Waves

wave is a disturbance from equilibrium position, which travels but particles do not travel.

wave is a energy / momentum which travels, particles oscillate.

It may be disturbance in the form of displacement variation, pressure variation & density & electric / magnetic field variation.

Types of waves (medium)

① **Mechanical wave** – The wave which requires a medium to travel.

Example – Sound waves

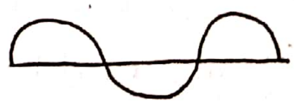
② **Non-mechanical waves** – The waves which do not require a medium to travel.

Example – Electromagnetic waves (E & B fields vibrates)

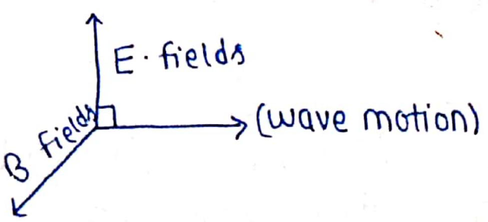
Types of waves (vibration / shape of disturbance)

① **Transverse waves** – The waves which travels perpendicular to the vibrations of particles is called transverse waves.

Example – ① waves of string,



② Electromagnetic waves are also transverse in nature.



(Transverse waves travels in Solids only & it travels in the liquid surface)

② **Longitudinal waves** – The waves which travels in the direction of vibration of particles or parallel to medium. these waves are called longitudinal waves.

Ex – Sound wave in air (density & Pressure variation of particles)

these waves are travel in Solid, liquid & gases

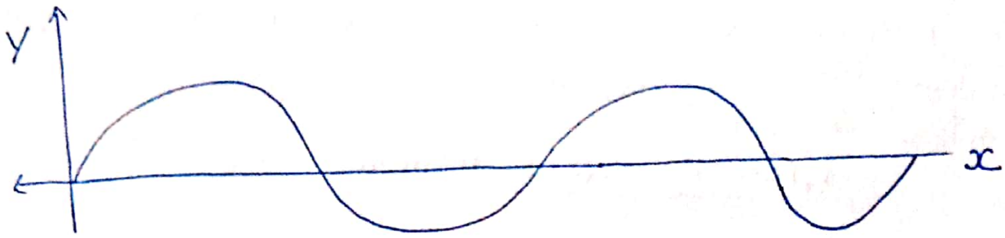
Ripples on water surfaces is the example of transverse & longitudinal waves.

General Equation of a wave

if some quantity (y) oscillates then —

'y' can be displacement, pressure, density or electric & magnetic fields.

y depends upon — (i) x & (ii) time (t)



$$\boxed{y = f(x, t)}$$

In all waves particle is not necessary but 99 % waves contain particles & produces SHM.

All functions of x & t are not the equation of waves.

Condition for wave motion —

(i) $$\boxed{\frac{\partial^2 y}{\partial t^2} = K \frac{\partial^2 y}{\partial x^2}}$$ $K \neq 0$, it is a constant.

(ii) y should be defined for all values of x & t.

(1) Check the given equation is a wave.

(a) $y = a \sin \omega t$

$$\frac{\partial y}{\partial t} = a\omega \cos \omega t \Rightarrow \frac{\partial^2 y}{\partial t^2} = -a\omega^2 \sin \omega t$$

$$\frac{\partial y}{\partial x} = 0$$ & $K \neq 0$, hence it is not a wave.

i.e. SHM is not a wave.

(b) $y = A \sin(\omega t - Kx)$

$$\frac{\partial y}{\partial t} = A\omega \cos(\omega t - Kx)$$

$$\frac{\partial^2 y}{\partial t^2} = -A\omega^2 \sin(\omega t - Kx)$$

$$\frac{\partial y}{\partial x} = -KA \cancel{\sin}\cos(\omega t - Kx)$$

$$\frac{\partial^2 y}{\partial x^2} = -K^2 A \sin(\omega t - Kx)$$

$$\frac{\partial^2 y}{\partial t^2} \times \frac{\partial t^2}{\partial x^2} = \frac{\omega^2}{K^2}$$

$$\frac{\partial^2 y}{\partial t^2} = \left(\frac{\omega^2}{K^2}\right) \frac{\partial x^2}{\partial t^2}$$

Hence it is a wave equation.

constant $= \omega^2 / K^2$

$$\boxed{\frac{\partial^2 y}{\partial t^2} = K \frac{\partial^2 x}{\partial t^2}}$$

it is a differential eq^N hence its solution is —

(i) $\boxed{y = f(ax \pm bt)}$ a & b are constants

(ii) y should be defined for all values of x & t.

General eq^N of wave $\Rightarrow$ $y = f(ax \pm bt)$

check given eq^N is a wave eq^N —

(i) $y = \log(x + 2t)$

$x + 2t < 0$ it is not a wave because y is not defined.

(ii) $y = e^{-(x - vt)^2}$

$x - vt = 0$, $y = e^0 = 1$

$x - vt \rightarrow \infty$, $y = e^{-\infty} = \frac{1}{e^\infty} = 0$

$x - vt \rightarrow -\infty$, $y = e^{-\infty} = \frac{1}{e^\infty} = 0$

yes it is a wave eq^N.

(iii) $y = (x - 2t)^2$

$x - 2t \rightarrow \infty$, $y = \infty$

displacement of particle is not ∞, it is not a wave.

(iv) $y = \frac{2}{(3x + 2t)^2 + 4}$

$3x + 2t \rightarrow \infty, -\infty$ $y = 0$

Hence it is a wave eq^N.

$$y = f(ax \pm bt)$$

wave speed (Energy) $\Rightarrow$ $v = \dfrac{\text{coefficient of } t\ (b)}{\text{coefficient of } x\ (a)}$

$$\boxed{V = \frac{b}{a}}$$

if ax & bt have same sign —

$[ax + bt \ / \ -ax - bt]$ — it means wave travelling in negative $(-)$ x-direction.

if ax & bt have different sign —

$[ax - bt \ / -ax + bt]$ — it means wave travelling in a positive $(+)$ x-direction.

Question - 1

if $y = \dfrac{10}{5 + (x-2t)^2}$, find — (i) wave velocity (ii) Amplitude of particle.

$\downarrow$

Max. displacement $\Rightarrow$ max. y

$a = 1$, $b = 2$ $\quad V = b/a = 2\ m/s$

if $(x-2t)^2$ is min. then y will be max —

$$(x-2t)^2 = 0$$

$$y = \frac{10}{5+0} = 2\ m$$

it is $(ax - bt)$, hence it is travelling in $+ve$ x-direction.

Question - 2

if $y = \dfrac{1}{1+x^2}$ at $t = 0$ sec & $y = \dfrac{1}{1+(x-1)^2}$ at $t = 2$ sec

find wave velocity.

$$y = \frac{1}{1 + (ax + bt)^2}$$

if $t = 0$ sec then

$$y = \frac{1}{1 + (ax)^2} \qquad \boxed{a = 1}$$

if $t = 2$ sec, $a = 1$

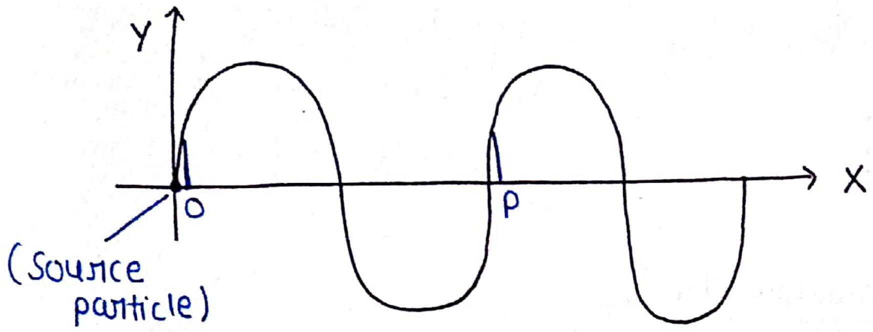
$$y = \frac{1}{1 + (x + 2b)^2} \qquad 2b = -1$$

$$\boxed{b = -1/2}$$

$$v = b/a = +1/2 = 0.5\ m/s$$

Plane Progressive Harmonic Equation

The wave motion in which each particle shows simple harmonic motion that wave is called plane progressive harmonic wave.



we can easily write the eq^N of y at o because $y \longrightarrow x, t$ but $x = 0$ at o.

$$y_0 = A\sin(\omega t + \phi)$$

The disturbance at point P is equal to disturbance at point O before some time.

let disturbance at point P cover 'x' distance with velocity 'v'.

time to cover x distance

$$t' = \frac{x}{v}$$

At time t, disturbance is at origin — before $t' = x/v$ 'P' disturbance is at 'O' hence —

$$(y_P)_t = (y_O)_{t-t'}$$

$$(y_P)_t = (y_O)_{t-\frac{x}{v}}$$

$$y_P = A\sin[\omega(t - \frac{x}{v}) + \phi]$$

point P is a general point —

$$\boxed{y = A\sin(\omega t - \frac{\omega x}{v} + \phi)}$$

wavelength is a distance covered by wave in one time period

$$\boxed{\lambda = V \times T} \Rightarrow \boxed{V = \lambda / T}$$

$$\boxed{\omega = \frac{2\pi}{T}}$$

Alternative Method

$$y = \frac{1}{1+x^2} \quad , \; t = 0\,s$$

$y = 1$

$$1 = \frac{1}{1+x^2}$$

$$x^2 = 0$$

$$\boxed{x = 0}$$

$$y = \frac{1}{1+(x-1)^2} \quad , \; t = 2\,s$$

$y = 1$

$$\boxed{x = 1}$$

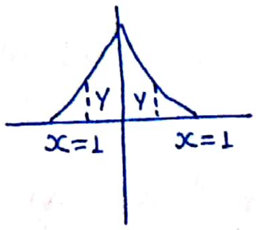
In 2 s wave cover 1 unit

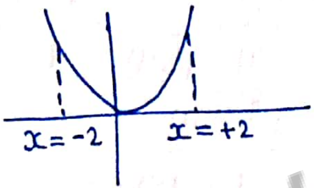
velocity $= \frac{1}{2} = 0.5$ unit/s

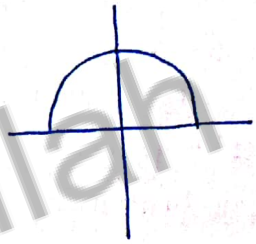
Symmetric Pulse

The pulse which is symmetric about y-axis is called symmetric pulse.

$\boxed{y(x) = y(-x)}$ i.e. (even function)







Question - 1

if pulse's eqN is — $y = \dfrac{0.8}{(4x+5t)^2+5}$ Find —

(i) direction of wave motion

(ii) y in 2 s

(iii) symmetric in nature

(iv) Maximum displacement of particle i.e. amplitude

$4x + 5t \Rightarrow ax + bt$ (same sign)

(i) $-ve$ x - direction

(ii) $v = b/a = 5/4 = 1.25$ m/s

In 2 s $y = 1.25 \times 2 = 2.5$ m

(iii) $y(x) = y(-x)$, t is variable hence $t = 0$,

$$5 + (4x+5t)^2 = (4x)^2 = 16x^2$$

(iv) y_{max} if $(4x+5t)^2 \longrightarrow 0$

$$y = \frac{0.8}{5} = 0.16 \text{ m}$$

on putting the values of ω & v

$$y = A \sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\cancel{F}} \times \frac{x}{\lambda} \times \cancel{F} + \phi\right)$$

$$y = A \sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x + \phi\right)$$

$$\boxed{y = A \sin(\omega t - Kx + \phi)}$$

$\because K = \frac{2\pi}{\lambda}$

$\because K$ = wave number

$$y = f(ax - bt)$$

$$y = f(\omega t - kx) \quad \text{(same sign)}$$

it travels in +ve x-direction.

$$\text{wave speed} = \frac{\text{coefficient of } t}{\text{coefficient of } x}$$

$$\boxed{V = \frac{\omega}{K}}$$

$\because \omega = \frac{2\pi}{T}$ & $K = \frac{2\pi}{\lambda}$

$$V = \frac{\cancel{2\pi}}{T} \times \frac{\lambda}{\cancel{2\pi}}$$

$$\boxed{V = \frac{\lambda}{T}}$$

$(\omega t - Kx + \phi) \Rightarrow$ phase

ϕ = initial phase at origin
($t = 0$ sec & $x = 0$)

we can write eq^N —

$$\boxed{y = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda} + \phi\right)}$$

General eq^N of wave —

$$\boxed{y = A \sin(\omega t \pm Kx + \phi)}$$

Question-1

if $y = 0.02 \sin\left(\frac{x}{0.01} + \frac{t}{0.05}\right)$, y is in m & x – m, t = sec

find — (i) A (ii) λ (iii) f (iv) v (v) initial phase ϕ at origin

$y = A\sin(\omega t \pm Kx + \phi)$

(i) $A = 0.02$ m

(ii) $\omega = \frac{1}{0.05}$ $\qquad K = \frac{1}{0.01}$

$K = 2\pi/\lambda \qquad \lambda = \frac{2\pi}{K}$

$\lambda = \frac{2\pi \times 0.01}{100} = \frac{\pi}{50}$ m

(iii) $\omega = 2\pi f$

$f = \omega/2\pi$

$f = \frac{100}{2\pi \times 0.05} = \frac{10}{\pi}$ sec^{-1}

(iv) $V = \omega/K = \frac{0.01}{0.05} = 1/5$ m/s

(v) initial phase $\phi = 0$

Question-2

if $y = 0.05 \sin\left[\frac{\pi}{2}(10t - 40x) - \frac{\pi}{4}\right]$ find —

(i) A (ii) λ (iii) f (iv) V (v) ϕ

$y = A\sin(\omega t \pm Kx + \phi)$

(i) $A = 0.05$

$y = 0.05 \sin\left(\frac{\pi}{2}10t - \frac{\pi}{2}\times 40x - \pi/4\right)$

$y = 0.05 \sin(5\pi t - 20\pi x - \pi/4)$

$\omega = 5\pi$ $\qquad$ (iii) $1/f = \frac{2\pi}{\omega} = \frac{2\pi}{5\pi}$ $\qquad f = 2.5$

$K = 20\pi$ $\qquad K = \frac{2\pi}{\lambda}$ $\qquad \lambda = \frac{2\pi}{20\pi}$

(ii) $\lambda = \frac{1}{10}$ $\qquad$ (iii) $\omega = 2\pi f$

$f = \frac{1}{10} \times 25 = 2.5$

$v = f\lambda$

$V = 2.5 \times \frac{1}{10} = 1/4$

$\boxed{\phi = -\pi/4}$

Question - 3

if $y = 2\sin\left(\frac{\pi}{2}(2t-5x)\right)$. what is the particle speed

(i) at origin, at $t = 0$ sec

(ii) at origin, at $t = 1$ sec

(iii) at $x = 2m$, at $t = 6$ sec

$$V_p = dy/dt$$

$$V = 2\pi\cos\left(\pi t - \frac{5}{2}x\pi\right)$$

(i) $x = 0,\ t = 0$

$$V = 2\pi\cos 0$$

$$V = 2\pi \text{ m/s}$$

(ii) $x = 0,\ t = 1$ sec

$$V = 2\pi\cos(\pi - 0)$$

$$V = -2\pi \text{ m/s}$$

(iii) $x = 2m,\ t = 6$ sec

$$V = 2\pi\cos\left(\pi\times 6 - \frac{5}{2}\times 2\pi\right)$$

$$V = 2\pi\cos(2\pi\times 3 - 5\pi)$$

$$V = 2\pi\cos\pi = -2\pi \text{ m/s}$$

(iii) Find the maximum velocity of particle.

$$V = \pm A\omega$$

$$\boxed{V = \pm 2\pi \text{ m/s}}$$

(iv) find acceleration of particle —

$$a_p = dv/dt$$

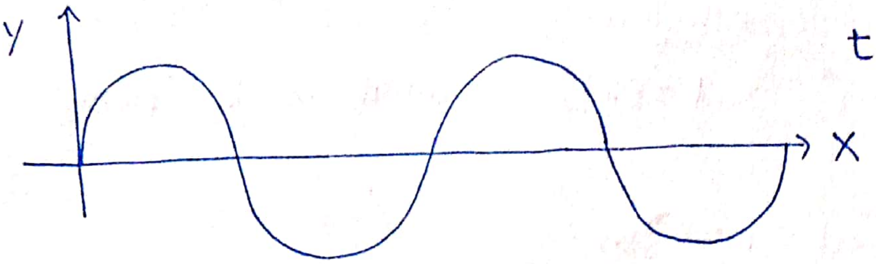
$$a_p = -a\omega^2\sin(\omega t \pm kx + \phi)$$

$$a_p = -2\cancel{4}\pi^2\sin\left(\pi/_2(2t-5x)\right)$$

Particle velocity

Particle velocity = − (wave velocity) × (slope of y v/s x curve)

$$\boxed{V_p = -v\times\left(\frac{\partial y}{\partial x}\right)}$$



t = constant (snapshot of wave)

$$y = A\sin(\omega t - Kx + \phi)$$

$$V_p = \frac{dy}{dt} = A\omega\cos(\omega t - Kx + \phi) \quad \text{—— (i)}$$

$$\frac{\partial y}{\partial x} = -KA\cos(\omega t - Kx + \phi) \quad \text{—— (ii)}$$

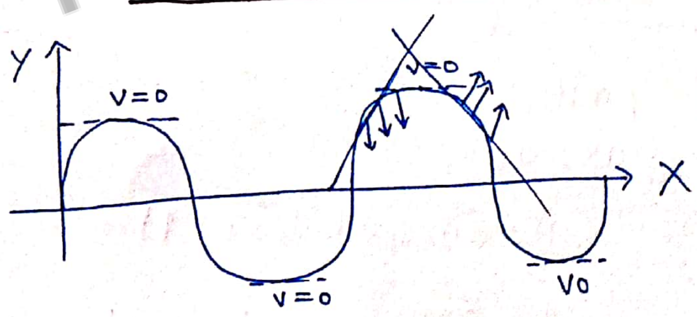
$$\frac{V_p}{\partial y/\partial x} = -\frac{\omega}{K}$$

$$\because \boxed{V = \omega/K} \text{ (wave velocity)}$$

$$\boxed{V_p = -V\left(\frac{\partial y}{\partial x}\right)} \quad \text{or} \quad \boxed{V_p = \pm\omega\sqrt{A^2 - y^2}}$$

($\partial y/\partial x$ = slope of y v/s X curve)

Direction of motion of particle



$$\boxed{V_p = -V(\partial y/\partial x)}$$

if slope ($\partial y/\partial x$) is +ve then

$\boxed{V_p = -ve}$ (−y −direction)

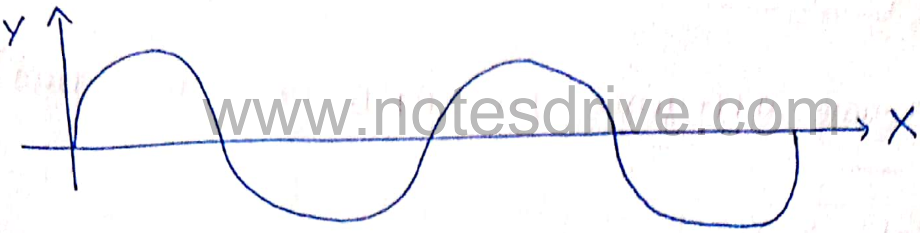
if slope is −ve then —

$\boxed{V_p = +ve}$ (+y − direction)

Acceleration of particle

$$y = A\sin(kx + \omega t + \phi)$$
$$v = A\omega\cos(\omega t + kx + \phi)$$
$$a_p = -A\omega^2 \sin(\omega t + kx + \phi)$$
$$\boxed{a_p = -\omega^2 y}$$

Direction of ~~di~~ acceleration of positive –



$a_p \longrightarrow$ depends upon $-y$

$y\ (+ve) \longrightarrow a_p\ (-ve)$

$y\ (-ve) \longrightarrow a_p\ (+ve)$

Phase

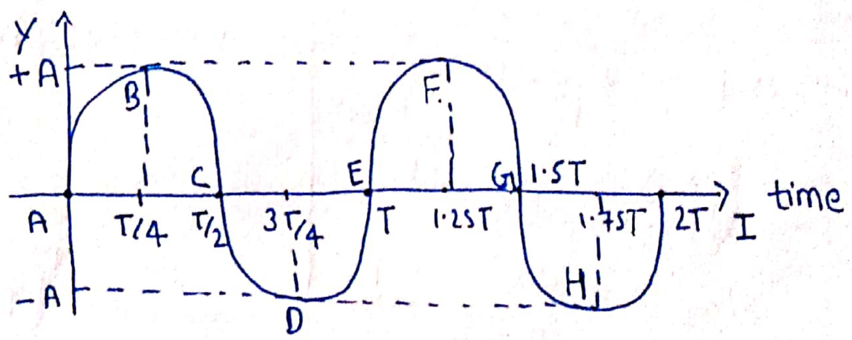
$$y = A\sin\underbrace{(\omega t - kx + \phi)}$$

y depens upon variable x & t

Phase $(\omega t - kx + \phi)$ gives complete information of particle.

if we fix x i.e. $x = 0$ or $x =$ constant

$y = A\sin(\omega t)$ (SHM)



(video Recording at fixed x)

$x =$ constant
$x =$ same

$t_1 = \phi_1$ $\qquad \phi_1 = \omega t_1 - kx$

$t_2 = \phi$ $\qquad \phi_2 = \omega t_2 - kx$

$$\phi_2 - \phi_1 = \omega t_2 - \omega t_1$$

$\Delta\phi$ = phase difference

$$\Delta\phi = \omega(t_2 - t_1)$$

$$\boxed{\Delta\phi = \frac{2\pi}{T} \times \Delta t}$$

phase difference for same particle at different time

$$\boxed{\Delta\phi = \frac{2\pi}{T}\Delta t}$$

Find the time difference b/w A & B & also find phase difference —

By graph of y v/s t —

$$\Delta t = T/4$$

$$\Delta\phi = \frac{2\pi}{T} \times \frac{T}{4}$$

$$\Delta\phi = \pi/2$$

B/w B & E —

$$\Delta t = T - T/4 = 3T/4$$

$$\phi = \frac{2\pi}{T} \times \Delta t = \frac{2\pi}{\cancel{T}} \times \frac{3\cancel{T}}{4} = \frac{3}{2}\pi$$

Question-1

if $y = 0.2 \sin 2\pi \left(\frac{t}{12} - \frac{x}{5}\right)$. find the difference in time b/w two positions of same particle which has 60° phase diffrence.

$$\Delta\phi = \frac{2\pi}{T}\Delta t$$

$$60 \times \frac{\cancel{\pi}}{180} = \frac{2\cancel{\pi}}{T}\Delta t$$

$$\Delta t = \frac{1}{3 \times 2} T$$

$$\Delta t = \frac{1}{3 \times 2} \times 12$$

$$\boxed{\Delta t = 2 \text{ seconds}}$$

$$y = 0.02 \sin\left(\frac{\pi t}{6} - \frac{2\pi x}{5}\right)$$

$$\omega = \pi/6$$

$$\frac{2\cancel{\pi}}{T} = \cancel{\pi}/6$$

$$\boxed{T = 12}$$

Same Phase

if particle repeats it velocity, acceleration & displacement or two particles are at same position. this is called same phase. After one time period (T) particle repeats its physical quantity hence after each time period phase difference is same.

$$\text{Same phase} = T, 2T, 3T, 4T \ldots\ldots + nT$$

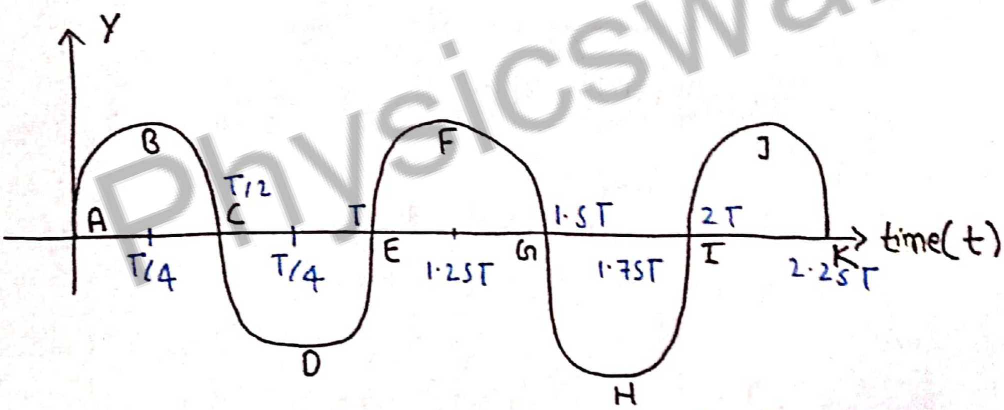
$$\text{phase diff. } (\Delta\phi) = \frac{2\pi}{T}\Delta t$$

$$\Delta\phi \text{ for same phase} = 2\pi, 4\pi, 6\pi \ldots\ldots, 2n\pi$$

Opposite phase

if particle — have

1. (i) Same distance but in opposite direction.
2. (ii) Same speed but in opp. direction.
3. (iii) Same Acceleration but in opp. direction.



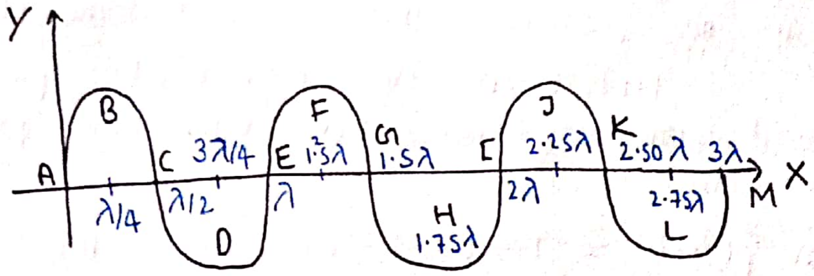
opposite phase $\Rightarrow$ (A & C), (A & G), (A & K)

$$\text{Time difference} = T/2, 3T/2, 5T/2, \ldots (2n+1)T/2$$

$$\text{phase difference } (\phi) = \frac{2\pi}{T}\Delta t$$

$$\phi = \pi, 3\pi, 5\pi, \ldots (2n+1)\pi$$

For different particles



(snapshot at an instant)

$$y = A\sin(\omega t - kx)$$

$$y = A\sin\left(\frac{2\pi}{T}t - kx\right)$$

For different particles — ~~time peri~~ is constant

let $\Rightarrow t = T/2$

$$y = A\sin\left(\frac{2\pi}{\cancel{T}} \times \frac{\cancel{T}}{2} - kx\right)$$

$$y = A\sin(\pi - kx)$$

$$y = A\sin kx$$

$$\boxed{y = A\sin\frac{2\pi}{\lambda}x}$$

y depends upon x

At position x_1

$x_1 = \phi_1 \qquad \phi_1 = \omega t - kx_1$

$x_2 = \phi_2 \qquad \phi_2 = \omega t - kx_2$

$$\phi_2 - \phi_1 = kx_1 - kx_2$$

$$\phi_2 - \phi_1 = k(x_1 - x_2)$$

$$\Delta\phi = k\Delta x$$

$$\boxed{\Delta\phi = \frac{2\pi}{\lambda}\Delta x}$$

Phase difference

For same particle — x = constant, t = variable $\qquad \boxed{\Delta\phi = \frac{2\pi}{T}\Delta t}$

For different particle – t = constant, x = variable $\qquad \boxed{\Delta\phi = \frac{2\pi}{\lambda}\Delta x}$

For Same phase

A & E & I are in same phase.

$\Delta x = \lambda, 2\lambda, 3\lambda \ldots\ldots, n\lambda$

$$\phi = \frac{2\pi}{\lambda}\Delta x$$

$$\phi = 2\pi, 4\pi, 6\pi$$

For Opposite phase

(A & C), (C & E), (B & D), (A & G), (B & H) are in opposite phase.

$$\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} \ldots\ldots + (2n+1)\lambda/2$$

$$\Delta\phi = \pi, 3\pi, 5\pi \ldots\ldots + (2n+1)\pi$$

Question-1

$y = 0.2 \sin 2\pi \left(t/2 - x/5\right)$, find the phase difference b/w two points 2.5 cm apart at same time.

$$\Delta x = 2.5 \text{ cm}$$

$$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x$$

$$K = 2\pi/5$$

$$K = \frac{2\pi}{\lambda} = \frac{2\pi}{5}$$

$$\boxed{K = 5}$$

$$\Delta\phi = \frac{2\pi}{5} \times 2.5$$

$$\boxed{\Delta\phi = \pi}$$

if x & t both are different -

$\phi_1 = \omega t_1 - kx_1$ & $\phi_2 = \omega t_2 - kx_2$

$$\boxed{\Delta\phi = \phi_2 - \phi_1}$$

Speed of wave on string (Transverse wave)

⇒ (one pulse of wave)

wave on lighterer mass travel fastly.

$$\text{speed of wave } (v) \propto \frac{1}{\text{mass of string}}$$

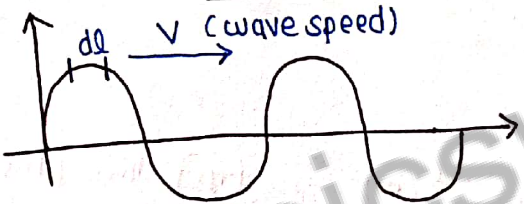
String / Rope has more tension mean the speed of wave is maximum.

$$\text{speed of wave } (v) \propto \text{Tension } (T)$$

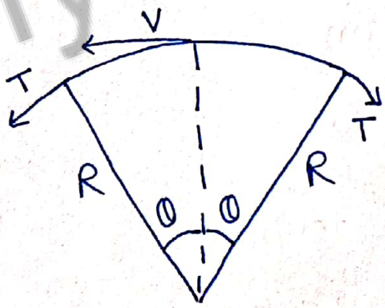
$$\boxed{V = \sqrt{\frac{T}{\mu}}}$$

V = speed of wave
T = Tension in string
μ = mass per unit length (linear density)

Derivation

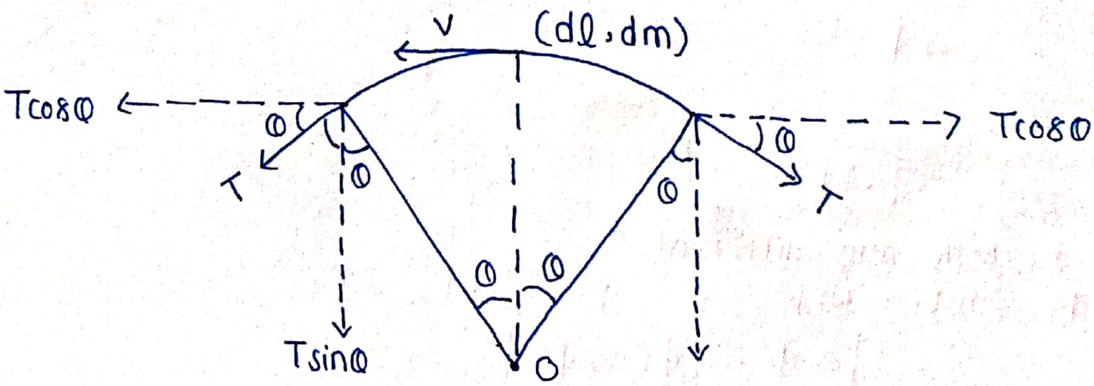


if we cut dl part —



if we assume that we are in the frame of wave & analyse than particle left back with the speed of wave v because dl is at rest

Tension is diffrent in all part of string because string have mass but dl is very small part hence tension is same.



$T\cos\theta$ cancelled each other & F_c acts towards centre O

$$\boxed{2T\sin\theta = F_c} \quad \text{——— (1)}$$

θ is very small because it has length dl & mass dm –

$$\sin\theta \approx \theta$$

$$2T\theta = F_c \quad \text{——— (i)}$$

(Arc = Radius × angle)

$$\boxed{dl = R(2\theta)}$$

μ = mass per unit length

$$\mu = \frac{dm}{dl}$$

$$\boxed{dm = \mu dl}$$

$$F_c = \frac{mv^2}{R} \quad \text{for dm mass —}$$

$$F_c = \frac{dm v^2}{R}$$

$$F_c = \frac{\mu dl v^2}{R}$$

$$F_c = \frac{\mu \cancel{R} 2\theta v^2}{\cancel{R}}$$

$$\boxed{F_c = \mu 2\theta v^2} \quad \text{——— (ii)}$$

From eq. (i) & (ii)

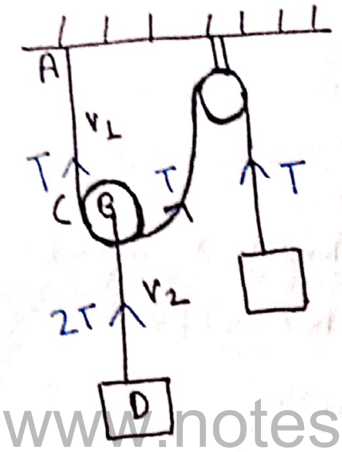
$$\cancel{2}T\cancel{\theta} = \mu\cancel{2}\cancel{\theta}v^2$$

$$v^2 = T/\mu$$

$$\boxed{v = \sqrt{T/\mu}}$$

Question - 1

All string have same material & same cross sectional Area Find the ratio of v_1 & v_2.



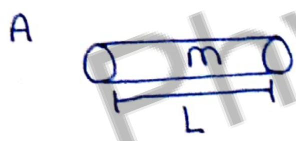
Tension is same for string & μ is same

$$\frac{v_1}{v_2} = \frac{\sqrt{\frac{T}{\mu}}}{\sqrt{\frac{2T}{\mu}}}$$

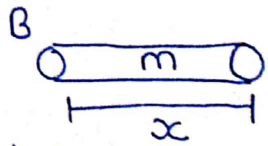
$$\boxed{\frac{v_1}{v_2} = \frac{\sqrt{2}}{1}}$$

Question - 2

There are two strings A & B both of them have same volum & material & stretched to same tension. if $R_A = 2R_B$. & A has v_A velocity & B has v_B velocity find the ratio of v_A & v_B.



$R_A = 2R_B$



$$\text{Volume (A)} = \text{volume (B)}$$

$$A \times L = A' \times x$$

$$\cancel{\pi}(2\cancel{R_B})^2 \times L = \cancel{\pi}(\cancel{R_B})^2 \times x$$

$$4L = x$$

$$\boxed{x = 4L}$$

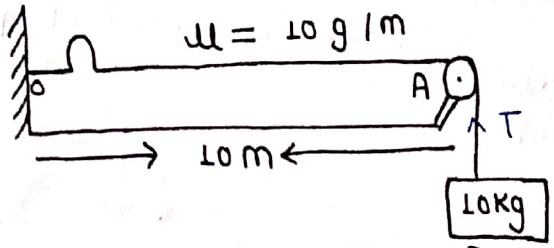
$\mu_A = m/L$ $\qquad$ $\mu_B = \frac{m}{4L}$

$$\frac{v_A}{v_B} = \frac{\sqrt{\frac{T}{\mu_A}}}{\sqrt{T/\mu_B}} = \sqrt{\frac{\mu_B}{\mu_A}}$$

$$\frac{v_A}{v_B} = \sqrt{\frac{\cancel{m}}{4\cancel{L}} \times \frac{\cancel{L}}{\cancel{m}}} = \frac{1}{2}$$

Ans ⇒ 1 : 2

Question-3



Given ⇒ $g = 10\ m/s^2$, find the time in which pulse travels from O to A.

$\mu = 10\,g/m$

$\mu = 10\times10^{-3}\ kg/m$

$T = mg$

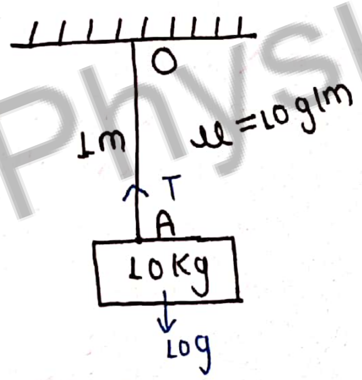
$T = 10g = 100\ N$

$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{10\cancel{0}}{1\cancel{0}\times10^{-3}}}$

$V = 100\ m/s$

$$\text{Time} = \frac{\text{Distance}}{\text{speed}} = \frac{10}{100} = 0.1\ m$$

Question-4



Assuming tension is constant. Find the time in which a pulse reaches from A to O.

$T = 10g = 100N$

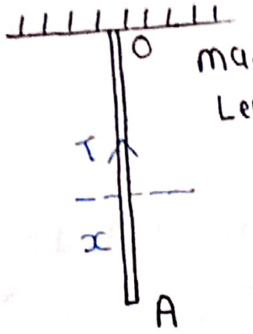
$\mu = 10g/m = 10\times10^{-3}\ kg/m$

$$\text{time} = \frac{1}{V}$$

$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{100}{10\times10^{-3}}} = 100\ m/s$$

$$t = \frac{1}{100} = 0.01\ sec$$

Question-5



mass (M)
Length (L)

Find the speed of pulse from A to O if tension is not constant.

$$V = \sqrt{\frac{T}{\mu}}$$

$T_A = 0$
$T_O = Mg$

V changes at every point because tension changes (due to mass of Rope)

$\because$ length L has mass M
if length x then — $\frac{M}{L}x$ (mass)

$$\boxed{T = \frac{M}{L}xg}$$

$$\boxed{\mu = \frac{M}{L}}$$

$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\cancel{\frac{M}{L}}xg}{\cancel{M/L}}}$$

$$\boxed{V = \sqrt{xg}}$$

$V \propto \sqrt{x}$
As x increases velocity of wave also increases.

$V = dx/dt$

$$\frac{dx}{dt} = \sqrt{xg}$$

$$\int_0^L \frac{dx}{\sqrt{x}} = \int \sqrt{g}\, dt$$

$$\left[\frac{x^{1/2}}{1/2}\right]_0^L = \sqrt{g}\,[t]_0^t$$

$$[2\sqrt{x}]_0^L = \sqrt{g}\, t$$

$$t = 2\sqrt{L}/\sqrt{g}$$

$$\text{time } t = 2\sqrt{\frac{L}{g}}$$

Alternative method

$$v = \sqrt{xg}$$

$$a = \frac{dv}{dt} \times \frac{dx}{dx} = \frac{v\,dv}{dx}$$

$$a = \sqrt{xg} \times \sqrt{g} \times \frac{1}{2\sqrt{x}}$$

$$\boxed{a = g/2 \ m/s^2}$$

acceleration is constant hence for time —

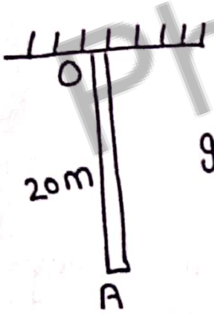
$$S = ut + \frac{1}{2}at^2$$

$$L = 0 + \frac{1}{2}\frac{g}{2}t^2$$

$$t^2 = \frac{4L}{g}$$

$$\boxed{t = 2\sqrt{L/g}}$$

Question-6



$g = 10 m/s^2$

if string is uniform & a pulse at free end is generated find the time at which pulse reached at point O.

$$t = 2\sqrt{L/g}$$

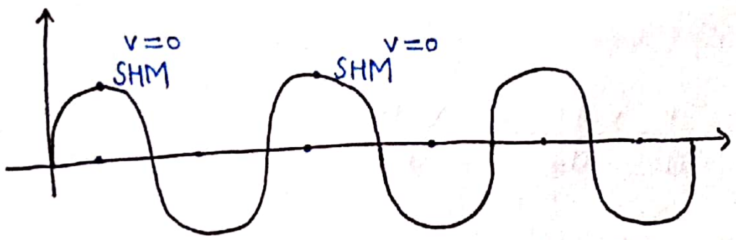
$$L = 20 m$$

$$g = 10 m/s^2$$

$$t = 2\sqrt{\frac{20}{10}}$$

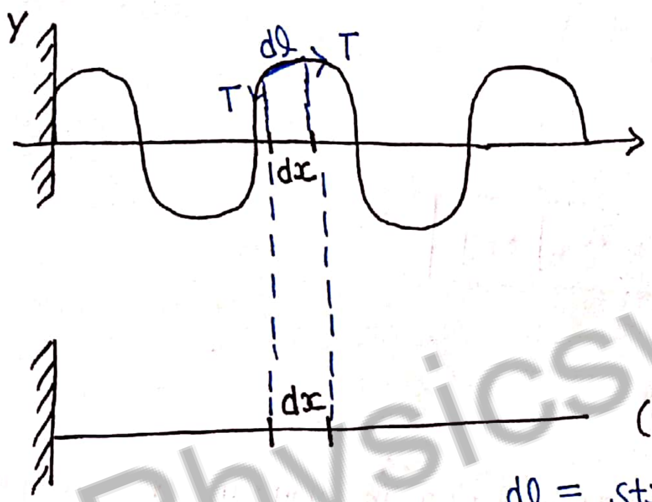
$$(t = 2\sqrt{2} \text{ seconds})$$

Energy of wave



In a wave, all particles are doing SHM & In SHM we know that total energy is constant but In SHM their is one particle, here all particles doing SHM.

Energy of a wave is variable & depends upon x & t.



(wave on string)

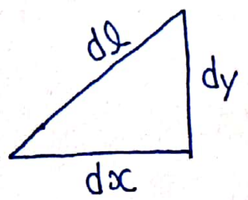
(unstretched string)

dl = streched length

dx = unstreched length

dy = height

$\because \; dl = \sqrt{dx^2+dy^2}$



Displacement in string due to tension $= dl - dx$

workdone by tension = Potential energy

$$P.E. = dU$$

$$dU = F \cdot ds$$

$$dU = T(dl - dx)$$

$$dU = T\left(\sqrt{dx^2+dy^2} - dx\right)$$

$$dU = T\,dx\left(\sqrt{1+\left(\frac{dy}{dx}\right)^2} - 1\right)$$

$\frac{dy}{dx}$ is very small due θ (very small)

By Binomial expansion —

$$(1+x)^n = 1+nx \quad \text{if} \quad x<<1$$

$$dU = T dx \left(\left(1+\left(\frac{dy}{dx}\right)^2\right)^{1/2} - 1\right)$$

$$dU = T dx \left(\cancel{1} + \frac{1}{2}\left(\frac{dy}{dx}\right)^2 - \cancel{1}\right)$$

$$\boxed{\frac{dU}{dx} = \frac{T}{2}\left(\frac{dy}{dx}\right)^2}$$

$\because y = A\sin(\omega t - Kx)$

$$\frac{dy}{dx} = -AK\cos(\omega t - Kx)$$

$$\boxed{\frac{dU}{dx} = \frac{T}{2} A^2K^2\cos^2(\omega t - Kx)}$$

velocity of string, $v = \sqrt{\frac{T}{\mu}}$

$\therefore T = v^2\mu$

$\because v = \omega/K \Rightarrow T = \frac{\omega^2}{K^2}\mu$

$$\frac{dU}{dx} = \frac{\omega^2\mu}{2\cancel{K^2}} A^2\cancel{K^2}\cos^2(\omega t - Kx)$$

$$\boxed{\frac{dU}{dx} = \frac{\mu A^2\omega^2}{2}\cos^2(\omega t - Kx)}$$

$\frac{dU}{dx} \Rightarrow$ (Potential energy per unit length)

Kinetic Energy

Kinetic energy of dx part $= \frac{1}{2}(dm)\, v_p^2$

$dK = \frac{1}{2}(dm)v_p^2 \qquad \because \frac{dm}{dx} = \mu$

$\therefore dm = \mu dx$

$dK = \frac{1}{2}\mu dx\, v_p^2$

$y = A\sin(\omega t - Kx)$

$v_p = \frac{dy}{dt} = A\omega\cos(\omega t - Kx)$

$$dK = \frac{1}{2}\mu dx\, A^2\omega^2 \cos^2(\omega t - Kx)$$

$$\boxed{\frac{dK}{dx} = \mu A^2\omega^2 \cos^2(\omega t - Kx)}$$

$\frac{dK}{dx}$ = Kinetic energy per unit length

$$\boxed{\frac{dK}{dx} = \frac{dU}{dx}}$$

Total energy in wave is not constant.

$$(DE)\ \frac{T.E.}{dx} = \frac{dK}{dx} + \frac{dU}{dx}$$

$$\boxed{\frac{DE}{dx} = \mu A^2\omega^2 \cos^2(\omega t - Kx)}$$

if all places Kinetic energy per unit length is equal to the potential energy per unit length.

$$\boxed{P.E = K.E. = \frac{DE}{2}}$$

Power in wave

Power. (P) = Energy (E) / Time(t)

$$P = \frac{dE}{dt} \times \frac{dx}{dx} \qquad \because \frac{dx}{dt} = \text{velocity of wave}$$

$$P = v\frac{dE}{dx}$$

$$\boxed{P_{instantaneous} = A^2\omega^2 \mu v \cos^2(\omega t - Kx)}$$

In one cycle –

Average of – $\sin x = 0$, $\cos x = 0$

Average of – $\sin^2 x$ / $\cos^2 x = 1/2$

Power in one complete cycle is called average power.

if $x = 0$

$$P_{inst.} = A^2 \omega^2 \mu v \cos^2 \omega t$$

In one cycle, $\cos^2 \omega t = 1/2$

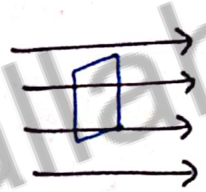
$$\boxed{P_{avg.} = \frac{A^2 \omega^2 \mu v}{2}}$$

Class 12th (AC) $\Rightarrow$

$$\boxed{P_{avg.} = \frac{\int P\,dt}{\int dt}}$$

Intensity of wave

The amount of energy passes through the unit area by unit time is called Intensity of a wave.

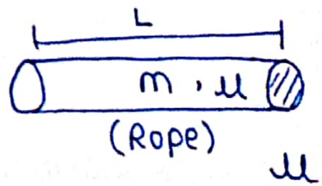


$$\boxed{I = \frac{E}{A \times t}}$$

time = 1s

this area is ⊥cular to energy.

$$I = P / \text{Area}$$



$$I = \frac{A^2 \omega^2 \mu v}{2 \times \text{Area}}$$

$$\mu = \frac{m}{L}$$

$$I = \frac{A^2 \omega^2 v\, m}{2 \times (\text{Area} \times L)}$$

Area × L = Volume

$$I = \frac{A^2 \omega^2 v}{2} \times \left(\frac{m}{\text{Volume}}\right)$$

$$I = \frac{A^2 \omega^2 \rho v}{2} \qquad \because \omega = 2\pi f$$

$$I = \frac{A^2 4\pi^2 \rho v f^2}{2}$$

$$\boxed{I = 2\pi^2 A^2 f^2 \rho v}$$

ρ = density of medium

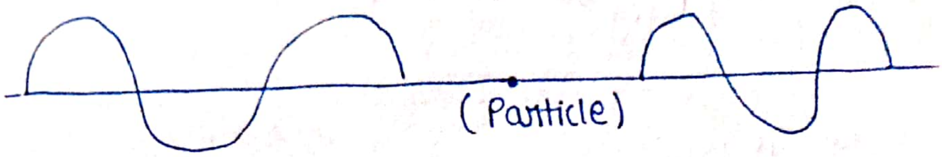
A = amplitude

f = frequency

v = velocity of wave

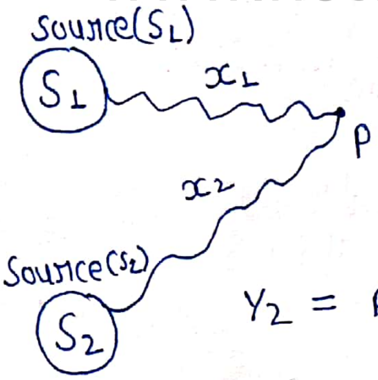
Interference of waves

When two or more waves meet at a point in a same medium is called interference of waves.



Net Displacement –

$$\vec{Y}_{net} = \vec{Y}_1 + \vec{Y}_2 \quad \text{(principal of superposition)}$$



$Y_1 = A_1 (\sin [\omega t + Kx])$
(wave eq^N)

$Y_2 = A_2 \sin (\omega t + Kx)$ (wave eq^N)

K & ω for both SHM eq^N are same. In this frequencies are also same. (coherent sources)

$f \rightarrow$ same

$\lambda = 2\pi f$

$\omega \rightarrow$ same $\quad \because K = 2\pi/\lambda$

$v = f\lambda$

$v \rightarrow$ same

SHM eq^N at particular $x \Rightarrow$

$$Y_1 = A_1 \sin (\omega t + Kx_1)$$

$$Y_2 = A_2 \sin (\omega t + Kx_2)$$

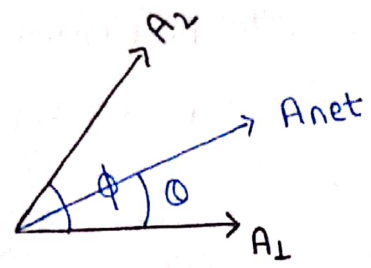
$$Y_{net} = Y_1 + Y_2$$

$$\boxed{Y_{net} = A_{net} \sin (\omega t + \theta)}$$

phase difference $(\Delta\phi) = Kx_2 - Kx_1$ (phase difference is due to path diff.)

$\phi = K(x_2 - x_1)$ Path diff. $= \Delta x$

$$\boxed{\phi = \frac{2\pi}{\lambda} \Delta x}$$



$$\boxed{A_{net}^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$\boxed{\tan\theta = \frac{A_2\sin\phi}{A_1 + A_2\cos\phi}}$$

Source S_1 & S_2 must be present : it may be sound source or light source.

<u>Coherent sources</u>– Coherent sources are those sources which can maintain a constant phase difference i.e. frequency is same for waves.

Intensity, $I = 2\pi^2 A^2 f^2 \rho v$

but here, f, ρ & v are constant.

$$\boxed{I = KA^2}$$

$I_1 = KA_1^2$, $\quad I_2 = KA_2^2$, $\quad I_{net} = KA_{net}^2$

$A_1^2 = \frac{I_1}{K}$ $\quad A_2^2 = \frac{I_2}{K}$ $\quad A_{net}^2 = \frac{I_{net}}{K}$

$$A_{net}^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$$

$$\frac{I_{net}}{\cancel{K}} = \frac{I_1}{\cancel{K}} + \frac{I_2}{\cancel{K}} + 2\sqrt{\frac{I_1}{\cancel{K}}} \times \sqrt{\frac{I_2}{\cancel{K}}}\cos\phi$$

$$\boxed{I_{net} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi}$$

<u>Types of Interference</u>

(i) Constructive Interference (ii) Destructive Interference

Constructive Interference	Destructive Interference
(i) A_{net} / I_{net} ⇒ maximum ($\cos\phi = 1$)	(i) A_{net} / I_{net} ⇒ minimum $\cos\phi = \phi - 1$
$A_{net} = (A_1 + A_2)$	$A_{net} = (A_1 - A_2)$
(ii) $I_{net} = (\sqrt{I_1} + \sqrt{I_2})^2$	(ii) $I_{net} = (\sqrt{I_1} - \sqrt{I_2})^2$
(iii) $\phi = 0, 2\pi, 4\pi \cdots 2n\pi$	(iii) $\phi = 180°(\pi), 3\pi, 5\pi, \ldots, (2n+1)\pi$
$\frac{2\pi}{\lambda}\Delta x = 0, 2\pi, 4\pi \cdots 2n\pi$	$\frac{2\pi}{\lambda}\Delta x = \pi, 3x, 5x \cdots (2n+1)\pi$
(iv) $\Delta x = 0, \lambda, 2\lambda, \ldots\ldots, n\lambda$	(iv) $\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \cdots (2n+1)\frac{\lambda}{2}$
(v) if amplitudes are same $A_1 = A_2 = A$	(v) $A_1 = A_2 = A$
$A_{net} = 2A$	$A_{net} = 0$
$I_{net} = 4I$	$I_{net} = 0$

Question-1

Two waves which have same frequencies, Intensities are in the ratio of 9:16. Find the ratio of Max I to Min. I of resultant wave if these are two superimpose.

$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2 = (\sqrt{9} + \sqrt{16})^2 = (3+4)^2 = 49$$

$$I_{min} = (\sqrt{I_1^2} - \sqrt{I_2^2})^2 = (3-4)^2 = 1$$

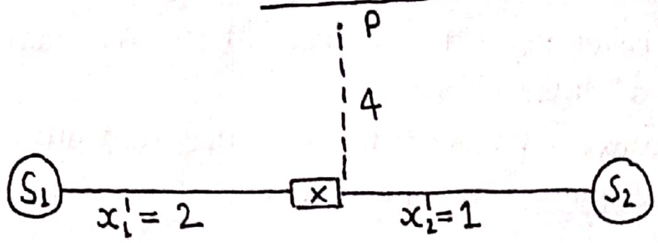
Ratio = 49 : 1

if ratio of amplitudes are 3 : 5. Find the ratio of Max I & ~~Max~~ Min I.

$$I_1 = KA_1^2 = 9K$$
$$I_2 = KA_2^2 = \cancel{16}\ 25K$$

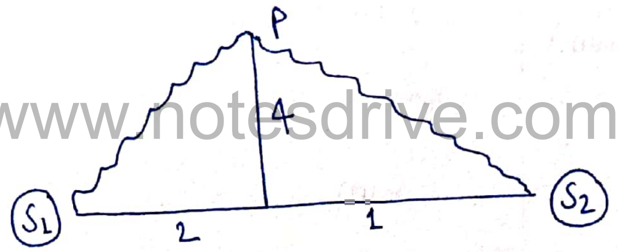
$$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2} = \frac{64}{4} = 16 : 1$$

Question-3



S_1 & S_2 are sound sources.

if speed of sound (V_{sound}) = $340\ m/s^2$ then for what frequencies will we hear a loud sound at P.



path difference (Δx) $= x_2 - x_1$

$$= \sqrt{4^2+2^2} + \sqrt{4^2+1^2}$$

$$= 4.8 - 4.1$$

$$\boxed{\Delta x = 0.7}$$

for constructive interference — $\cos\phi = 1$

$$\phi = 0, 2\pi, 4\pi, 6\pi \dots 2n\lambda\pi$$

$$\frac{2\pi}{\lambda}\Delta x = 0, 2\pi, 4\pi \dots 2n\lambda\pi$$

$$\Delta x = \lambda, 2\lambda, 3\lambda \dots n\lambda$$

$$0.7 = n\lambda$$

$$\lambda = 0.7/n$$

$$\lambda = 0.7\ m,\ 0.7/2\ m,\ 0.7/3\ m,\ 0.7/4\ m$$

$$v = f\lambda$$

$$\lambda = v/f$$

$$\lambda = \frac{340}{0.7}, \frac{340 \times 2}{0.7}, \frac{340 \times 3}{7} \dots\dots$$

Reflection & Transmission of wave on a string

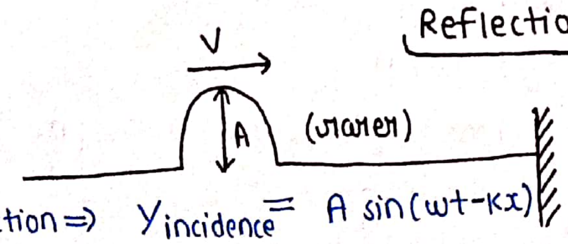
if wave collides with boundary & comes back in same medium this phenomenon is called Reflection.

if wave passes from one medium to another medium is called ~~reflection~~ the phenomenon of transmission.

Properties of Reflection & Transmission

Property	Reflection	Transmission
① velocity of wave (depends upon medium) $v = f\lambda$	Same	change
② frequency (f) $\omega = 2\pi f$, $f = \frac{1}{T}$	Same	Same
③ wavelength λ, $K = \frac{2\pi}{\lambda}$	Same	change
④ phase diff. (ϕ)	$\phi = 0$ if wave comes back in denser medium. $\phi = \pi$ if wave comes back to rare medium	$\phi = 0$ (in all medium)
⑤ A, ($I \propto A^2$)	if energy is conserved Amplitude $= A$	$A = 0$ if energy is not transfered.

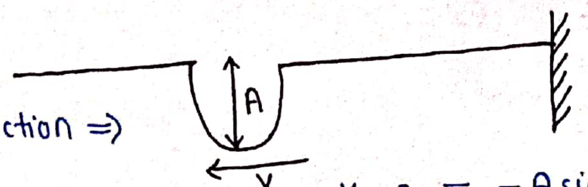
① **Reflection from fixed End**



$+x$ - direction $\Rightarrow$ $y_{incidence} = A\sin(\omega t - kx)$

Energy loss $= 0$
Transmission $= 0$

$\phi = \pi$

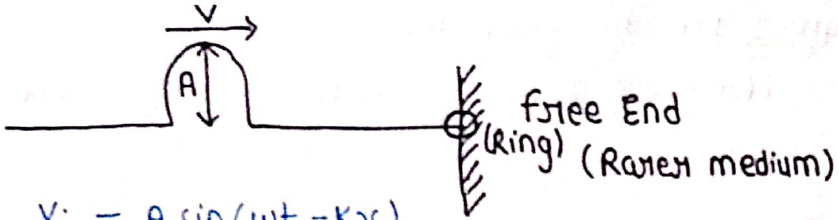


$-x$ - direction $\Rightarrow$

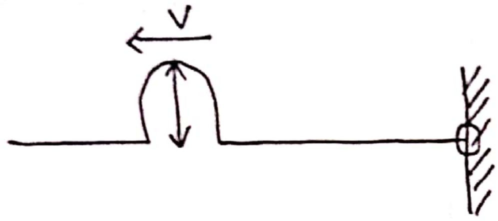
$y_{refl.} = -A\sin(\omega t + kx)$

$y_r = A\sin(\omega t + kx + \pi)$

Reflection from Free End



$y_i = A\sin(\omega t - kx)$



$y_R = A\sin(\omega t + kx)$

Question - 1

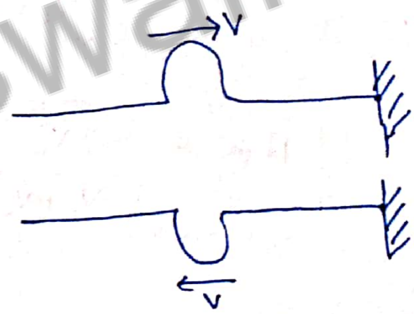
$y_i = 2\sin(4x - 8t)$ is reflected at $x=0$ from —

(i) fixed End (ii) Free End

$y_R = ?$

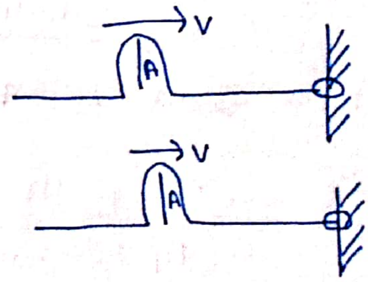
(i) $y_i = 2\sin(4x - 8t)$

$y_R = -2\sin(4x + 8t)$



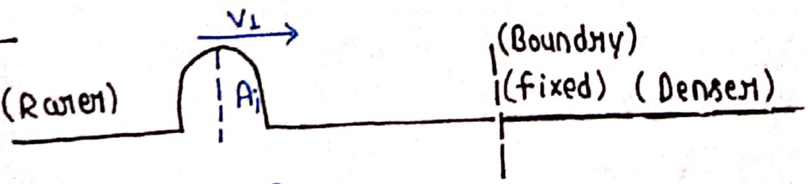
(ii) $y_i = 2\sin(4x - 8t)$

$y_R = 2\sin(4x + 8t)$

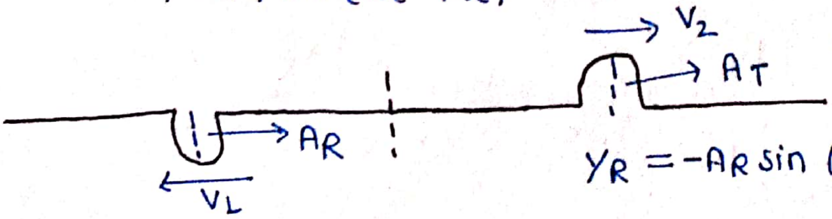


Transmission & Reflection

Case (i)



$y = A_i\sin(\omega t - kx)$

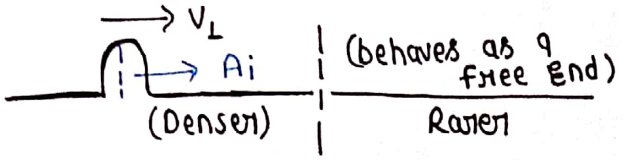


$y_T = A_T\sin(\omega t - k'x)$

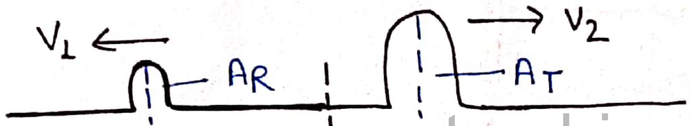
$y_R = -A_R\sin(\omega t + kx)$

velocity v_1 is same for reflection because medium is same.
ω is same due to frequency.
wavelength changes due to which K also differs.

Case (ii)



$$Y_i = A_i \sin(\omega t - Kx)$$



$$y_R = A_R \sin(\omega t + Kx)$$

$$y_T = A_T \sin(\omega t - K'x)$$

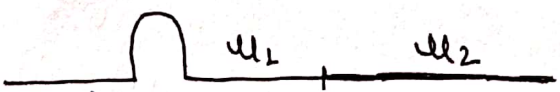
$$\boxed{A_R = \left(\frac{v_2 - v_1}{v_1 + v_2}\right) A_i}$$

$$\boxed{v = f\lambda}$$

$$\boxed{A_T = \left(\frac{2v_2}{v_1 + v_2}\right) A_i}$$

Question-1

if $y_i = 3\sin(2x - 4t)$ & $\mu_2 = 4\mu_1$. Find y_R & y_T.



$(A_i = 3, \ \omega_1 = 4, \ K_1 = 2)$

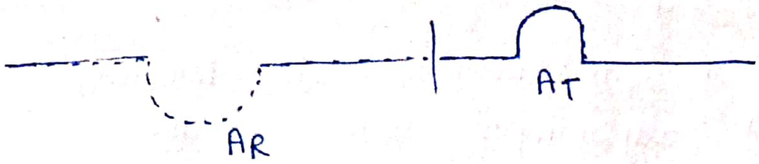
$$v_1 = \omega/K = 4/2 = 2$$

$$v = \sqrt{\frac{T}{\mu}} \Rightarrow \frac{v_2}{v_1} = \sqrt{\frac{\cancel{T}}{4\cancel{\mu_1}} \times \frac{\cancel{\mu_1}}{\cancel{T}}}$$

$$\boxed{v_2 = 2/2 = 1}$$

$$A_R = \left(\frac{v_2 - v_1}{v_1 + v_2}\right) A_i \Rightarrow \left(\frac{1-2}{1+2}\right) \times 3 \Rightarrow -1$$

$$A_T = \left(\frac{2v_2}{v_1 + v_2}\right) A_i \Rightarrow \frac{2\times 1}{3} \times 3 \Rightarrow 2$$



$$\boxed{Y_R = -\sin(2x + 4t)}$$

$$Y_T = A_T \sin(\omega t - K'x)$$

$$V_1 = f\lambda_1$$
$$V_2 = f\lambda_2$$

$$\frac{V_2}{V_1} = \frac{\lambda_2}{\lambda_1} = \frac{1}{2}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{1}{2} \qquad K = \frac{2\pi}{\lambda}$$

$$K \propto \frac{1}{\lambda}$$

$$K_1 \propto \frac{1}{\lambda_1} \quad \& \quad K_2 \propto \frac{1}{\lambda_2}$$

$$\frac{K_2}{K_1} = \frac{\lambda_2}{\lambda_1} = \frac{2}{1}$$

$$\boxed{K_2 = 2K_1}$$

$$K_2 = 2 \times 2 = 4$$

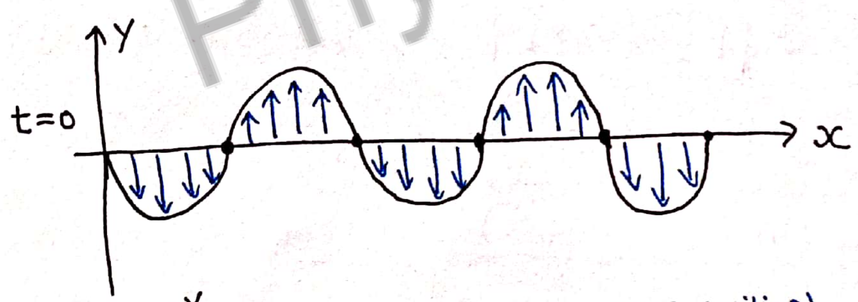
$$\boxed{Y_T = 2\sin(\omega t - 4x)}$$

Standing wave

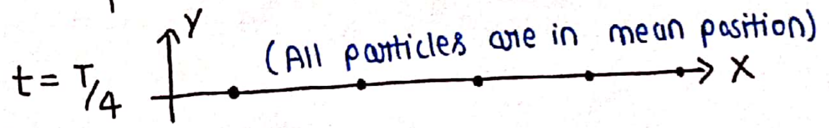
When two waves of same amplitude, same frequency & (in opposite direction) super impose with each other.

Standing waves are found in string & organ pipe.

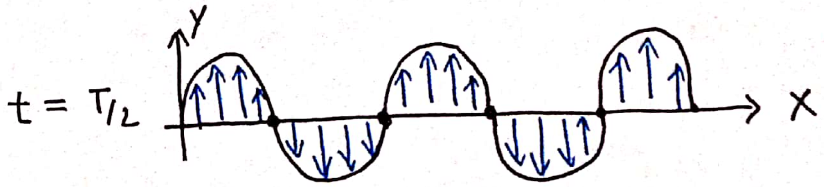
t	$y_i = A\sin(\omega t - kx)$	$y_r = -A\sin(\omega t + kx)$	$\vec{y}_{net} = \vec{y_i} + \vec{y_r}$
0	$-A\sin kx$	$-A\sin kx$	$-2A\sin kx$
$T/4$	$+A\sin\left(\frac{2\pi}{T}\times\frac{T}{4} - kx\right)$ $A\cos kx$	$-A\sin\left(\frac{\pi}{2} + kx\right)$ $-A\cos kx$	0
$T/2$	$A\sin\left(\frac{2\pi}{T}\times\frac{T}{2} - kx\right)$ $A\sin kx$	$-A\sin\left(\frac{2\pi}{T}\times\frac{T}{2} + kx\right)$ $A\sin kx$	$2A\sin kx$
$3T/4$	$A\sin\left(\frac{2\pi}{T}\times\frac{3T}{4} - kx\right)$ $-A\cos kx$	$-A\sin\left(\frac{3\pi}{2} + kx\right)$ $+A\cos kx$	0
T	$A\sin\left(\frac{2\pi}{T}\times T - kx\right)$ $-A\sin kx$	$-A\sin(2\pi + kx)$ $-A\sin kx$	$-2A\sin kx$



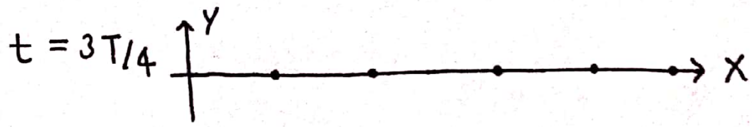
$y_{net} = -2A\sin kx$



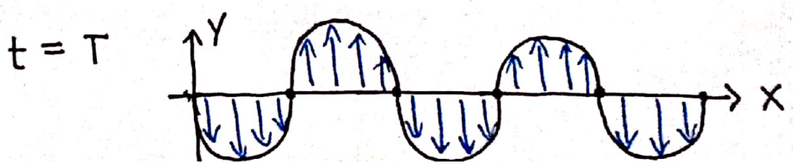
$y_{net} = 0$



$y_{net} = +2A\sin kx$



$y_{net} = 0$



$y_{net} = -2A\sin kx$

In all 4 graphs we can see some particles whose displacement is always 0 irrespective of time. These particles are nodes.

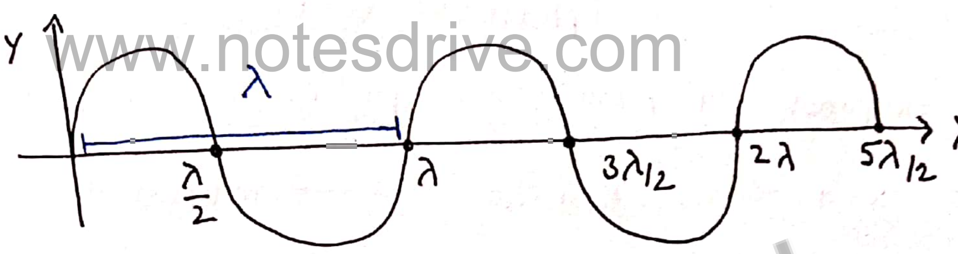
Some particle doing SHM at the height of amplitude.

All particles are doing SHM but with different amplitude.

Those particles which have maximum displacement are called Antinodes.

In real ways standing waves are not a waves these are the SHM of particles. There is no transfer of energy.

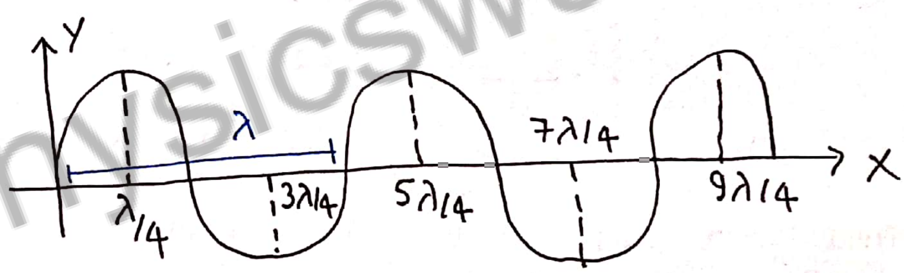
Nodes



Distance b/w two consecutive nodes is always constant.

— Distance b/w two consecutive nodes $= \lambda/2$

Antinodes –



Distance b/w two consecutive Antinodes

$= \lambda/2$

Distance b/w nodes & Antinodes

$= \lambda/4$

Fixed End Reflection

$y_i = A\sin(\omega t - kx)$

$y_R = -A\sin(\omega t + kx)$

$$\vec{y}_{net} = \vec{y_i} + \vec{y_R}$$

$$Y_{net} = A[\sin(\omega t - kx) - \sin(\omega t + kx)]$$

$$Y_{net} = A \times 2\cos(\omega t) \times \sin(-kx)$$

$$\boxed{Y_{net} = \underline{-2A \sin kx}\, \cos \omega t}$$

Equation standing waves (stationary waves) —

$$\boxed{Y_{net} = -2A \sin kx \cos \omega t}$$

$$\boxed{\text{Amplitude} = -2A \sin kx}$$

$$\boxed{\text{phase} = \cos \omega t}$$

Amplitude of particle is dependent on the position of particle.

<u>Nodes</u> – $Y_{net} = 0$ $t \rightarrow$ independent

$$Y_{net} = -2A \sin kx \underline{\cos \omega t}$$

$$+2A \sin kx = 0 \qquad \sin 0 \Rightarrow 0^\circ$$

$$kx = 0, \pi, 2\pi, 3\pi \ldots\ldots$$

$$\frac{2\pi}{\lambda} x = 0, \pi, 2\pi, 3\pi \ldots\ldots$$

$$\boxed{x = 0, \lambda/2, \lambda, 3\lambda/2, 2\lambda}$$

<u>Antinodes</u> Amplitude must be maximum $\Rightarrow$

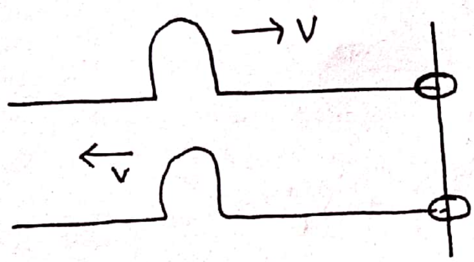
$$\sin kx \longrightarrow \text{maximum } (1)$$

$$kx = \pi/2, 3\pi/2, 5\pi/2 \ldots\ldots$$

$$\frac{2\pi}{\lambda} x = \pi/2, 3\pi/2, 5\pi/2 \ldots\ldots$$

$$\boxed{x = \lambda/4, 3\lambda/4, 5\lambda/4 \ldots\ldots}$$

<u>Free End Reflection</u>



$$Y_i = A\sin(\omega t - kx)$$

$$Y_r = A\sin(\omega t + kx)$$

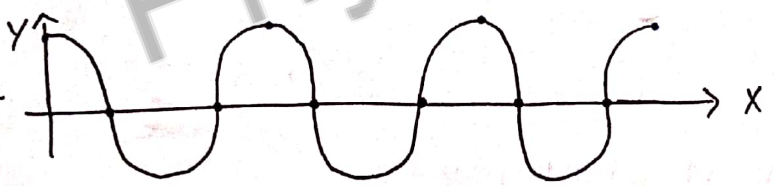
t	$y_i = A\sin(\omega t - kx)$	$y_r = A\sin(\omega t + kx)$	$\vec{y}_{net} = \vec{y}_i + \vec{y}_r$
0	$-A\sin kx$	$A\sin kx$	0
$T/4$	$A\sin\left(\frac{2\pi}{T}\times\frac{T}{4} - kx\right)$ $A\cos kx$	$A\sin\left(\frac{\pi}{2} + kx\right)$ $A\cos kx$	$2A\cos kx$
$T/2$	$A\sin\left(\frac{2\pi}{T}\times\frac{T}{2} - kx\right)$ $A\sin kx$	$A\sin(\pi + kx)$ $-A\sin kx$	0
$3T/4$	$A\sin\left(\frac{2\pi}{T}\times\frac{3T}{4} - kx\right)$ $-A\cos kx$	$A\sin\left(\frac{3\pi}{2} + kx\right)$ $-A\cos kx$	$-2A\cos kx$
T	$A\sin\left(\frac{2\pi}{T}\times T - kx\right)$ $-A\sin kx$	$A\sin(2\pi + kx)$ $+A\sin kx$	0

Snapshot of Resultant wave —

$t = 0$

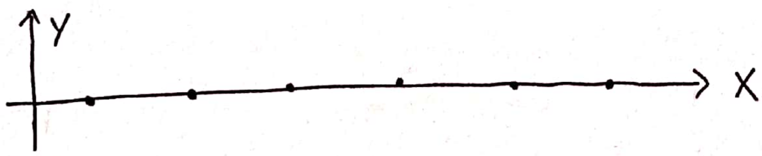
$y_{net} = 0$

$t = T/4$



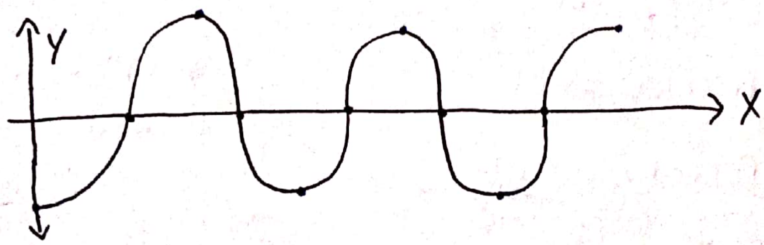
$y_{net} = 2A\cos kx$

$t = T/2$



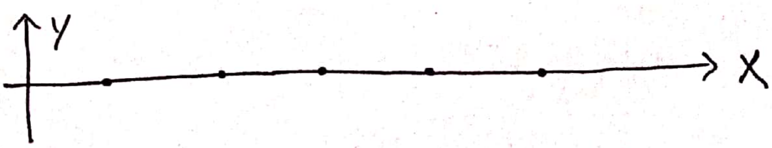
$y_{net} = 0$

$t = 3T/4$



$y_{net} = -2A\cos kx$

$t = T$



$y_{net} = 0$

$$\vec{Y}_{net} = \vec{y}_i + \vec{y}_r$$

$$Y_{net} = A\sin(\omega t - Kx) + A[\sin(\omega t + Kx)]$$

$$Y_{net} = A[\sin(\omega t - Kx) + \sin(\omega t + Kx)]$$

$$Y_{net} = 2A \sin\omega t \cos Kx$$

$$\boxed{Y_{net} = 2A\cos Kx \sin\omega t}$$

Amplitude $= 2A\cos Kx$

Phase $= \sin\omega t$

Nodes – $Y_{net} = 0$ $t \rightarrow$ independent

$$Kx = \pi/2, 3\pi/2, 5\pi/2 \ldots$$

$$\frac{2\pi}{\lambda} x = \pi/2, 3\pi/2, 5\pi/2 \ldots$$

$$\boxed{x = \lambda/4, 3\lambda/4, 5\lambda/4 \ldots}$$

Antinodes – Amplitude $\longrightarrow$ max.

$$\cos Kx = 1$$

$$Kx = 0, \pi, 2\pi, 3\pi, 4\pi \ldots$$

$$\frac{2\pi}{\lambda} x = 0, \pi, 2\pi, 3\pi, 4\pi \ldots$$

$$\boxed{x = 0, \lambda/2, \lambda, 3\lambda/2, 2\lambda}$$

All equations of standing wave

$$y = \underbrace{2A\sin Kx}_{\text{Amplitude}}\ \underbrace{\cos\omega t}_{\text{phase}}$$

$$y = -2A\sin Kx \sin\omega t$$

$$y = +2A\cos Kx \cos\omega t$$

$$y = -2A\cos Kx \cos\omega t$$

these all are eq^N of standing waves.

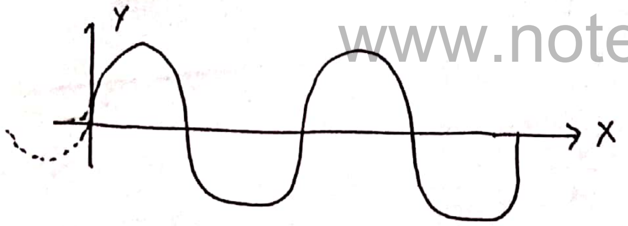
Many eq^N can be formed.

Travelling wave

① Energy is transferred in these waves.

② Amplitude (Max. displacement) of each particle is same.

③ At any time, all the particles passes from mean position simultaneously.



$Y = A\sin(\omega t - Kx)$

$Y = A\sin[\omega(t+t_1) - Kx]$

Any phase difference b/w two particles –
$(0, 2\pi) \rightarrow \Delta\phi$

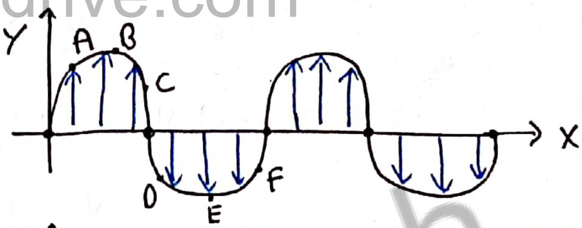
Standing wave / stationary wave

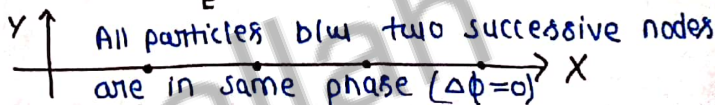
① Energy is not carried / transferred.

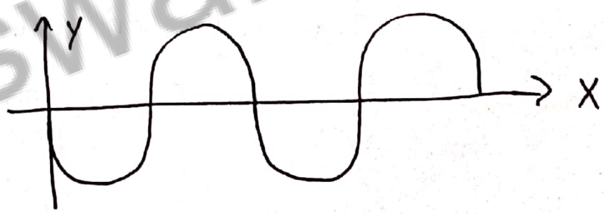
② Amplitude is different for different particles.

Antinodes → Max. Nodes → Min.

③ At each time period, All the particles passes from mean-position simultaneously.



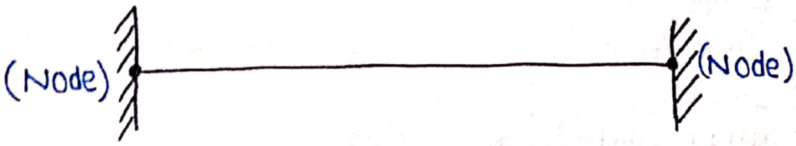




A & B, B & C are in same phase because these are in b/w two successive nodes.
i.e. $\Delta\phi = 0$

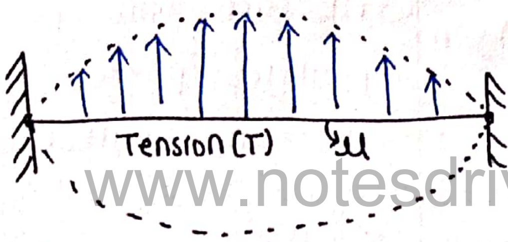
Standing wave have $\Delta\phi = 0$ or $\Delta\phi = 0$

Standing waves on a string fixed at both Ends



Normal modes of string —

(i) Fundamental mode/ tone —



$$\text{Frequency } (f) = \frac{v}{\lambda}$$

$$\because v = \sqrt{\frac{T}{\mu}}$$

Distance b/w two nodes $= \lambda/2$

let length of string $= l$

$$l = \lambda/2$$

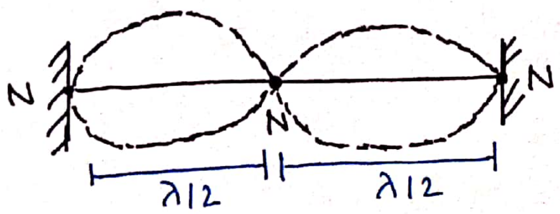
$$\boxed{\lambda = 2l}$$

$$f_1 = \frac{\sqrt{\frac{T}{\mu}}}{\lambda}$$

$$\boxed{f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}}$$

f = fundamental frequency or first harmonic frequency.

(ii) First over tone / mode —



$\boxed{\text{wavelength} = 2\,\lambda/2}$

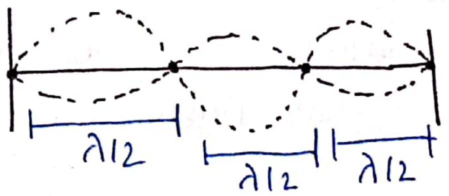
$$f_2 = \frac{v}{\lambda}$$

$$\boxed{f_2 = \frac{2}{2l}\sqrt{\frac{T}{\mu}}}$$

f_2 = Second harmonic frequency

$$\boxed{f_2 = 2f_1}$$

(iii) Third harmonic mode / Second overtone ⇒



no. of lobes = 3

$$l = 3\lambda/2 \qquad \lambda = 2l/3$$

$$f_3 = \frac{v}{\lambda} = \frac{3}{2l}\sqrt{\frac{T}{\mu}}$$

$$\boxed{F_3 = \frac{3}{2l}\sqrt{\frac{T}{\mu}}}$$

F_3 = third harmonic frequency

$$\boxed{F_3 = 3F_1}$$

(iv) $(n-1)^{th}$ overtone ⇒ For n^{th} harmonic —

no. of lobes = n

$$\frac{n\lambda}{2} = l \qquad \lambda = \frac{2l}{n}$$

$$f_n = \frac{v}{\lambda}$$

$$\boxed{f_n = \frac{n}{2l}\sqrt{\frac{T}{\mu}}}$$

F_n = n^{th} harmonic frequency.

$$\boxed{F_n = nF_1}$$

(no. of nodes = n+1)

Find 9^{th} overtone —

$$F_8 = \frac{8}{2l}\sqrt{\frac{T}{\mu}}$$

$$F_8 = \frac{4}{l}\sqrt{\frac{T}{\mu}}$$

8^{th} harmonic = 9^{th} overtone

Question-1

In a normal mode of vibration of a string tied at both ends, the difference in frequencies of fifth harmonic & second overtone is 54 Hz. Calculate fundamental frequency.

$$F_5 = 5F_1 \qquad F_5 - F_3 = 54 \text{ Hz}$$

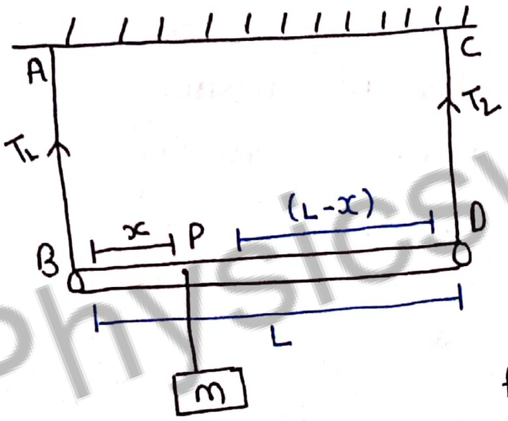
$$F_3 = 3F_1$$

$$5F_1 - 3F_1 = 54$$

$$(F_1 = 27 \text{ Hz})$$

— Fundamental frequency is 27 Hz.

Question-2



BD is a massless rod

AB & CD are massless string (identical)

Fundamental frequency of left wire is twice the fundamental frequency of left wire. Find x.

$$\tau_p = 0$$

due to mass m torque is 0 because it passes at P.

$$T_1 x = T_2 (L-x) \quad \text{—— (1)}$$

$$f_1 = 2f_2$$

$$\frac{1}{\cancel{2l}}\sqrt{\frac{T_1}{\cancel{\mu}}} = \frac{2}{\cancel{2l}}\sqrt{\frac{T_2}{\cancel{\mu}}}$$

$$\boxed{T_1 = 4T_2}$$

$$4\cancel{T_2}x = \cancel{T_2}(L-x)$$

$$5x = L$$

$$\boxed{x = L/5}$$

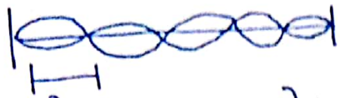
Question-3

A string fixed at both ends has consecutive standing wave modes for which distances between adjacent nodes are 18 cm & 16 cm respectively. Find —

(i) what is the mode of vibration ?

(ii) what is the minimum possible length of string ?

let First harmonic is nth —



$\lambda_1/2$ $\qquad \frac{\lambda_1}{2} = 18 \qquad \frac{n\lambda_1}{2} = l \qquad l = 18n$ —— (i)

let 2nd harmonic is $(n+1)^{th}$

$\lambda_2/2$ $\qquad \frac{\lambda_2}{2} = 16 \qquad (n+1)\frac{\lambda_2}{2} = l$

$l = 16(n+1)$ —— (ii)

$$18n = 16(n+1)$$
$$2n = 16$$
$$n = 8$$

(i) 8^{th} harmonic & 9^{th} harmonic

(ii) $l = 18n \qquad l = 18 \times 8 = 144$ cm

Question-4

A horizontal stretched string fixed at two ends, is vibrating in 5th harmonic, According to eqN — $\qquad \pi = 3.14$

$$y = 0.01 \sin(62.8\, m^{-1} x) \cos(628\, s^{-1} t)$$

Find —

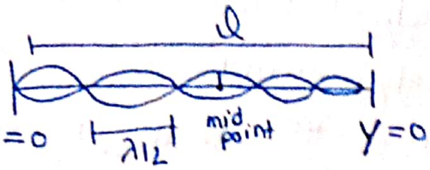
(i) no. of nodes (ii) length of string (l)

(iii) Maximum displacement of mid point of string from it mean position.

(iv) fundamental frequency (f_1)

(i) no. of nodes = $(n+1)$ $\because n =$ harmonic no.

no. of nodes = $(5+1) = 6$



(ii) $$y = 0.01 \sin(62.8\, m^{-1} x) \cos(628\, s^{-1} t)$$

$$K = 62.8$$

$$\frac{2\pi}{\lambda} = 62.8$$

$$\lambda = 2\pi / 62.8$$

$$\lambda = \frac{2 \times \cancel{3.14}}{\cancel{62.8}\,20}$$

$$\boxed{\lambda = 1/10}$$

$$5\,\lambda/2 = l$$

$$l = \frac{5 \times 1}{20} = 0.25\, m$$

(iii) $A = 0.001\, m$

(iv) $$f_1 = \frac{V}{\lambda} = \frac{\omega / K}{\lambda} \qquad \because \lambda/2 = l$$
$$\lambda = 2l$$

$$f_1 = \frac{\omega}{K \times 2l}$$

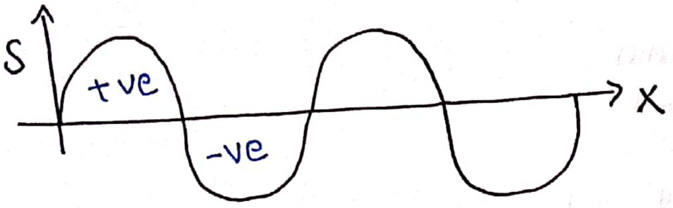
$$f_1 = \frac{\cancel{628000}\;1000}{\cancel{62.8} \times 0.25 \times 2}$$

$$f_1 = 40/2 = 20\, Hz$$

$$\boxed{f_1 = 20\, Hz}$$

Sound Wave

longitudinal wave is a sound wave & waves on a string is transverse wave.



$$\boxed{S = A\sin(\omega t - Kx)}$$

S = displacement of particle

Sound wave travels due to pressure & density variation.

Compression – Pressure → Maximum, Density → Maximum

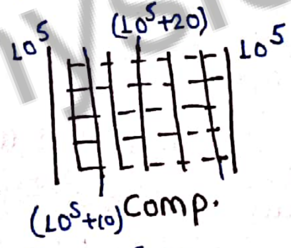
Rarefaction – Pressure → Minimum, Density → Minimum

Normal pressure = 1 atm (10^5 pa)

$$\pm \Delta P = 20 \text{ pa}$$

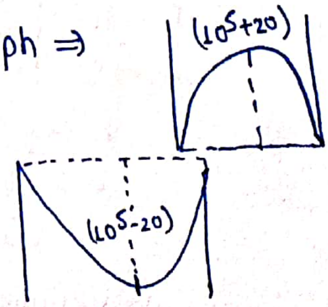
At compression — $P = (10^5 + 20)$ Pa (max)

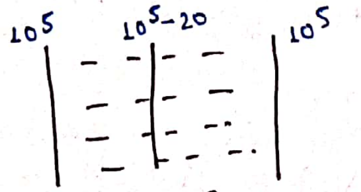
At Rarefaction — $P = (10^5 - 20)$ Pa (min)



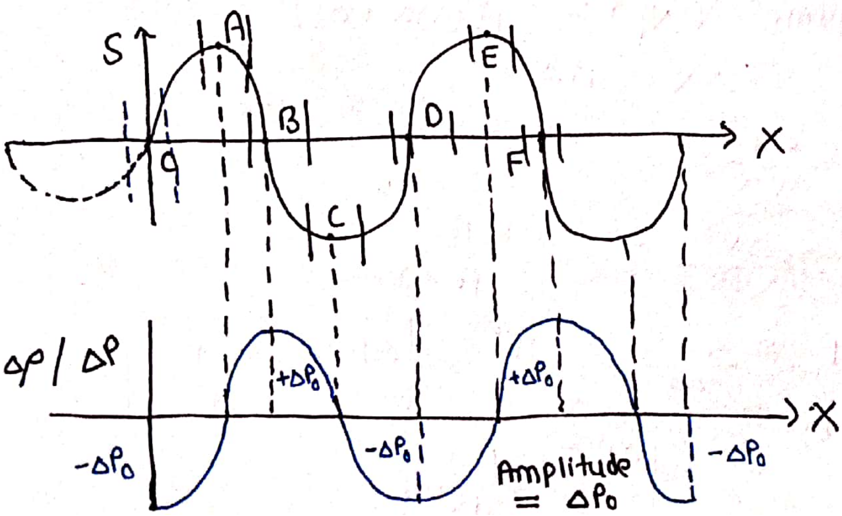
ΔP is maximum in mid of compression

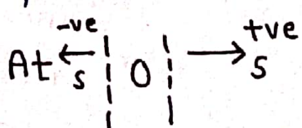
Graph ⇒





Rarefaction

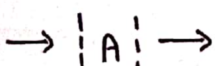




Particle stretched (Rarefaction)

$P \rightarrow$ minimum

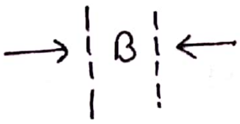
ΔP = most –ve (max)



No ΔP

$\Delta P = 0$

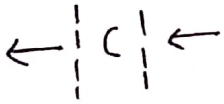
Normal Pressure



Particle compressed hence —

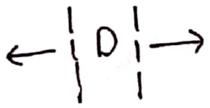
$\Delta P =$ maximum (+ve)

$P =$ max



$P =$ min

$\Delta P = 0$



Rare faction

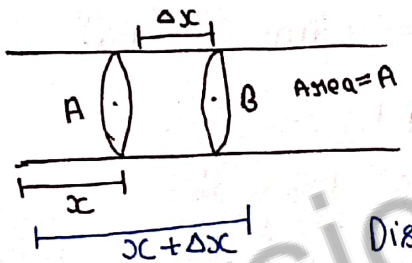
$\Delta P =$ min

$P =$ min

ΔP & $\Delta \rho$ have same graph.

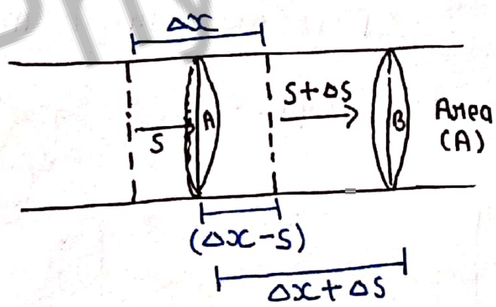
Pressure wave & density wave have phase difference of $(\pi/2)$ with displacement of sound wave's particle.

Proof



undisturbed medium
(wave is not present)

Displacement of particle A & B are different.



disturbed medium
(wave is present)

initial volume $(V_i) = A\Delta x$

final volume $(V_f) = A(\Delta x + \Delta S)$

$\Delta V = A\Delta S$

Bulk modulus (K) $\Rightarrow -\dfrac{\Delta P}{\Delta V / V}$

$$\Delta P = \frac{K\Delta V}{V} \Rightarrow -\frac{K\cancel{A}\Delta S}{\cancel{A}\Delta x}$$

$$\Delta P = -\frac{K\Delta S}{\Delta x} \quad \text{or} \quad \boxed{\Delta P = -\frac{K\,ds}{dx}}$$

Bulk modulus represent by K & β $\Rightarrow$ $\boxed{\Delta P = -\dfrac{\beta\,ds}{dx}}$

$$S = A\sin(\omega t - Kx)$$

$$\frac{ds}{dx} = -KA\cos(\omega t - Kx) \quad \text{--- (i)}$$

$$\Delta P = -\beta\frac{ds}{dx}$$

$$\Delta P = \beta KA\cos(\omega t - Kx)$$

$$\boxed{\Delta P = \beta KA\cos(\omega t - Kx)}$$

Amplitude $= \beta KA$ Phase $= \cos(\omega t - Kx)$

$$\Delta P_0 = \beta KA$$

$$\boxed{\Delta P = \Delta P_0\cos(\omega t - Kx)}$$

$$\sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta$$

$$\because \theta = \omega t - Kx$$

$$\boxed{\cos(\omega t - Kx) = \sin\left(\frac{\pi}{2} + \omega t - Kx\right)}$$

$$\boxed{\Delta P = \Delta P_0\sin\left(\omega t - Kx + \pi/2\right)}$$ (Pressure wave Equation)

$$\boxed{S = A\sin(\omega t - Kx)}$$

$$\Delta\phi = \left(\cancel{\omega t} \cancel{-Kx} + \pi/2 \cancel{-\omega t} \cancel{+Kx}\right)$$

$$\boxed{\Delta\phi = \pi/2}$$

Question - 1

if a sound wave travelling in air then $\lambda = 35\,cm$ & Amplitude of SHM particle is $5.5 \times 10^{-6}\,m$. At any point pressure varies as $(10^5 \pm 14)$ Pa. Find bulk modulus of medium.

$$\Delta P = \pm 14$$

$$A_{mp} = \beta K A$$

$$\Delta P_0 = \beta K A \qquad \lambda = 35 \times 10^{-2}\,m$$

$$\because K = \frac{2\pi}{\lambda} \Rightarrow \frac{2\pi}{35 \times 10^{-2}}$$

$$\beta = \Delta P_0 / KA$$

$$\beta = \cancel{10^5}\ \frac{14 \times 35 \times 10^{-2}}{5.5 \times 10^{-6} \times 2\pi}$$

$$\beta = \frac{\cancel{14}^{7} \times \cancel{35}^{7} \times 10^5 \times 7}{\underset{11}{\cancel{55}} \times \underset{11}{\cancel{22}} \times 2}$$

$$\beta = \frac{343}{242} \times 10^5 = 1.41 \times 10^5\ N/m^2$$

Density wave Equation

if Pressure maximum $\longrightarrow$ Density maximum

$$\Delta P = \Delta P_0 \sin(\omega t - kx + \pi/2)$$

$$\boxed{\Delta \rho = \Delta \rho_0 \sin(\omega t - kx + \pi/2)}$$

Proof

$$\text{density}(\rho) = \frac{\text{mass}(m)}{\text{volume}(V)}$$

$$m = \rho \times V$$

on differentiating both side
mass is constant.

$$0 = \frac{\rho\, dv}{dx} + \frac{v\, d\rho}{dx}$$

$$-\rho\, dv = v\, d\rho$$

$$dv = \Delta V$$

$$d\rho = \Delta \rho$$

$$-\rho \Delta V = V \Delta \rho$$

$$\Delta \rho = -\rho \frac{\Delta V}{V}$$

$$\Delta V = A \Delta S = A ds$$
$$V = A \Delta x = A dx$$

$$\Delta \rho = -\rho \left(\frac{ds}{dx}\right)$$

$$S = A \sin(\omega t - Kx)$$

$$\frac{ds}{dx} = -KA\cos(\omega t - Kx)$$

$$\Delta \rho = +\rho KA\cos(\omega t - Kx)$$

$$\boxed{\Delta \rho = \Delta \rho_0 \cos(\omega t - Kx)}$$

$$\boxed{\Delta \rho_0 = \rho KA}$$

$$\boxed{\Delta \rho = \rho KA \sin\left(\omega t - Kx + \pi/2\right)}$$

$$\boxed{S = A \sin(\omega t - Kx)}$$

$$\boxed{\Delta \phi = \pi/2}$$

Speed of Sound (Longitudinal wave)

$$\boxed{v = \sqrt{\frac{E}{\rho}}}$$

E = coefficient of Elasticity
ρ = density of medium

For liquid / gases ⇒ In liquid modulus of Elasticity is called Bulk modulus.

$$\boxed{E = \beta} \qquad \because \beta = \frac{-P}{\Delta V/V}$$

$$\boxed{v = \sqrt{\frac{\beta}{\rho}}}$$

For Solid – In solid coefficient of elasticity is called the young modulus.

$$\boxed{E = Y} \qquad \because Y = \frac{Fl}{A\Delta l}$$

$$\boxed{v = \sqrt{\frac{Y}{\rho}}}$$

Speed of Sound waves In air (By Newton)

$$\boxed{v = \sqrt{\frac{\beta}{\rho}}}$$

Assume, if Sound travels in air in constant temperature i.e. process is isothermal.

$$\Delta T = 0$$

According to Boyle's law —

$$PV = \text{constant}$$

on differenting —

$$Pdv + vdp = 0$$

$$Pdv = -vdp$$

$$P = -vdp/dv$$

$$P = \frac{-dp}{v/dv}$$

Bulk modulus (K)/(β) $= \frac{-\Delta P}{V/\Delta V}$

$P = $ Bulkmodulus

$$\boxed{P = \beta_{isothermal}}$$

$$V_{sound} = \sqrt{\frac{\beta}{\rho_{air}}}$$

$$V = \sqrt{\frac{P_{air}}{\rho_{air}}}$$

$\because$ Pressure of air $= 1.01 \times 10^5$ pascal

$\because$ Density of air $= 1.29\ kg/m^3$

$$V = \sqrt{\frac{1.01 \times 10^5}{1.29}}$$

$$\boxed{V \simeq 280\ m/s}$$

But according to Experiment —

Speed of sound in air at $0°c$ = 332 m/s

—— Hence velocity of sound in air given by Sir Isaac Newton is wrong.

Speed of Sound waves In air

(Laplace correction)

According to Laplace — Rarefaction & compression make very fast i.e. process is very fast i.e. there is no time to exchange heat & process is adiabatic.

At compression —— Temperature increases due to particle compression.

At Rarefaction —— Temperature decreases – pressure & density will decreases.

In Adiabatic — $\Delta Q = 0$

$$PV^{\gamma} = \text{constant}$$

$\because \gamma = cp/cv = 1 + \frac{2}{F}$

on differentiating partially ——

$$P\gamma V^{\gamma - 1} dV + V^{\gamma} dP = 0$$

$$v^{\gamma} dp = -p\gamma v^{\gamma-1} dv$$

$$dp = \frac{-\gamma p v^{\gamma-1} dv}{v^{\gamma}}$$

$$dp = \frac{-\gamma P dv}{V}$$

$$dp = \frac{-\gamma P}{V/dv}$$

$$\text{Bulk modulus } (\beta) = \frac{-\Delta P}{\Delta V/V}$$

$$\frac{-dp}{dv/V} = +\gamma P$$

$$\boxed{+\gamma P = \beta_{adiabatic}}$$

$$V = \sqrt{\frac{\beta}{\rho}}$$

$$\boxed{V = \sqrt{\frac{\gamma P}{\rho}}}$$

$$\gamma_{air} = 1 + \frac{2}{F} = 1 + \frac{2}{5}$$

DOF of gases is 5 because Nitrogen has 78% constituent of air & it is diatomic

$$\gamma_{air} = 1.4 \ N/m^2 \qquad \rho_{air} = 1.29 \ kg/m^3$$

$$P_{air} = 1.01 \times 10^5 \ Pa$$

$$V = \sqrt{\frac{1.4 \times 1.01 \times 10^5}{1.29}}$$

$$V \simeq 333 \ m/s$$

$$\boxed{V_{experimental} = 332 \ m/s}$$

$$\boxed{V_{\text{sound in air}} = \sqrt{\frac{\gamma P}{\rho}}}$$

$PV = nRT$

$PV = \frac{m}{M} RT$

$\frac{PM}{RT} = \frac{m}{V}$ $\quad \because \rho = m/V$

$\frac{PM}{RT} = \rho$

$\boxed{\frac{P}{\rho} = \frac{RT}{M}}$ $\quad \because V = \sqrt{\frac{\gamma P}{\rho}}$

$\boxed{V = \sqrt{\frac{\gamma RT}{M}}}$

R = 8.314 J/mol K
T = temperature (K)
M = mass (in kg)

$\boxed{\gamma = 1 + \frac{2}{F}}$

Question - 1

Find the speed of sound in ——

(i) O_2 gas at 0°C

(ii) He gas at 27°C

(i) $T = 0°C = 273\ K$

$F = 5$, $\gamma = 1 + \frac{2}{5} = 1.4\ N/m^2$

$M = 32\ gm = 32 \times 10^{-3}\ kg$

$R = 8.314\ J/mol K$

$V = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{1.4 \times 8.314 \times 273}{32 \times 10^{-3}}} = 316\ m/s$

(ii) $T = 27°C = 300\ K$

$F = 2$, $\gamma = 1 + \frac{2}{2} = (2)\ \gamma$

$M = 2\ gm = 2 \times 10^{-3}\ kg$

$V = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{\cancel{2} \times 8.314 \times 300}{\cancel{2} \times 10^{-3}}}$

$V = \sqrt{8314 \times 300} = \sqrt{2494200} = \backsimeq 1580\ m/s$

Question - 2

At what temp. speed of sound in H_2 gas will be same as speed of sound in O_2 gas at $27°C$.

For H_2 gas — Temp. = T , $\gamma = 1 + \frac{2}{F} = 1 + \frac{2}{5} = 1.4$

$M = 1\ gm = 1 \times 10^{-3}\ kg$

$R = 8.314\ J/mol\ K$

For O_2 gas — Temp. = $27°C = 300\ K$

$\gamma = 1.4 \quad M = 32 \times 10^{-3}\ kg$

$$V_{H_2} = V_{O_2}$$

$$\sqrt{\frac{\gamma RT}{M_1}} = \sqrt{\frac{\gamma RT'}{M_2}}$$

$$\sqrt{\frac{T}{2 \times 10^{-3}}} = \sqrt{\frac{300}{32 \times 10^{-3}}}$$

$$\sqrt{T} = \sqrt{\frac{300}{32}}$$

$$T = \frac{2 \times \cancel{300}}{\cancel{32}} \frac{75}{8}$$

$$T = 2 \times 9.375\ K$$

$$\boxed{T = 18.750\ K}$$

Question - 3

Find Ratio of RMS speed to speed of sound —

$$V_{sound} = \sqrt{\frac{\gamma RT}{M}}$$

$$V_{RMS} = \sqrt{\frac{3RT}{M}}$$

$$\boxed{\frac{V_{sound}}{V_{RMS}} = \sqrt{\frac{\gamma}{3}}}$$

Factors on which speed of sound in Air depends

(i) **Temperature** – $V = \sqrt{\frac{\gamma RT}{M}}$ T = absolute temp.

$$\boxed{V \propto \sqrt{T}}$$

Ratio of speed of sound in $t^\circ C$ to speed of sound in air at $0^\circ C$.

$$\frac{V_{t^\circ C}}{V_{0^\circ C}} = \sqrt{\frac{t+273}{0+273}} \qquad \because V_{t^\circ C} = \sqrt{\frac{\gamma R(t+273)}{M}}$$

$$\because V_{0^\circ C} = \sqrt{\frac{\gamma R(273)}{M}}$$

$$\frac{V_{t^\circ C}}{V_{0^\circ C}} = \sqrt{1 + t/273}$$

$$\frac{V_t}{V_0} = \left(1 + t/273\right)^{1/2}$$

if $t << 273$

$(1+x)^n \Rightarrow (1+nx)$ if $x << 1$

$$\frac{V_{t^\circ C}}{V_{0^\circ C}} = 1 + \frac{t}{273 \times 2}$$

$$\frac{V_{t^\circ C}}{V_{0^\circ C}} = 1 + t/546$$

$$\boxed{V_{0^\circ C} = 332 \text{ m/s}}$$

$$\boxed{V_{t^\circ C} = 332 + 0.61t}$$ [valid upto $25^\circ C$]

$$\boxed{\frac{V_1}{V_2} = \sqrt{\frac{T_2}{T_1}}}$$ (Always valid)

$V = \sqrt{\frac{\gamma RT}{M}}$ γ, R & M are constant, if temperature is constant velocity of sound in air also constant

Velocity of sound in air = constant at constant temp.

(ii) Pressure – $v = \sqrt{\gamma P/\rho}$

$v \propto \sqrt{P}$ & $v \propto \frac{1}{\sqrt{\rho}}$

— if we increase pressure, density also increases

P/ρ = constant

if $\Delta T = 0$ i.e. temperature is constant then there is no effect of P & ρ. in speed of sound in air.

(iii) Humidity –

$$\boxed{\rho_{\text{dry air}} > \rho_{\text{moist air}}}$$

Reason : Molar mass of dry air is 29 gm/mole but In moist gases some particles of (29 gm/mol) converted into water molecule (18 gm/mol).

$$\boxed{\rho \propto M}$$

Hence ρ of dry air is more than that of moist air.

$$\boxed{v \propto \frac{1}{\sqrt{\rho}}}$$

if pressure is same —

$$\boxed{V_{\text{sound in dry air}} > V_{\text{sound in moist air}}}$$

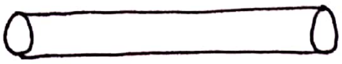
— In rainy days, we can observed that we can easily here sound with low velocity.

Organ Pipe

organ pipe is a pipe which is made up of metal, wood. & sometimes with glass.

There are two types of organ pipe —

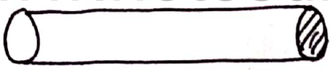
(i) Open organ pipe —



(Both ends are open)

(Sound standing waves)

(ii) Closed organ pipe —



(one end is closed)

(Sound standing waves)

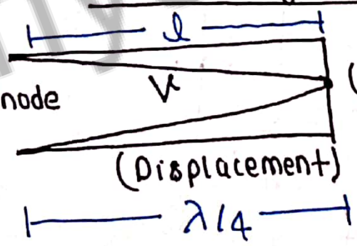
Open End — Both ends have antinode in open organ pipe.

Closed End — In closed organ pipe one side is closed hence one end (open) has antinode & closed end has node.

Closed organ pipe

(i) Ist harmonic

Fundamental tone :



(Fundamental tone ⇒ Ist harmonic)

$\lambda/4 = L$, $\lambda = 4L$

$$f_1 = \frac{V}{\lambda} = \frac{V}{4L}$$

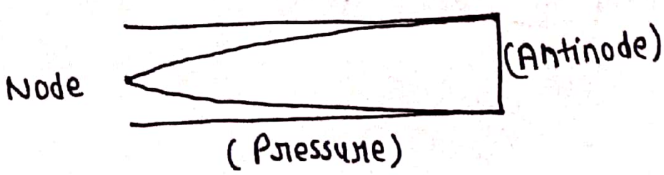
$$\boxed{f_1 = \frac{V}{4L}}$$

$\boxed{\text{fundamental frequeny} = f_1}$

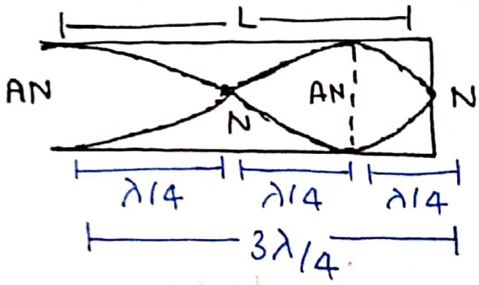
V = speed of air
L = length of closed organ pipe

closed organ pipe for pressure standing wave.

$\Delta\phi = \pi/2$ with displacement



(ii) First overtone – Third harmonic



$$L = \frac{3\lambda}{4}, \quad \lambda = \frac{4L}{3}$$

$$f = \frac{V}{\lambda}$$

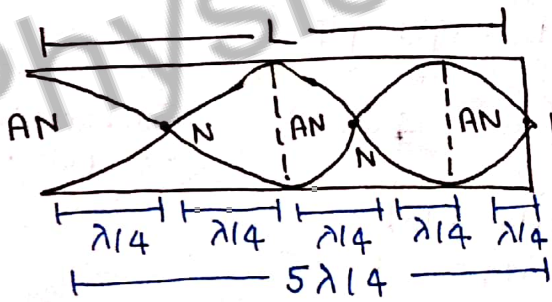
$$f = \frac{V}{4L/3}$$

$$\boxed{f_3 = 3\left(\frac{V}{4L}\right)}$$

Third harmonic frequency $= f_3$

$$\boxed{f_3 = 3f_L}$$

(iii) Second over tone – (Fifth harmonic)



$$L = 5\lambda/4 \qquad \lambda = \frac{4L}{5}$$

$$f = \frac{V}{\lambda} = \frac{V}{4L/5}$$

$$\boxed{f = 5\left(\frac{V}{4L}\right)}$$

fifth harmonic frequency $= f_5$

$$\boxed{F_5 = 5F_L}$$

In closed organ pipe Even harmonic are absent —

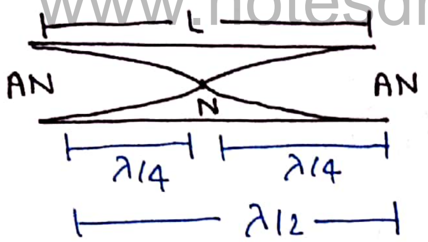
$$\boxed{n^{th} \text{ overtone} = (2n+1)^{th} \text{ harmonic}}$$

$$\text{Fundamental frequency } (f_1) = \frac{V}{4L}$$

$$\boxed{\cancel{f_n = n F_1}} \qquad \boxed{f_{(2n+1)} = (2n+1) f_1}$$

Open organ pipe

(i) Fundamental tone (first harmonic) —



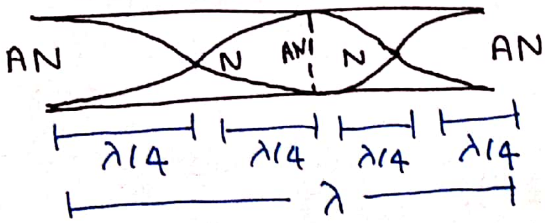
$$L = \lambda/2$$

$$\boxed{\lambda = 2L}$$

$$f_1 = \frac{V}{\lambda} = \frac{V}{2L}$$

$$\boxed{f_1 = \frac{V}{2L}}$$

(ii) first overtone (second harmonic) —



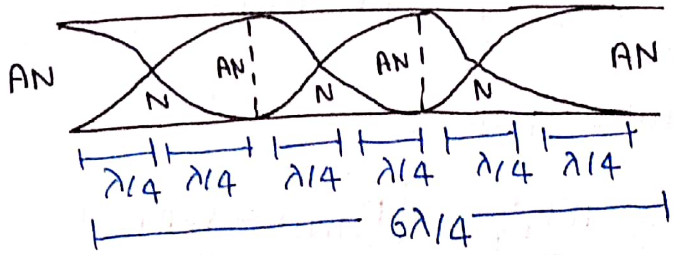
$$\lambda = L$$

$$f_2 = \frac{V}{\lambda} = \frac{V}{L}$$

$$\boxed{f_2 = 2\left(\frac{V}{2L}\right)}$$

$$\boxed{f_2 = 2 f_1}$$

(iii) Second overtone (3rd harmonic) –



$$\frac{6\lambda}{4} = L$$

$$\lambda = \frac{4L}{6} = \frac{2L}{3}$$

$$f_3 = \frac{v}{\lambda} = \frac{v}{2L/3}$$

$$\boxed{f_3 = 3\left(\frac{v}{2L}\right)}$$

$$\boxed{f_3 = 3F_1}$$

All harmonic are exist –

$$\boxed{n\text{th overtone} = (n+1)\text{th harmonic}}$$

$$\boxed{f_n = nF_1}$$

$$\text{Fundamental frequency } (f_1) = \frac{v}{2L}$$

Question-1

Third overtone of a closed organ pipe is in resonance with 4th harmonic of an open organ. Find the ratio of length of closed organ pipe to open organ pipe.

$$n^{th} \text{ overtone} = (2n+1)^{th} \text{ harmonic (closed)}$$

$$(\text{closed})\ 7^{th} \text{ harmonic} = 4^{th} \text{ harmonic (open)}$$

$$f_7 \text{ (closed)} = f_4 \text{ (open)}$$

$$7(f_1)_{closed} = 4(f_1)_{open}$$

$$7\left(\frac{v}{4L_c}\right) = 4\left(\frac{v}{2L_o}\right)$$

$$\frac{7\cancel{v}}{4L_c} = \frac{\overset{2}{\cancel{4}}\cancel{v}}{\cancel{2}L_o}$$

$$\boxed{\frac{L_c}{L_o} = \frac{7}{8}}$$

Question-2

An open organ pipe has fundamental frequency of 300 Hz. The first overtone of a closed organ pipe has the same frequency as the first overtone of open organ pip. How long is each pipe? (V_{sound} in air = 330 m/s)

$$(f_1)_{open} = 300$$

$$(f_3)_{closed} = (f_2)_{open}$$

$$(f_3)_c = (2f_1)_{open}$$

$$(f_3)_{closed} = 600 \text{ Hz}$$

$$3(f_1)_{closed} = 600$$

$$(f_1)_c = 200$$

$$\frac{v}{4L_c} = 200 \qquad L_c = \frac{\overset{\overset{33}{\cancel{66}}}{\cancel{330}}}{\underset{\underset{20}{\cancel{40}}}{\cancel{200}\times 4}} = \frac{33}{80}$$

$$= 0.41 \text{ m}$$

$$(f_1)_0 = \frac{V}{2L_0}$$

$$300 = \frac{330}{2L_0}$$

$$L_0 = \frac{33\cancel{0}}{60\cancel{0}}$$

$$L_0 = 0.55 \text{ m}$$

Question - 3

A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillation of air column in the pipe whose frequency lies below 1250 Hz. ($V_{sound\ in\ air} = 340$ m/s)

closed organ pipe —

$$f_1 = \frac{V}{4L}$$

$$f_1 = \frac{\cancel{340}\ 4 \times 100}{\cancel{4 \times 0.85}}$$

$$f_1 = 100\,s^{-1} = 100 \text{ Hz}$$

$$1250 = (2n+1)\,100$$

$$12.5 = (2n+1)$$

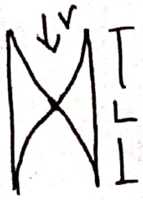
$$\cancel{2}n = \frac{11.5}{2}$$

$$n = 5.7$$

number of oscillations $= 6$

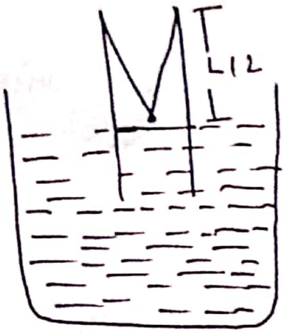
Question - 4

A pipe open at both ends has a fundamental frequency 'f' in air, the pipe is dipped vertically in water so that half of its length is in water. The fundamental frequency of air column now is —



fundamental frequency

$$f_1 = \frac{V}{2L}$$



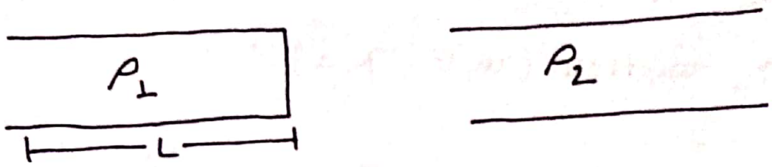
on dipping in water it behaves like a closed organ pipe

$$f_1 = \frac{V}{4L'}$$

$$f_1 = \frac{V}{4(L/2)}$$

$$\boxed{f_1 = \frac{V}{2L}}$$

Question - 5



compressibility of each gases are equal. Both vibrating in first overtone with same frequency. Find the length of open organ pipe.

$$\text{compressibility} = \frac{1}{\text{Bulk modulus } (\beta)}$$

first overtone (closed) $= (2n+1)$
$= 3^{rd}$ harmonic

first overtone (open) $= (n+1) = 2^{nd}$ harmonic

$$(3 f_1)_c = 2 (f_1)_o$$

$$\frac{3 \times V_c}{4L_c} = 2 \times \frac{V_o}{2L_o} \qquad V = \sqrt{\beta/\rho}$$

$$\frac{3\sqrt{\beta/\rho_1}}{4L} = \cancel{2} \times \frac{\sqrt{\beta/\rho_2}}{\cancel{2} L_o}$$

$$\boxed{L_o = \frac{4L}{3}\sqrt{\frac{\rho_1}{\rho_2}}}$$

Beats

The superposition of two sound waves having small difference in frequency this superposition / interference is called beats.

$(f_2 - f_1 < 10\,Hz)$

Standing waves	Beats
(i) Two waves travel in opposite direction. (Amplitude is same)	(i) Travels in same direction. (Amplitude is same)
(ii) $f_1 = f_2$ (same)	(ii) $f_1 \neq f_2$ (different)

Beats :

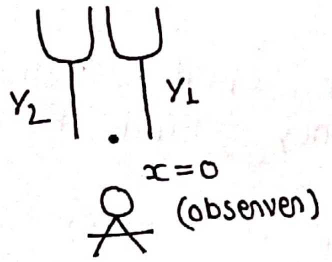
$$y_1 = A\sin(\omega_1 t - K_1 x)$$

$$y_2 = A\sin(\omega_2 t - K_2 x)$$

$\because v = \dfrac{\omega}{K}$ (ω is different hence K is also different)

$\because \omega = 2\pi f$

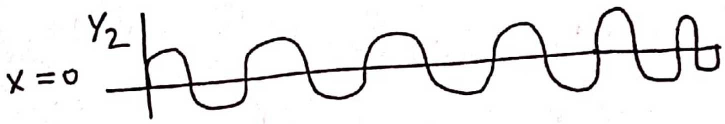
medium is same hence $v \rightarrow$ same



$$y_1 = A\sin\omega_1 t$$

$$y_2 = A\sin\omega_2 t$$

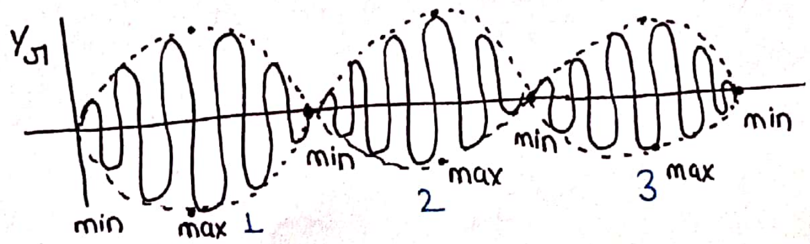
$(f_1) = 3$



$(f_2) = 6$

$f_2 - f_1 = 3$

$$y_{net} = y_1 + y_2$$



No. of beats in one second $= |(f_2 - f_1)| = 3$

No. of beats in one second is called beat frequency.

Beat frequency

The number of beats in one second is called beat frequency.

$$y_1 = A\sin(\omega_1 t - K_1 x)$$
$$y_2 = A\sin(\omega_2 t - K_2 x)$$

when we observe at $x = 0$.

$$y_1 = A\sin\omega_1 t$$
$$y_2 = A\sin\omega_2 t$$

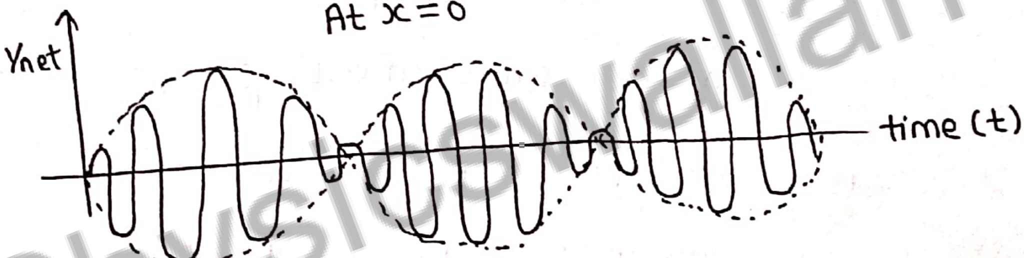
$$\vec{y}_{net} = \vec{y_1} + \vec{y_2}$$

$$y_{net} = A(\sin\omega_1 t + \sin\omega_2 t)$$

$$y_{net} = 2A\sin\left(\frac{\omega_1 t + \omega_2 t}{2}\right)\cos\left(\frac{\omega_1 t - \omega_2 t}{2}\right)$$

$$\boxed{y_{net} = 2A\cos\left(\frac{\omega_1 - \omega_2}{2}\right)t \cdot \sin\left(\frac{\omega_1 + \omega_2}{2}\right)t}$$

At $x = 0$

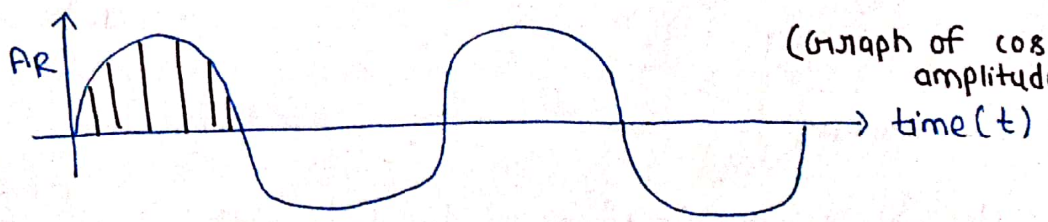


$$y_{net} = \underbrace{2A\cos\left(\frac{\omega_1 - \omega_2}{2}\right)t}_{A_R} \ \underbrace{\sin\left(\frac{\omega_1 + \omega_2}{2}\right)t}_{B}$$

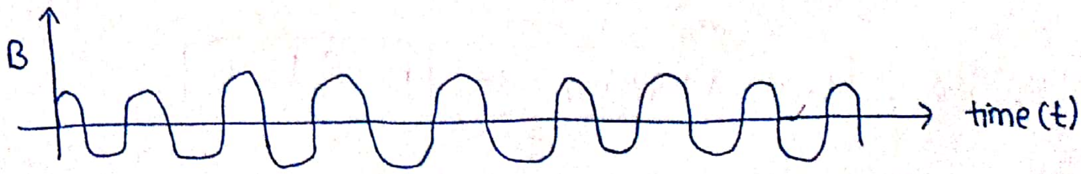
$$\omega_1 = 2\pi f_1 \qquad \omega_2 = 2\pi f_2$$

Graph of $\cos\left(\frac{\omega_1 - \omega_2}{2}\right)t$ have less frequency.

Graph of $\sin\left(\frac{\omega_1 + \omega_2}{2}\right)t$ have more frequency.



(Graph of cos affect the amplitude)



$$\boxed{y_{net} = A_R \sin\left(\frac{\omega_1+\omega_2}{2}\right)t}$$

$$y_R = 2A\cos\left(\frac{\omega_1-\omega_2}{2}\right)t \, \sin\left(\frac{\omega_1+\omega_2}{2}\right)t$$

<u>Maxima : Loud sound</u> $\qquad A_R \longrightarrow \max$

$$\left(\frac{\omega_1-\omega_2}{2}\right)t = 0, \pi, 2\pi, 3\pi, 4\pi \ldots\ldots, n\pi$$

$$\cancel{2}\pi \frac{(f_1-f_2)}{\cancel{2}} t = 0, \pi, 2\pi, 3\pi, 4\pi \ldots n\pi$$

$$t_1 = 0, \; t_2 = \frac{1}{f_1-f_2}, \; t_3 = \frac{2}{f_1-f_2}, \; t_4 = \frac{3}{f_1-f_2} \; \ldots \; t_n = \frac{n}{f_1-f_2}$$

After how much time loud sound is heard —

$$t_2 - t_1 = \frac{1}{f_1-f_2}$$

one maxima comes after $\frac{1}{f_1-f_2}$ seconds.

In one second $= (f_1 - f_2)$ Hz

No. of beats in 1 second $= (f_1 - f_2)$ Hz

$$\boxed{\text{Beat frequency} = (f_1 - f_2) \text{ Hz}}$$

<u>Minima : faintsound</u> $\qquad A_R - \text{Min}$

$$\left(\frac{\omega_1-\omega_2}{2}\right)t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots\ldots$$

$$\cancel{2}\pi \frac{(f_1-f_2)}{\cancel{2}} t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots\ldots\ldots$$

$$t_1 = \frac{1}{2(f_1-f_2)}, \; t_2 = \frac{3}{2(f_1-f_2)}, \; t_3 = \frac{5}{2(f_1-f_2)}$$

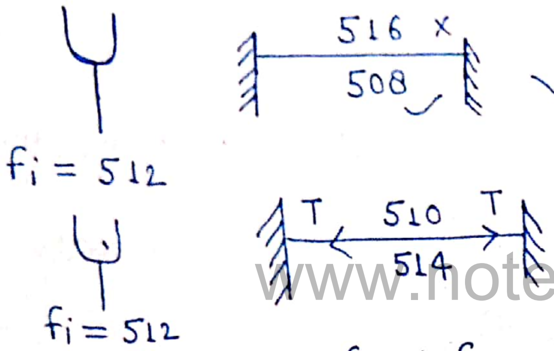
1 minima $\longrightarrow \frac{1}{(f_1-f_2)}$ seconds

In one second $\longrightarrow (f_1 - f_2)$ Hz

$$\boxed{\text{Beat frequency} = |f_1 - f_2| \text{ Hz}}$$

Question-1

A tunning fork has natural frequency of 512 Hz, makes 4 beats / second with a piano. The beat frequency decreases to 2 beats / second. when tension in piano string is increased The initial frequency of piano string wave. was —



Beats / second

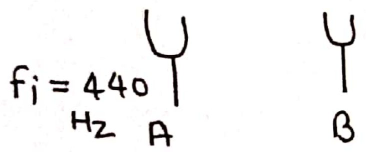
4

Freq. incr.

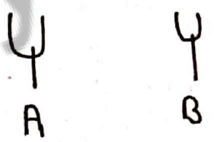
$\therefore f = \frac{n}{2l}\sqrt{\frac{T}{\mu}}$ ⇒ 2 beats / second

— final frequency is 508 Hz because on increasing tension frequency also increases.

Question-2



4 beats with B — Arm of B is loaded with wax & beat frequency becomes 6 Find the natural frequency of B before loading fork.



		Beats / second
A	B	4 beats
440 Hz	444 Hz / 436 Hz	

B: 434 Hz / 446 Hz	6 beats

Concept –

Loaded with wax mean frequency (natural) ↓ ⇒ ± 5 Hz

Scrapping / Peel off mean frequency (natural) ↑ ⇒ ± 5 Hz

436 is the natural frequency because +2 decrease in frequency after waxing.

Doppler's Effect

"when there is relative motion b/w Source & observer, the frequency of sound heard by observer is different from actual frequency of sound source". This phenomenon is called Doppler's Effect.

"The frequency which is different from actual frequency of sound source is called apparent frequency".

case I ⇒ when source is at rest & observer is in motion

$$f_s = \frac{V}{\lambda}$$

Speed of sound for observer $= (V + V_0)$

f_s = frequency of sound.

$$f_0 = \frac{V+V_0}{\lambda}$$

$$\lambda = \left(\frac{V+V_0}{f_0}\right)$$

$$f_s = \frac{V}{(V+V_0)} f_0$$

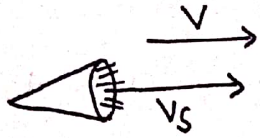
$$\boxed{f_0 = \left(\frac{V+V_0}{V}\right) f_s}$$

f_0 = Apparent frequency

if observer goes away from source –

$$\boxed{f_0 = \left(\frac{V-V_0}{V}\right) f_s}$$

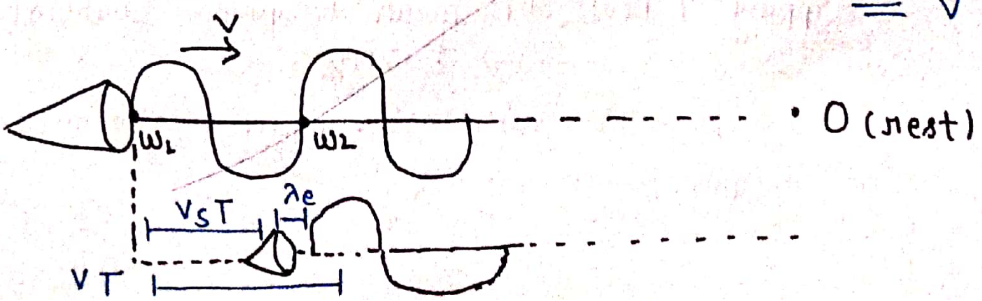
case II ⇒ when observer is at rest & source is in motion –



O (rest)

V_S = velocity of source

Speed of sound for observer $= V$



After one time period 'T'

Distance covered by source $= V_S T$

Distance covered by sound $= VT$

effective wavelength $(\lambda_e) = VT - V_S T$

$$\lambda_e = T(V - V_S)$$

$$\lambda_e = \frac{1}{f_S}(V - V_S)$$

$$\lambda_e = \left(\frac{V - V_S}{f_S}\right)$$

$$f_0 = \frac{V}{\lambda_e}$$

$$\boxed{f_0 = \left(\frac{V}{V - V_S}\right) f_S}$$

if source goes away –

$$\boxed{f_0 = \left(\frac{V}{V + V_S}\right) f_S}$$

Case III ⇒ when source & observer moves —

$$\boxed{f_0 = \left(\frac{V \pm V_0}{V \mp V_S}\right) f_S}$$

(i) when observer & source comes closer —

$S \rightarrow \quad \leftarrow O$

frequency increases

$$\boxed{f_0 = \left(\frac{V + V_0}{V - V_S}\right) f_S}$$

(ii) when observer & source are in same direction.

$S \rightarrow \quad O \rightarrow$

Source wants to increase frequency & observer decreases frequency.

$$\boxed{f_0 = \left(\frac{V - V_0}{V - V_S}\right) f_s}$$

(ii) when observer comes closer to source & source goes away.

$\xleftarrow{\;}$ S $\qquad\qquad$ $\xleftarrow{\;}$ O

observer wanted to increase frequency —

$$\boxed{f_0 = \left(\frac{V + V_0}{V + V_S}\right) f_s}$$

(iii) when source & observer are moving in opposite direction

Both wanted to decrease frequency

$$\boxed{f_0 = \left(\frac{V - V_0}{V + V_S}\right) f_s}$$

Question- 1

(i) A policeman stands in a ground & whistling by his whistle suddenly a man comes closer with speed 36 km/hr. if frequency of whistle is 2KHz & speed of sound is 340 ~~k~~ m/s. find —

(i) λ for policeman & observer

(ii) f as heard by observer.

$$\lambda = \frac{V}{f} = \frac{340}{2 \times 10^3} = 170 \times 10^3 = 1.70 \times 10^5 \text{ m}$$

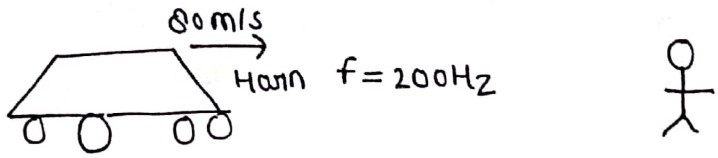
$$f_0 = \left(\frac{V + V_0}{V \pm V_S}\right) f_s$$

$$f_0 = \left(\frac{V + V_0}{V}\right) f_s$$

$$f_0 = \left(\frac{340 + 10}{340}\right) 2 = \frac{35\cancel{0}}{34\cancel{0}} \times 2$$

$$= 2.06 \text{ KHz}$$

Question-2



V_{sound} in air $= 340$ m/s

(i) λ b/w source & observer.

(ii) f heard by observer.

Source is in motion so λ changed.

Distance travelled by sound $= 340 \times T$

Distance travelled by car $= 80 \times T$

$$\lambda_e = 340T - 80T$$

$$\lambda_e = \frac{1}{f_s}(260)$$

$$\Rightarrow \frac{260}{200} = 1.3\,m$$

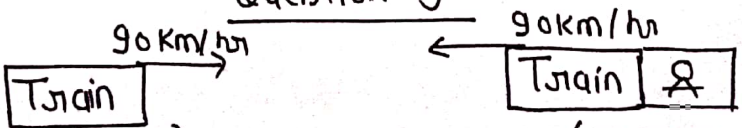
$$f_o = \left(\frac{V}{V - V_s}\right) f_s$$

$$= \left(\frac{340}{340 - 80}\right) \times 200$$

$$= \frac{340}{26\cancel{0}} \times 20\cancel{0}$$

$$= 261.5\ Hz$$

Question-3



(Horn)
$f_s = 500\,Hz$

(observer is in train)

V_{sound} in air $= 350$ m/s

find the f_o.

$$f_o = \left(\frac{V + V_o}{V - V_s}\right) f_s \Rightarrow \left(\frac{350 + 25}{350 - 25}\right) 500$$

$$= \frac{375}{\cancel{325}_{13}} \times \cancel{500}^{20} = 576.92\ Hz$$