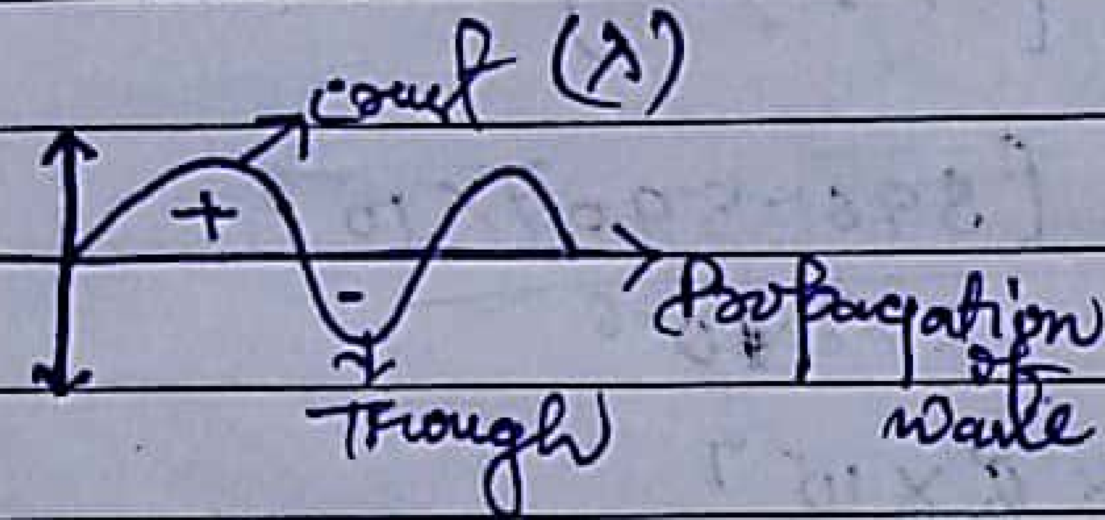
$$\left[\text{Linear width of central maxima} = \frac{2D\lambda}{a}\right]$$

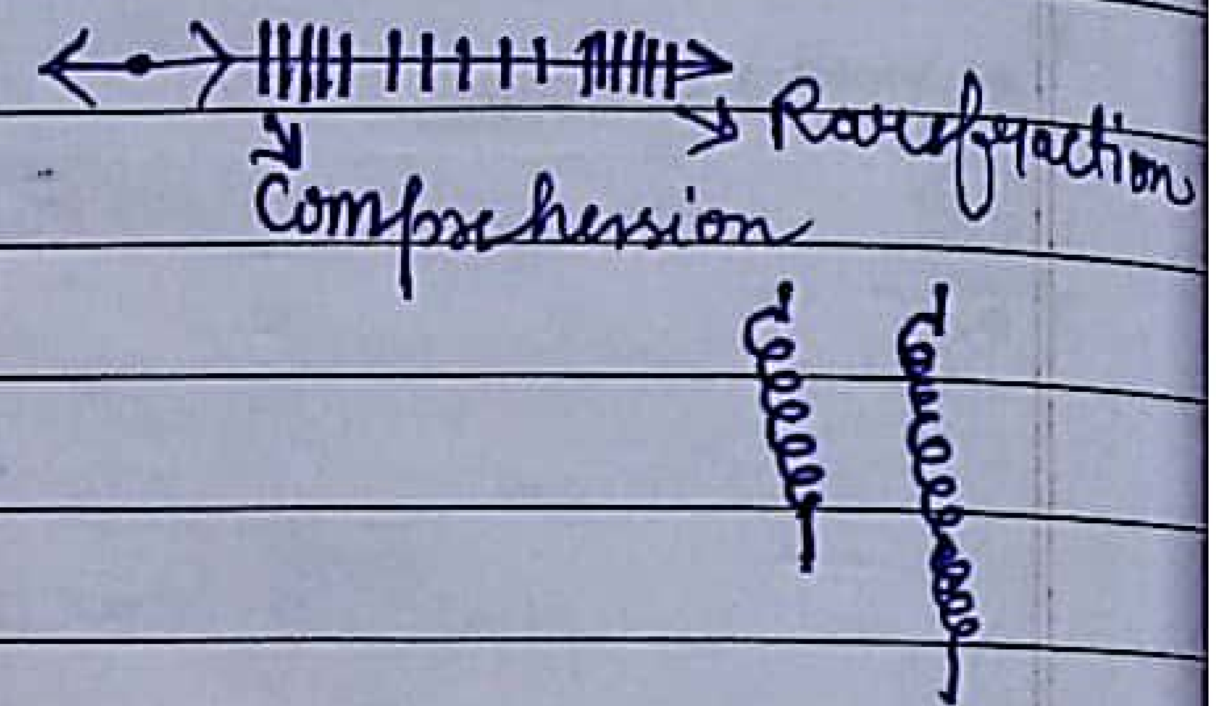
Polarisation

$\vec{E} = 75\%$
$\vec{B} = 25\%$

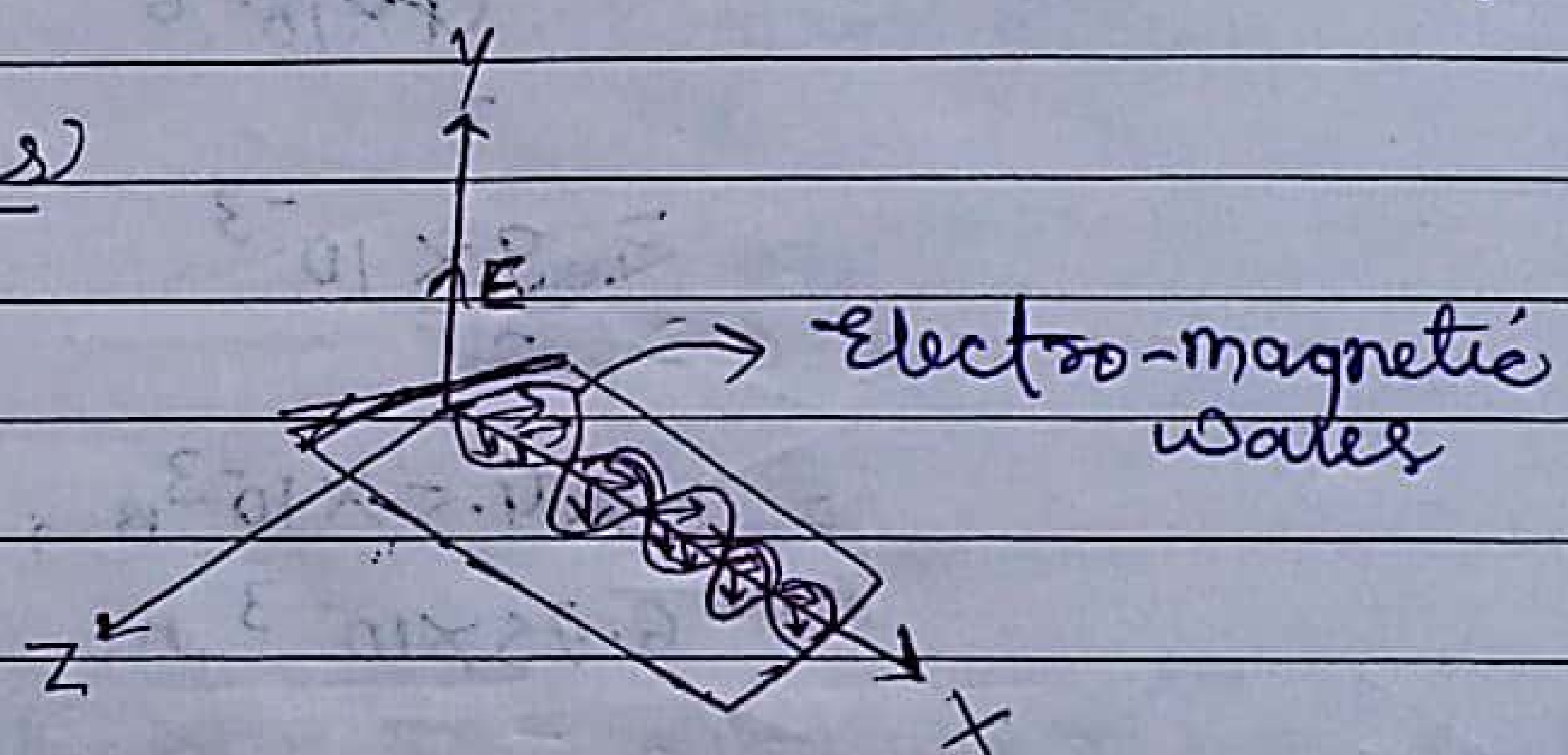
Waves

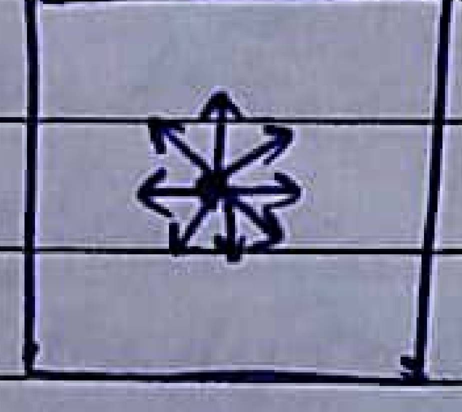
- Transverse waves
- Longitudinal waves





Light waves





unpolarised light
(simple light)

Light is an electromagnetic wave in which electric & magnetic field vector vary sonios. Sinosuadly ⊥ to each other

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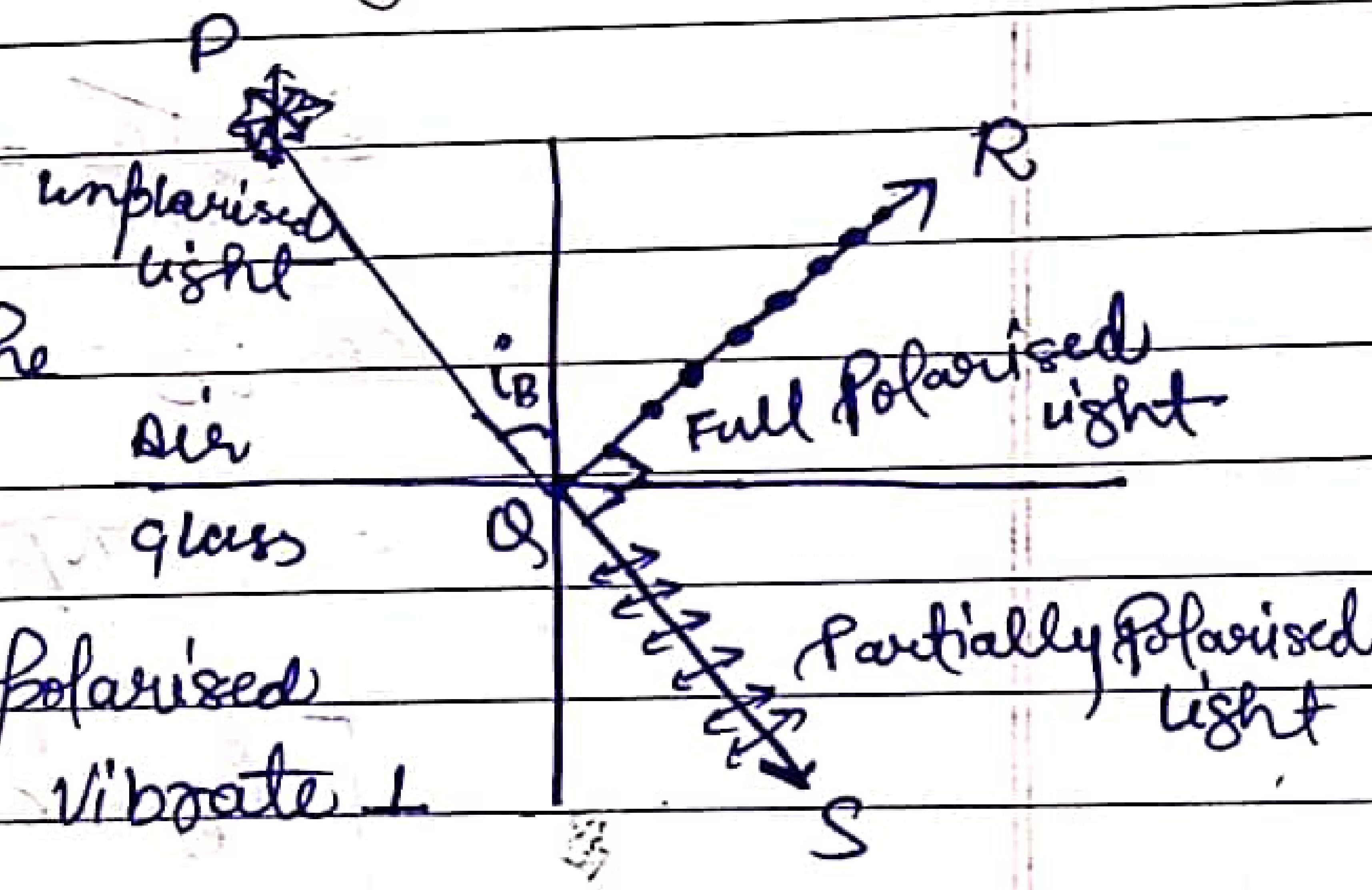
and as well as ⊥ to the direction of propagation of light.

If the vibration of electric vector occur symmetrically in all possible direction in a plane ⊥ to the direction of propagation of light then it is called unpolarised or ordinary light.

The phenomenon of restricting the vibration of light in a particular direction ⊥ to the direction of wave motion is called polarisation of light. And then the light obtain is called plane polarised light.

Imp Polarisation of light by reflection



⇒ When an unpolarised light is incident on the boundary b/w two transparent media the reflected light will be polarised and its electric vector vibrate ⊥ to the plane of incident.

⇒ i_B = Brewster angle or Angle of polarisation.

⇒ The refractive index of medium

$$\boxed{n = \tan i_B}$$

This is called Brewster's Law

The Reflected Rays & Refracted Rays will be perpendicular.

Proof – According snell law –

$$n = \frac{\sin i_B}{\sin r} \quad \text{— (1)}$$

According to Brewster's law

$$n = \tan i_B = \frac{\sin i_B}{\cos i_B} \quad \text{— (2)}$$

from eq (1) & (2)

$$\frac{\cancel{\sin i_B}}{\sin r} = \frac{\cancel{\sin i_B}}{\cos i_B}$$

$$\cos i_B = \sin r$$

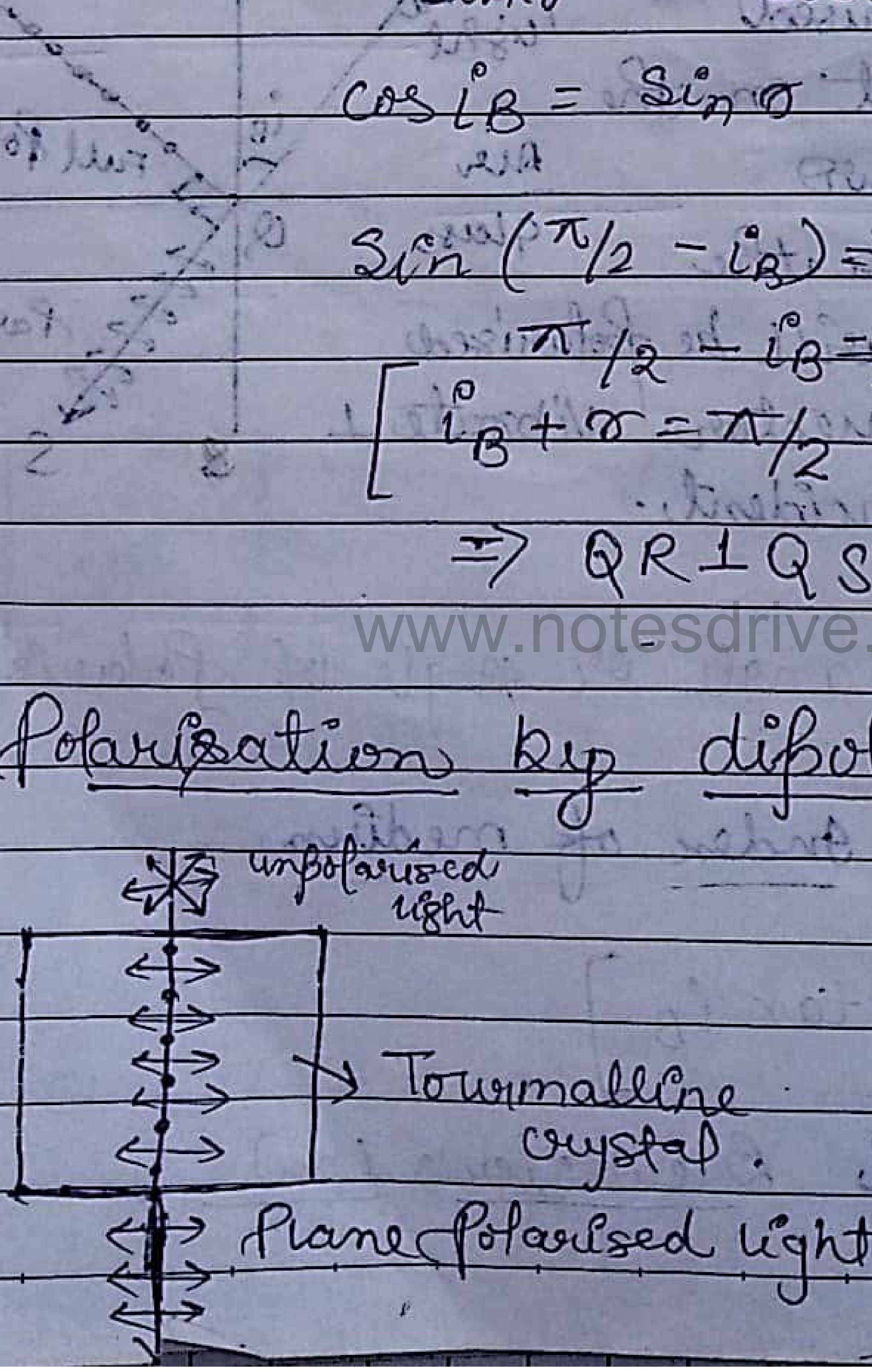
$$\sin(\pi/2 - i_B) = \sin r$$

$$\pi/2 - i_B = r$$

$$\left[i_B + r = \pi/2 \right]$$

$$\Rightarrow QR \perp QS$$

Polarisation by dipolarism



when an unpolarise light incident on the Surface of Tourmalline Crystal it absorb by the crystal & split out in two polarise light and by selective absorbtion it absorb one polarise light & emerge another polarise light. This phenomenon is known as dipolarism

Resolving Power of Optical Instruments

Resolving power of an optical instrument is the ability of the instrument to produce distinctly separate images of two close objects.

(i) Resolving power of microscope

$$\boxed{= \frac{1}{\Delta d} = \frac{2\mu \sin\beta}{\lambda}}$$

(ii) Resolving power of a telescope $\boxed{= \frac{1}{d\theta} = \frac{D}{1.22\lambda}}$

$d\theta$ = angle subtended by the two distinct objects of objective.

β = half angle of cone of light from the point object.

D = diameter of the objective.

Difference between the interference pattern and the diffraction pattern

Characteristics	Interference	Diffraction
Fringe width	All bright and dark fringes are of equal width.	The central bright fringe have got double width to that of width of secondary maxima or minima.
Intensity of bright fringes	All bright fringes are of same intensity.	Central fringe is the brightest and intensity of secondary maxima, decreases with the increase of order of secondary maxima on either side of central maxima.

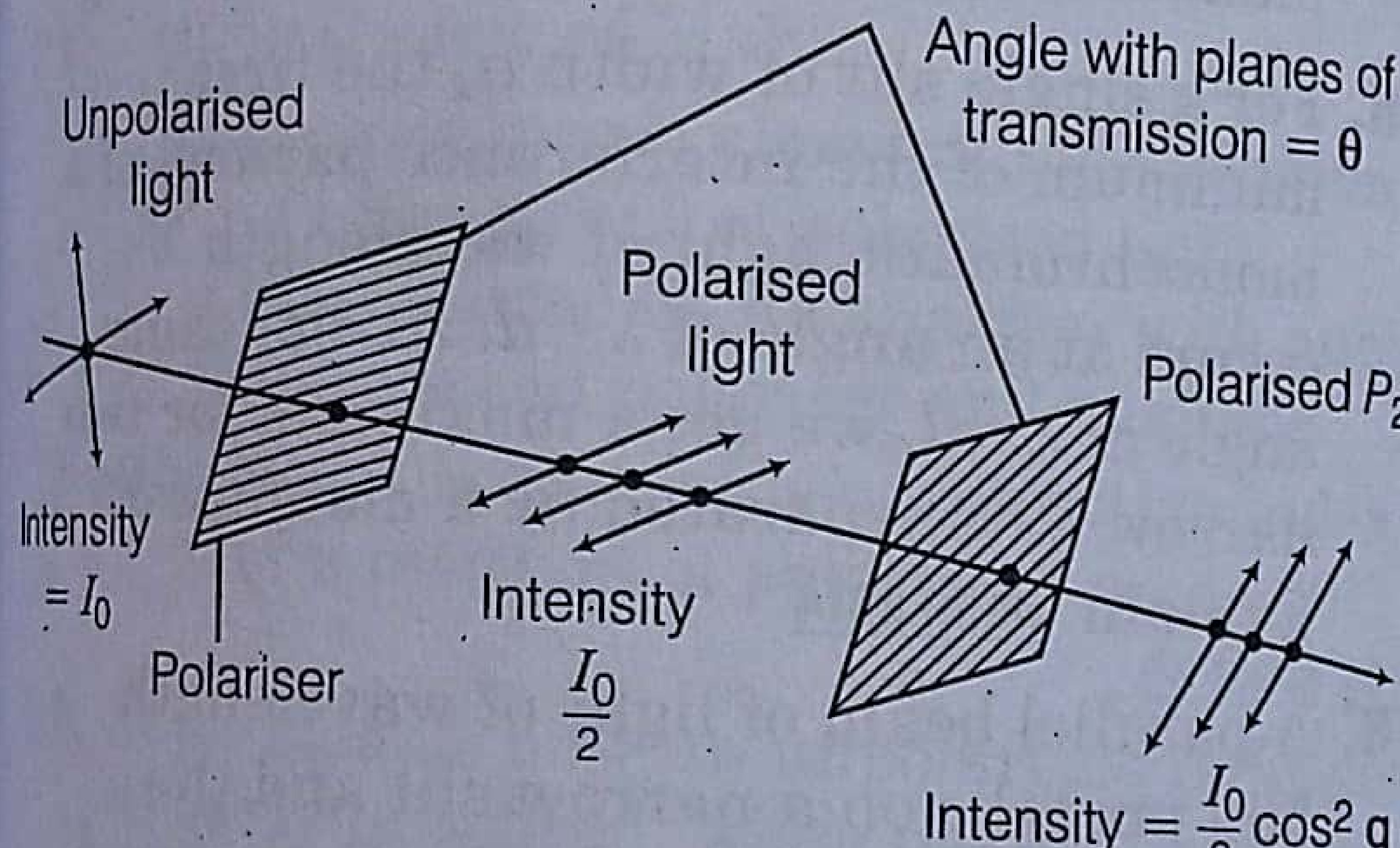
Diffraction at a Circular Aperture

Angular spread of central maxima $= 1.22\lambda/d$,

Linear spread $= D\theta'$, Areal spread $= D^2\theta^2$

Malus' Law

According to law of Malus', when a beam of completely plane polarised light is incident on an analyser, the resultant intensity of light (I) transmitted from the analyser varies directly as the square of the cosine of the angle θ between the plane of transmission of analyser and polariser.



Polarisation of light through polariser

i.e.,

$$\boxed{I \propto \cos^2\theta \Rightarrow I = I_0 \cos^2\theta}$$

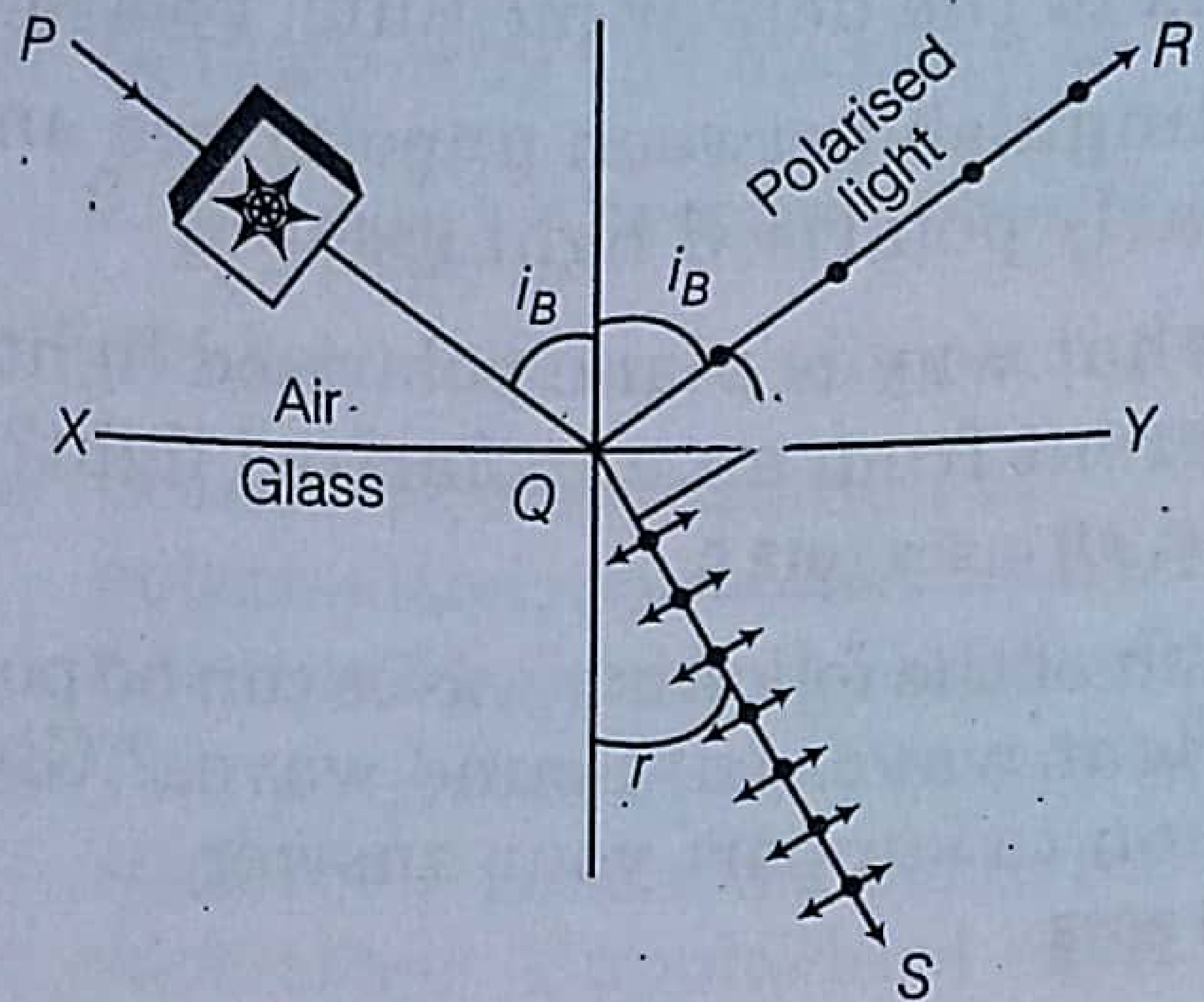
This rule is also called **cosine square rule.**

where, I_0 = intensity of plane polarised light after passing through P_1.

Brewster's Angle and Brewster Law

(i) The angle of incidence at which the reflected light is completely plane polarised is called polarising angle or Brewster's angle (i_B).

(ii) According to this law, when unpolarised light is incident at polarising angle, i_B on an interface separating air from a medium of refractive index μ, then the reflected light is plane polarised (perpendicular to the plane of incidence), provided, $\mu = \tan i_B$



Polarisation by reflection

where, i_B = Brewster's angle and μ = refractive index, $i_B + r = 90°$, i.e. reflected plane polarised light is at right angle from refracted light.

From Snell's law,

$$\boxed{\mu = \frac{\sin i_B}{\sin r_B} = \frac{\sin i_B}{\sin(90 - i_B)} = \frac{\sin i_B}{\cos i_B} = \tan i_B}$$

Polaroids

Polaroids are the commercial devices to produce plane polarised light making use of selective absorption. Polaroids are used in sunglasses, wind screen, window panes of aeroplane and to make the 3-D movies to make images vivid and clear.

Modes of production of plane polarised light are below:

(i) Reflection (Brewster's law) (ii) Scattering

(iii) Double refraction (calcite)

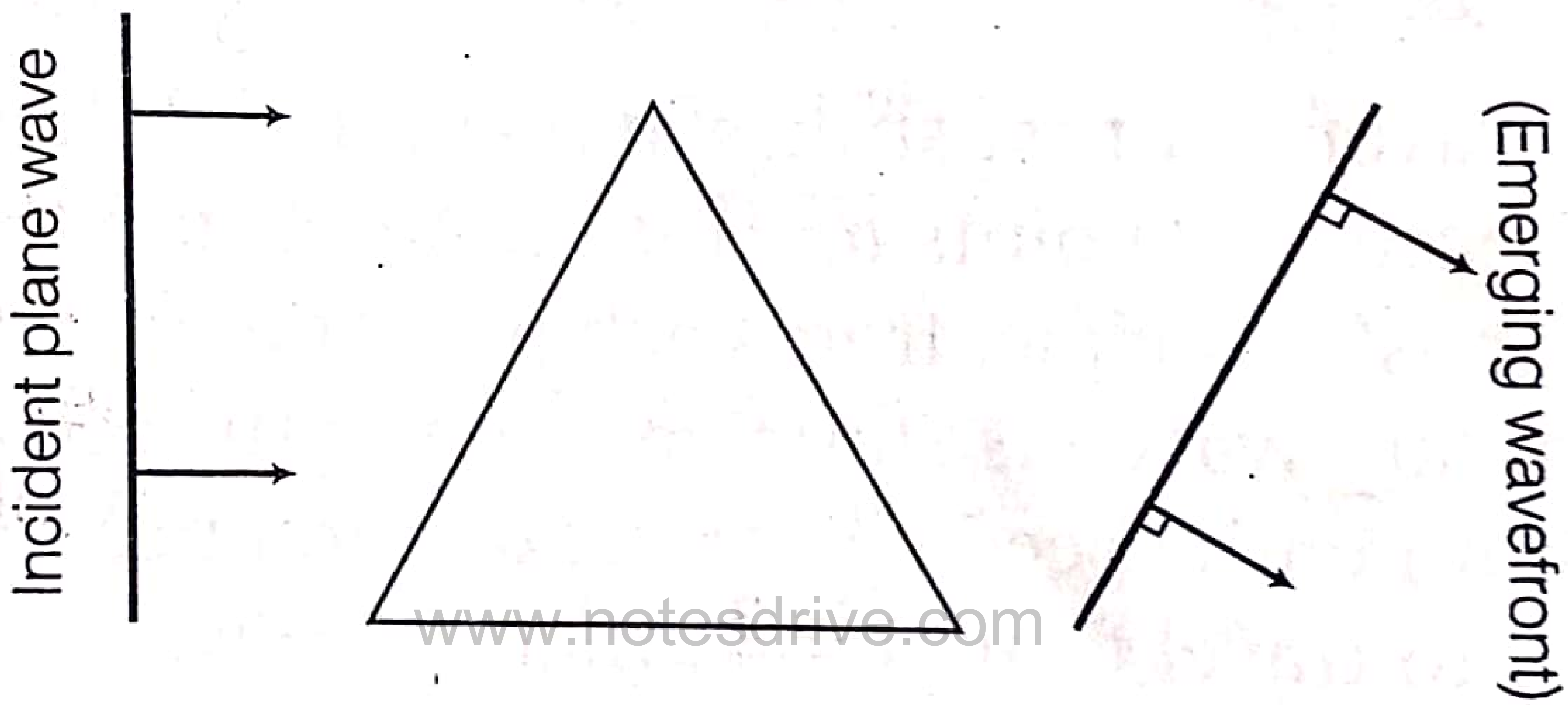
(iv) Selective absorption (dichroism)

Behaviour of a Prism, Lens and Spherical Mirror towards Plane Wavefront

There are following behaviour of a prism, lens and spherical mirrors towards plane wavefront

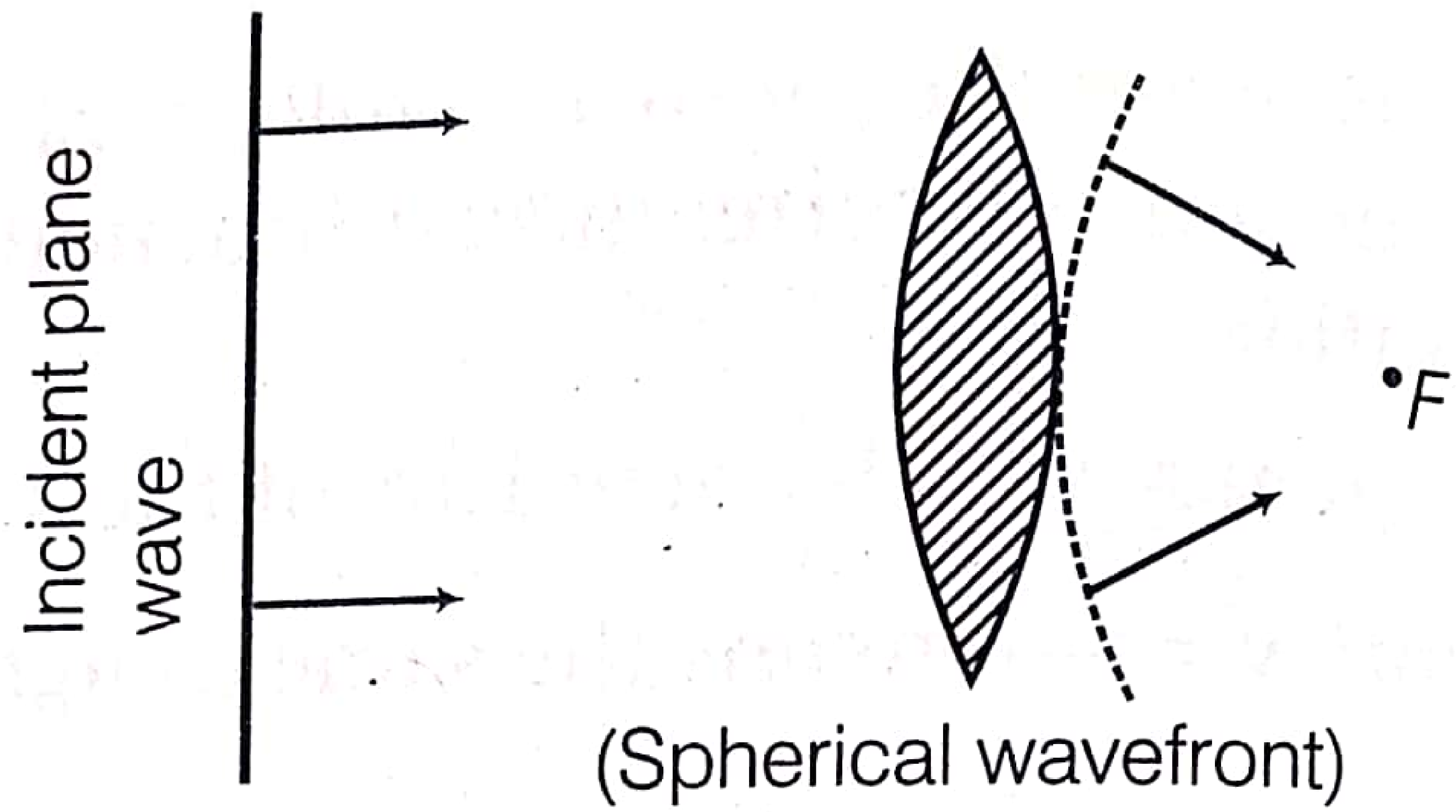
Behaviour of a Prism

Emerging wavefront will be tilted.



Behaviour of a Lens

Emerging wavefront will be spherical.



Behaviour of a Spherical Mirror

Reflected wavefront will be spherical.