

# CO-ORDINATE

# GEOMETRY

The branch of mathematics which deals with the study of position of a point in Cartesian plane or Co-ordinate plane is called Co-ordinate Geometry.

The  $x$ -co-ordinate of a point is also called as abscissa and  $y$ -co-ordinate of it is also called as ordinate.

## DISTANCE FORMULA

If  $P(x_1, y_1)$  &  $Q(x_2, y_2)$  are any two points in co-ordinate plane then the distance between points  $P$  and  $Q$  is given by,

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$(ii) (-5, 7), (-1, 3)$$

→ let A (-5, 7) & B (-1, 3) be given points  
By distance formula,

$$AB = \sqrt{(-5 + 1)^2 + (7 - 3)^2}$$

$$AB = \sqrt{(-4)^2 + (4)^2}$$

$$AB = \sqrt{16 + 16}$$

$$AB = \sqrt{32} = \sqrt{2 \times 2 \times 2 \times 2 \times 2}$$

$$AB = 4\sqrt{2} \text{ units}$$

$$(iii) (a, b), (-a, -b)$$

→ let A (a, b) & B (-a, -b) be given points  
By distance formula,

$$AB = \sqrt{(a + a)^2 + (b + b)^2}$$

$$AB = \sqrt{(2a)^2 + (2b)^2}$$

$$AB = \sqrt{4a^2 + 4b^2}$$

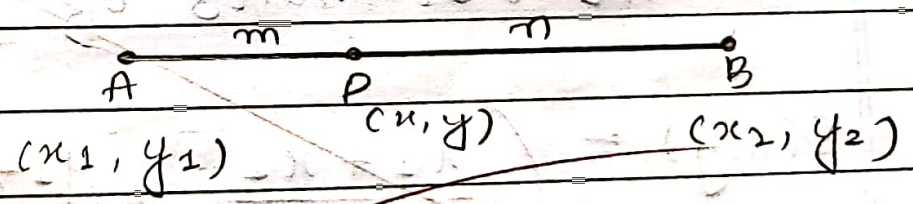
$$AB = 2\sqrt{a^2 + b^2} \text{ units}$$

# SECTION FORMULA

If the point  $P(x, y)$  divides the line segment joining the points  $A(x_1, y_1)$  &  $B(x_2, y_2)$  internally in the ratio  $m : n$  then by section formula coordinates of point  $P$  is given by

$$P(x, y) \equiv \left( \frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

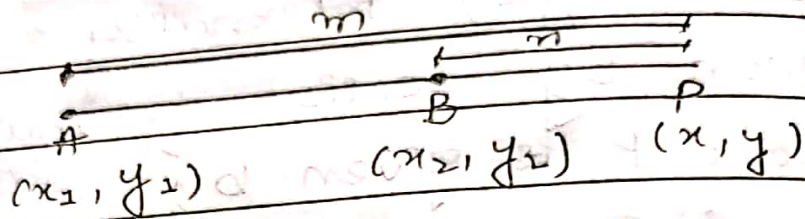
NOTE :- If the point  $P$  divides line segment  $AB$  & if the ratio is not given then it is consider as  $k : 1$ .



If the point  $P$  divides the line segment joining the points  $A(x_1, y_1)$  &  $B(x_2, y_2)$  externally in the ratio  $m : n$  then by section formula the coordinates of point  $P$  is given by

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$$P(x, y) \equiv \left( \frac{mx_2 - ny_1}{m-n}, \frac{my_2 - ny_1}{m-n} \right)$$



If  $\lambda$  is positive then the ratio is for internal division & if  $\lambda$  is negative then the ratio is for external division.

If the point P is the mid point of line segment AB where A  $(x_1, y_1)$  & B  $(x_2, y_2)$  then the coordinates of point P is given by

$$D(x, y) \equiv \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$



Find the coordinates of the point which divides the point of  $(-1, 7)$  and  $(4, -3)$  in the ratio  $2:3$ .

Let  $P(x, y)$  divide the join of  $A(-1, 7)$  &  $B(4, -3)$  in the ratio  $2:3$ .

∴ By Section Formula,

$$P(x, y) = P\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

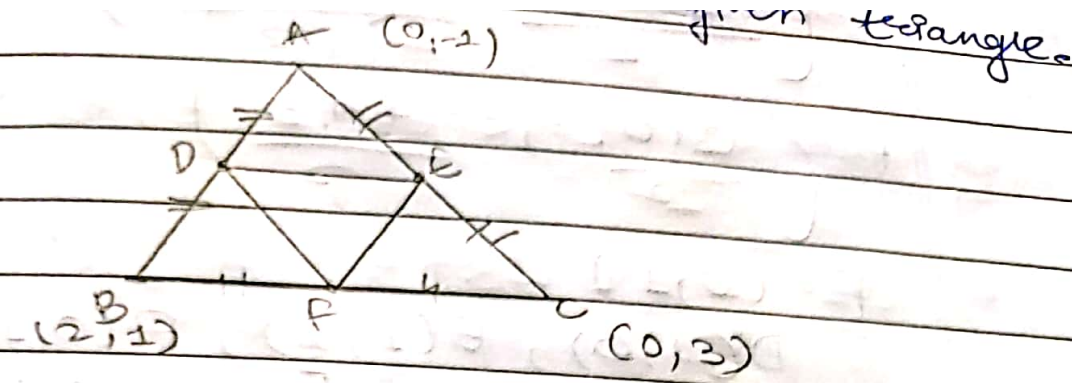
$$\equiv P\left(\frac{2(4) + 3(-1)}{2+3}, \frac{2(-3) + 3(7)}{2+3}\right)$$

$$\equiv P\left(\frac{8-3}{5}, \frac{-6+21}{5}\right)$$

$$\equiv P(1, 3)$$

Find the coordinates of points of intersection of line segment joining  $(4, -1)$  and  $(-2, -3)$ .





Let D, E, F be the mid points of sides AB, BC, AC of  $\triangle ABC$ .

By mid point formula,

$$D \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$D \left( \frac{0 + (-2)}{2}, \frac{-1 + 1}{2} \right)$$

$$D(1, 0)$$

$$E \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$E \left( \frac{2 + 0}{2}, \frac{1 + 3}{2} \right)$$

$$E = (1, 2)$$

$$F \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$F \left( \frac{0+0}{2}, \frac{-1+3}{2} \right)$$

$$F = (0, 1)$$

$$D(1, 0), E(1, 2), F(0, 1)$$

$$ae(DEF) = \frac{1}{2} [1(2-1) + 1(0-1) + 0(0-2)]$$

$$ae(DEF) = \frac{1}{2} [1(1) + 1(0) + 0(-2)]$$

$$ae(DEF) = \frac{1}{2} [1 + 0 + 0]$$

$$ae(DEF) = \frac{2}{2} = 1$$

$$ae(ABC) = \frac{1}{2} [0(1-3) + 2(3+1) + 0(-1-1)]$$

$$ae(ABC) = \frac{1}{2} [0 + 8 + 0]$$

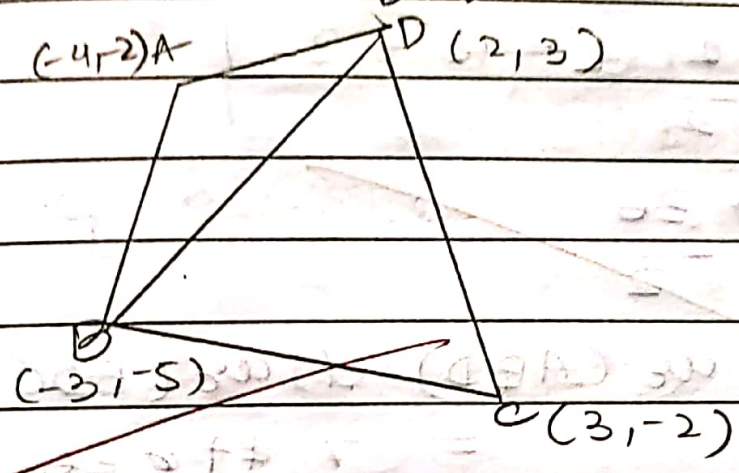
$$ae(ABC) = 4$$

$$\frac{ae(DEF)}{ae(ABC)} = \frac{1}{4}$$

$$ae(DEF) : ae(ABC) = 1 : 4$$

Find the area of quadrilateral whose vertices taken in order, are  $(-4, -2)$ ,  $(-3, -5)$ ,  $(3, -2)$  and  $(2, 3)$ .

Let  $A(-4, -2)$ ,  $B(-3, -5)$ ,  $C(3, -2)$  &  $D(2, 3)$  be the vertices of quadrilateral.



Let BD be the diagonal.

$$\text{ar}(ABD) = \frac{1}{2} [-4(-5-3) - 3(3+5) + 2(-2+5)]$$

$$= \frac{1}{2} [-4(-8) - 3(8) + 2(3)]$$

$$= \frac{1}{2} [32 - 24 + 6]$$

$$= \frac{1}{2} [14]$$

$$= 7 \text{ sq. unit}$$



$$ae(BDC) = \frac{1}{2} [-3(3+2) + 2(-2+5) + 3(5-3)]$$

$$ae = \frac{1}{2} [-3(6) + 2(3) + 3(-8)]$$

$$= \frac{1}{2} [-18 + 6 - 24]$$

$$= \frac{1}{2} [-48 - 36]$$

$$= \frac{-84}{2}$$

So  $ae(ABD) + ae(BDC)$

$$= 7 + (-42)$$

$$= \frac{50}{2} = 25$$

iii)  $(4,5), (7,6), (4,3), (1,2)$

→ Let A, B, C, D be the given points.

By Distance formula,

$$AB = \sqrt{(7-4)^2 + (6-5)^2}$$

$$AB = \sqrt{(3)^2 + (1)^2}$$

$$AB = \sqrt{9+1}$$

$$AB = \sqrt{10}$$

$$BC = \sqrt{(4-7)^2 + (3-6)^2}$$

$$BC = \sqrt{(-3)^2 + (-3)^2}$$

$$BC = \sqrt{9+9}$$

$$BC = \sqrt{18}$$

$$CD = \sqrt{(1-4)^2 + (2-3)^2}$$

$$CD = \sqrt{(-3)^2 + (-1)^2}$$

$$CD = \sqrt{9+1}$$

$$CD = \sqrt{10}$$

$$AD = \sqrt{(1-4)^2 + (2-5)^2}$$

$$AD = \sqrt{(-3)^2 + (-3)^2}$$

$$AD = \sqrt{9 + 9}$$

$$AD = \sqrt{18}$$

All opposite sides are equal