INTRODUCTION TO TRIGONOMETRY

The word Trigonometry is derived from three Greek words

TRI, GONO, METRON

Tri :- Three

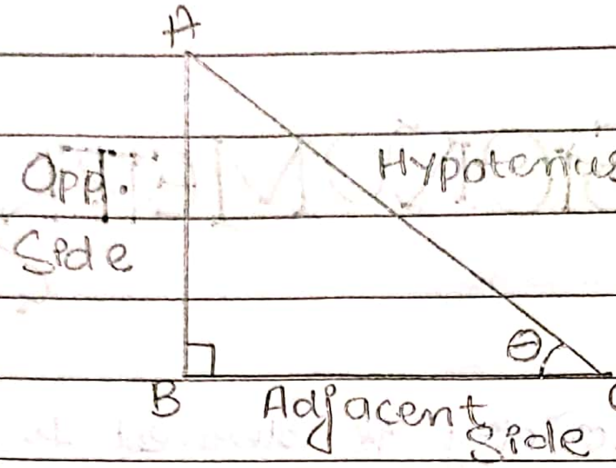
Gono :- Sides or Angles

Metron :- Measurement

Trigonometry is a branch of mathematics which deals with the study of three sides and three angles of a triangle.

Trigonometry is defined for a right triangle where hypotenuse is defined.

TRIGONOMETRIC RATIOS



Consider a right triangle ABC in which $\angle B = 90°$ & $\angle C = \theta$.

∴ Trigonometric Ratios for ΔABC are defined as follows :-

$\sin \theta = \dfrac{\text{Opp. side}}{\text{Hypotenuse}}$ $\qquad$ $\operatorname{cosec} \theta = \dfrac{\text{Hypo}}{\text{Opp. side}}$

$\cos \theta = \dfrac{\text{Adjacent side}}{\text{Hypotenuse}}$ $\qquad$ $\sec \theta = \dfrac{\text{Hypo.}}{\text{Adj. side}}$

$\tan \theta = \dfrac{\text{Opp. side}}{\text{Adj. side}}$ $\qquad$ $\cot \theta = \dfrac{\text{Adj. side}}{\text{Opp. side}}$

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RELATION BETWEEN TRIGONOMETRIC RATIOS

① $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

$\sin\theta = \dfrac{1}{\operatorname{cosec}\theta}$

$\sin\theta \times \cos\theta = 1$

② $\cos\theta = \dfrac{1}{\sec\theta}$

$\sec\theta = \dfrac{1}{\cos\theta}$

$\sec\theta \times \cos\theta = 1$

③ $\tan\theta = \dfrac{1}{\cot\theta}$

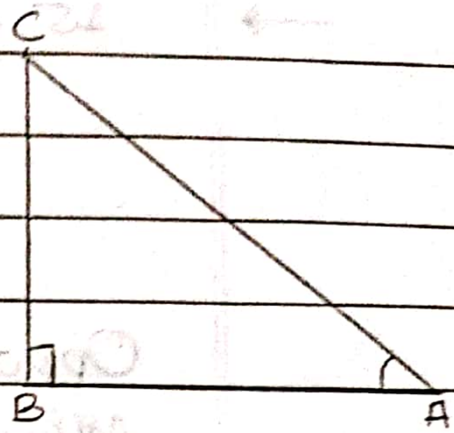
$\cot\theta = \dfrac{1}{\tan\theta}$

$\cot\theta \times \tan\theta = 1$

(4) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

(5) $\tan\theta = \dfrac{\sec\theta}{\operatorname{cosec}\theta}$ $\cot\theta = \dfrac{\operatorname{cosec}\theta}{\sec\theta}$

If $\sin A = \frac{3}{4}$, calculate $\cos A$ and $\tan A$.



Opposite side $= 3k = AB$

Hypotenuse $= 4k = AC$

Adjacent side $= ? = BC$

By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$(4k)^2 = (3k)^2 + BC^2$$

$$16k^2 = 9k^2 + BC^2$$

$$BC^2 = 16k^2 - 9k^2$$

$$BC^2 = 7k^2$$

$$BC = \sqrt{7}\,k$$

$$\cos A = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$$

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* TRIGNOMETRIC RATIOS OF SOME SPECIFIC ANGLES *

$\angle A°$	$0°$	$30°$	$45°$	$60°$	$90°$
sin A	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos A	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
tan A	0	$1/\sqrt{3}$	1	$\sqrt{3}$	NOT DEFINED
cosec A	NOT DEFINED	2	$\sqrt{2}$	$2/\sqrt{3}$	1
sec A	1	$2/\sqrt{3}$	$\sqrt{2}$	2	Not Defined
cot A	NOT DEFINED	$\sqrt{3}$	1	$1/\sqrt{3}$	0

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$\rightarrow (\sqrt{3}/2)(\sqrt{3}/2) + (1/2)(1/2)$

$3/4 + 1/4$

1

(ii) $2\tan^2 45^\circ + \cos 30^\circ - \sin^2 60^\circ$

$\rightarrow \cancel{3}\, 2 + 3/4 - 3/4$

$2 + 0$

2

$$\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$$

$$\frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1}$$

$$\frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{3}{2} + \frac{2}{\sqrt{3}}}$$

$$\frac{\frac{3\sqrt{3} - 4}{2\sqrt{3}}}{\frac{3\sqrt{3} + 4}{2\sqrt{3}}}$$

$$\frac{3\sqrt{3} - 4}{3\sqrt{3} + 4}$$

$$\frac{(3\sqrt{3} - 4)^2}{27 - 16}$$

$\sin 2A = 2 \sin A$ is true when A

when, $A = 0^\circ$

$\sin 2 \cdot 0^\circ = 2 \sin 0^\circ$

$2 \times 0 = 2 \times 0$

$0 = 0$

So $\rightarrow$ (A) 0°

If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = 1/\sqrt{3}$, $0° < A+B \leq 90°$; $A > B$, find A and B.

Given,

$\tan(A+B) = \sqrt{3}$

$\tan(A+B) = \tan 60°$

$A+B = 60°$ — (1)

$\tan(A-B) = 1/\sqrt{3}$

$\tan(A-B) = \tan 30°$

$A-B = 30°$ — (2)

Subtract,

$A + \cancel{B} = 60°$

$A - \cancel{B} = 30°$

$2A = 90°$

$A = 45°$

$A+B = 60°$

$45° + B = 60°$

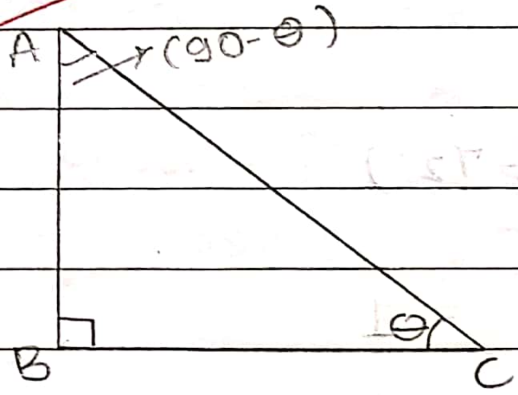
$B = 15°$

$\therefore A = 45°$

$B = 15°$

TRIGNOMETRIC RATIOS OF COMPLEMENTARY
* ANGLES *

Trignometric ratios for complementary angles are defined as follows



$\sin(90^\circ - \theta) = \cos \cancel{A}\, \theta$

$\cos(90^\circ - \theta) = \sin \theta$

$\tan(90^\circ - \theta) = \cot \theta$

$\cot(90^\circ - \theta) = \tan \theta$

$\sec(90^\circ - \theta) = \operatorname{cosec} \theta$

$\operatorname{cosec}(90^\circ - \theta) = \sec \theta$

$$\frac{\tan 26^\circ}{\cot 64^\circ}$$

$$\frac{\tan 26^\circ}{\tan (90^\circ - 64^\circ)}$$

$$\frac{\tan 26^\circ}{\tan 26^\circ}$$

1

$\cos 48^\circ - \sin 42^\circ$

$\cos 48^\circ - \cos (90^\circ - 42^\circ)$

$\cos 48^\circ - \cos 48^\circ$

0

$\operatorname{cosec} 31^\circ - \sec 59^\circ$

$\operatorname{cosec} 31^\circ - \operatorname{cosec} (90^\circ - 59^\circ)$

$\operatorname{cosec} 31^\circ - \operatorname{cosec} 31^\circ$

0

Show that :-

) $\tan 48^\circ . \tan 23^\circ . \tan 42^\circ . \tan 67^\circ = 1$

LHS =

$\tan 48^\circ \times \tan 42^\circ \times \tan 23^\circ \times \tan 67^\circ$

$\cot (90^\circ - 48^\circ) \times \tan 42^\circ \times \tan 23^\circ \times \cot (90^\circ -$

$\cot 42^\circ \times \tan 42^\circ \times \tan 23^\circ \times \cot 23^\circ$

$\frac{1}{\cancel{\tan 42^\circ}} \times \cancel{\tan 42^\circ} \times \cancel{\tan 23^\circ} \times \frac{1}{\cancel{\tan 23^\circ}}$

1

$\therefore$ LHS = RHS

) $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ = 0$

LHS =

$\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ$

$\sin (90^\circ - 38^\circ) \times \sin (90^\circ - 52^\circ) - \sin 38^\circ \sin 52^\circ$

$\sin 52^\circ \times \sin 38^\circ - \sin 38^\circ \times \sin 52^\circ$

$= 0$

$\therefore$ LHS = RHS

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6] If A, B and C are interior angles of triangle ABC, then show that,

$$\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$$

→ We have,

$$A + B + C = 180^\circ$$

$$B + C = 180^\circ - A$$

Divide by 2,

$$\frac{B+C}{2} = \frac{180^\circ - A}{2}$$

$$\frac{B+C}{2} = 90° - \frac{A}{2}$$

$$\sin\left(\frac{B+C}{2}\right) = \sin\left(90° - \frac{A}{2}\right)$$

$$\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$$

7] Express $\sin 67° + \cos 75°$ in terms of trignometric ratios of angles betw 0° and 45°.

→ $\sin 67° + \cos 75°$

$\cos(90° - 67°) + \cancel{\cos} \sin(90° - 75°)$

$\cos 23° + \sin 15°$

TRIGONOMETRIC IDENTITIES

In $\triangle ABC$ if

$\angle ACB = \theta$

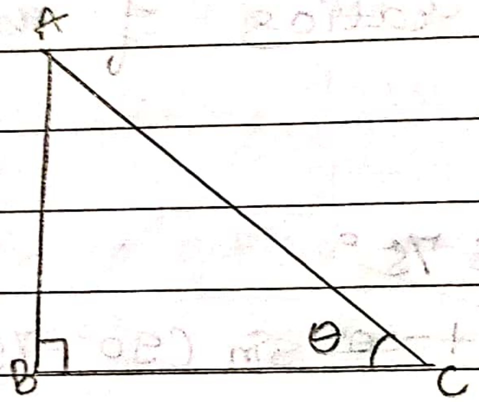
$\angle ABC = 90^\circ$

then,

i) $\sin^2\theta + \cos^2\theta = 1$

ii) $1 + \tan^2\theta = \sec^2\theta$

iii) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$



$\sin\theta = \dfrac{AB}{AC}$ $\qquad$ $\operatorname{cosec}\theta$

$\cos\theta = \dfrac{BC}{AC}$ $\qquad$ $\sec\theta$

$\tan\theta = \dfrac{AB}{BC}$ $\qquad$ $\cot\theta$

In ΔABC;
By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

Divide by AC^2

$$\frac{AC^2}{AC^2} = \frac{AB^2}{AC^2} + \frac{BC^2}{AC^2}$$

$$1 = \left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2$$

$$\cos^2\theta + \sin^2\theta = 1$$

Similarly,
Divide by AB^2

$$\frac{AC^2}{AB^2} = \frac{AB^2}{AB^2} + \frac{BC^2}{AB^2}$$

$$\sec^2\theta = 1 + \tan^2\theta$$

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Divide by BC^2

$$\frac{AC^2}{BC^2} = \frac{AB^2}{BC^2} + \frac{BC^2}{\cancel{AB}\,BC^2}$$

$$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$$

Q) Write all the other trigonometric ratios of $\angle A$ in terms of $\sec A$.

→ Using trigonometric identities,

$$\sin^2 A + \cos^2 A = 1$$

$$\sin^2 A + \frac{1}{\sec^2 A} = 1$$

$$\sin^2 A = 1 - \frac{1}{\sec^2 A}$$

$$\sin^2 A = \frac{\sec^2 A - 1}{\sec^2 A}$$

$$\sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

also,

$$\cos A = \frac{1}{\sec A}$$

$1 + \tan^2 A = \sec^2 A$

$\tan^2 A = \sec^2 A - 1$

$\tan A = \sqrt{\sec^2 A - 1}$

$\overset{\cot A}{\cancel{\tan A}} = \dfrac{1}{\underset{\tan A}{\cancel{\cot A}}}$

$\cot A = \dfrac{1}{\sqrt{\sec^2 A - 1}}$

$\text{cosec}^2 A = \dfrac{1}{\sin A}$

$\text{cosec}\, A = \dfrac{\cancel{1}}{} \quad \dfrac{\sec A}{\sqrt{\sec^2 A - 1}}$

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$$\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$$

LHS =

$\cos / 1 + \sin A \quad + \quad 1 + \sin A / \cos A$

$$\frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)\cos A}$$

$$\frac{\cos^2 A + 1 + 2\sin A + \sin^2 A}{(1+\sin A)\cos A}$$

$$\frac{1 + 1 + 2\sin A}{(1+\sin A)\cos A}$$

$$\frac{2 + 2\sin A}{(1+\sin A)\cos A}$$

$$\frac{2\cancel{(1+\sin A)}}{\cancel{(1+\sin A)}\cos A}$$

$2\sec A$

$\therefore$ LHS = RHS

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta \operatorname{cosec}\theta$$

$$\frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{(1-\tan\theta)}$$

$$\frac{\tan\theta}{\frac{\tan\theta-1}{\tan\theta}} + \frac{1}{\tan\theta(1-\tan\theta)}$$

$$\frac{\tan^3\theta}{(\tan\theta-1)} - \frac{1}{\tan\theta(\tan\theta-1)}$$

$$\frac{\tan^3\theta}{\tan\theta(\tan\theta-1)} - \frac{1}{\tan\theta(\tan\theta-1)}$$

$$\frac{\tan^3\theta - 1}{\tan\theta(\tan\theta 1)}$$

$$\frac{(\tan\theta-1)(\tan^2\theta+\tan\theta+1)}{\tan\theta(\tan\theta-1)}$$

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$$\frac{\tan^2\theta}{\tan\theta} + \frac{\tan\theta}{\tan\theta} + \frac{1}{\tan\theta}$$

$$1 + \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$1 + \frac{1}{\sin\theta\cos\theta}$$

$$1 + \sec\theta \operatorname{cosec}\theta$$

$$\frac{1 + \cancel{\cot}\,\sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$$

LHS =

$$\frac{1 + \sec A}{\sec A}$$

$$\frac{1}{\sec A} + \frac{\cancel{\sec A}}{\cancel{\sec A}}$$

$$1 + \cos A$$

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$$\frac{1+\cos A}{1} \times \frac{1-\cos A}{1-\cos A}$$

$$\frac{1-\cos^2 A}{1-\cos A}$$

$$\frac{\sin^2 A}{1-\cos A}$$

~~$\therefore$ LHS = RHS~~

$$\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$$

using the identity $\operatorname{cosec}^2 A = 1 + \cot^2 A$

Divide by ~~sin~~ $\sin A$, LHS

$$\cancel{\frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}}$$

$$\frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A - \operatorname{cosec} A + 1}$$

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$$\frac{\cot A + \csc A - (\csc A + \cot A)(\csc A - \csc ...}{\cot A + 1 - \csc A}$$

$$\frac{(\cot A + \csc A)(1 - \csc A + \cot A)}{\cot A - \csc A + 1}$$

$\cancel{\csc A + \cot A}$

$\therefore$ LHS = RHS

(vii) $$\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$$

$\longrightarrow$ LHS

$$\sqrt{\frac{1 \times \sin A \times (1+\sin A)}{(1-\sin A)\ (1+\sin A)}}$$

$$\sqrt{\frac{(1+\sin A)^2}{1^2 - \sin^2 A}}$$

$$\sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$$

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$$\frac{1 + \sin A}{\cos A}$$

$$\frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$\sec A + \tan A$

$\therefore$ LHS = RHS

$(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$
$= 7 + \tan^2 A + \cot^2 A$

$(\sin^2 A + 2\sin A \operatorname{cosec} A + \operatorname{cosec}^2 A) +$
$(\cos^2 A + 2\cos A \sec A + \sec^2 A)$

$(\sin^2 A + 2\sin A \operatorname{cosec} A + \operatorname{cosec}^2 A) +$
$(\cos^2 A + 2\cos A \sec A + \sec^2 A)$

$\sin^2 A + 2\sin A \operatorname{cosec} A + \operatorname{cosec}^2 A + \cos^2 A +$
$2\cos A \sec A + \sec^2 A$

$1 + 2\cancel{\sin A} \times \frac{1}{\cancel{\sin A}} + 1 + \cot^2 A + 2\cancel{\cos A} \times \frac{1}{\cancel{\cos A}}$

$+ 1 + \tan^2 A$

$7 + \tan^2 + \cot^2 A$

$\therefore$ LHS = RHS