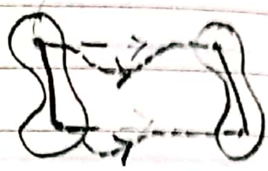
Rigid body – When 2 points in a body have constant distance.

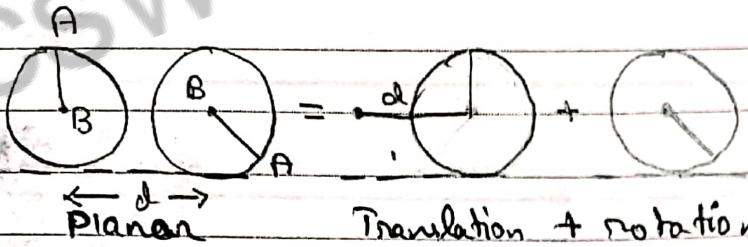
① Translation motion :
→ when ~~two points~~ the line between 2 points of a rigid body are parallel throughout the motion.
When path is a straight line = Rectilinear
when path is a curved line = Curvilinear



pivot

② Rotational motion :
→ When all the particles (internal) move is a circular path about a fixed axis.
The line joinging 2 internal points does not remain parallel.

③ Planar motion :
→



Quantity	Linear	Angular
Displacement	S	θ
Velocity	v	ω
Acceleration	a_t	α
Force	F	τ (Torque)
Mass	m	I (Moment of Inertia)
momentum	p	L (Angular momentum)

ω – It is same for all points of a rotating body. (except $\omega_{pivot} = 0$)

$$\boxed{\vec{V} = \vec{\omega} \times \vec{r}} \quad \boxed{a_c = \frac{v^2}{R}} \quad \boxed{a_t = \frac{d|v_t|}{dt}} \quad \boxed{a_{net} = \sqrt{a_c^2 + a_t^2}}$$

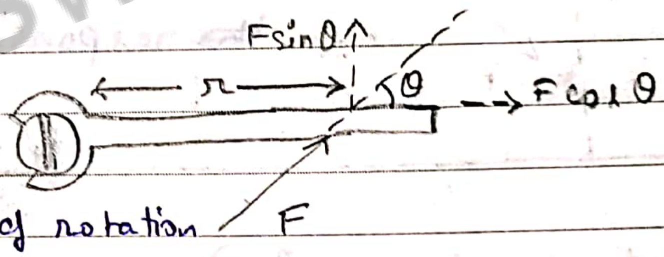
$\boxed{a_t = R\alpha}$ To have a circular/rotational motion, a_t & α may be absent. a_c must be present.

If α and ~~ω~~ ω have the same ~~size~~ sign, ω increases else decreases.

Linear	Angular
$V = u + at$	$\omega_f = \omega_i + \alpha t$
$V^2 = u^2 + 2aS$	$\omega_f^2 = \omega_i^2 + 2\alpha\theta$
$S = ut + \frac{1}{2}at^2$	$\theta = \omega_i t + \frac{1}{2}\alpha t^2$

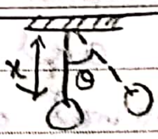
Torque (τ)

$$\boxed{\tau = r(F\sin\theta)} \quad \boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$

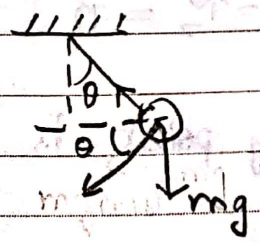


The component of τ along axis of rotation only produces turning effect.

Q Find the torque of gravitational force about X. ($r = 1m$, $\theta = 30°$, $m = 1kg$)



→



$\therefore \tau = r \times mg \sin\theta = 1 \times 1 \times 10 \times \frac{1}{2} = 5$

Q. A Force $(F = 2\hat{i} + 3\hat{j} - \hat{k})$ is acting at a point $(1, 2, 2)$. Find the moment of force about point $(0, 1, 3)$.

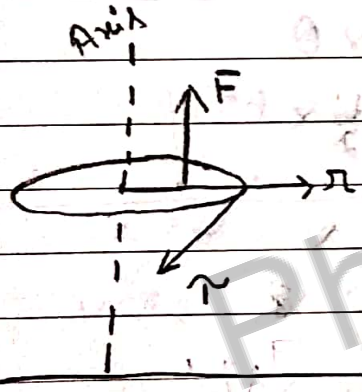
→ $\vec{r} = (1-0)\hat{i} + (2-1)\hat{j} + (2-3)\hat{k} = \hat{i} + \hat{j} - \hat{k}$

$\vec{F} = 2\hat{i} + 3\hat{j} - \hat{k}$

$$\therefore \vec{\tau} = \begin{array}{c} \\ r \\ F \end{array}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 2 & 3 & -1 \end{vmatrix} = [-1-(-3)]\hat{i} + [-2+1]\hat{j} + [3-2]\hat{k}$$

$$= 2\hat{i} - \hat{j} + \hat{k}$$

$\therefore |\tau| = \sqrt{2^2 + (-1)^2 + 1^2} = \cancel{3}\sqrt{6}$



If torque is present, ~~tho~~ then rotation is not compulsory, The component of Torque parallel to the axis of rotation will only ~~rotate~~ be ~~the~~ responsible for the body's rotation.

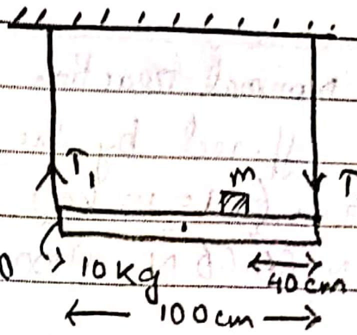
Static Equilibrium : When the body is in both Translation equilibrium and rotational equilibrium.

(i) Translational equilibrium is when $F_x = F_y = F_z = 0$

(ii) Rotational equilibrium is when $\tau_x = \tau_y = \tau_z = 0$

If static equilibrium is present, τ_{net} about any point is zero.

Q. Find T_1 and T_2 if $m = 5kg$, $g = 10ms^{-1}$ and the System is in equilibrium.



(a) 80 N, 80 N (b) ~~80N, 70N~~ (c) 60N, 80N (d) ~~70, 80~~

$\rightarrow$ COM = 50 cm $\therefore r = 10$ cm

$\therefore \tau = r \cdot F = 10 \times 5 \times 10 = 500$ rad s^{-2}

Anticlockwise Torque = Clockwise Torque

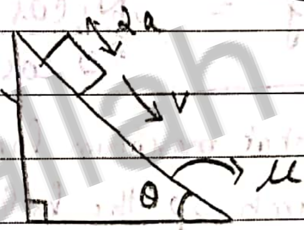
$\Rightarrow T_1 \cdot 50 + 500 = T_2 \cdot 50$

$\Rightarrow T_1 + 10 = T_2$ —— (ii)

$T_1 + T_2 = mg + Mg$
$= 150$ —— (1)

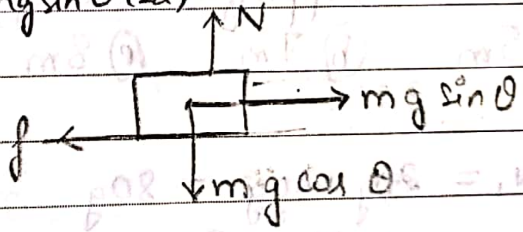
* Normal Reaction does not pass through COM.

Q. Rough inclined surface has a block on it with constant velocity 'v'. What is the torque due to 'N' reaction about COM.



(a) $mg\sin\theta \frac{a}{2}$ (b) $mg\sin\theta \frac{a}{3}$ (c) ~~$mg\sin\theta\, a$~~ (d) $mg\sin\theta\,(2a)$

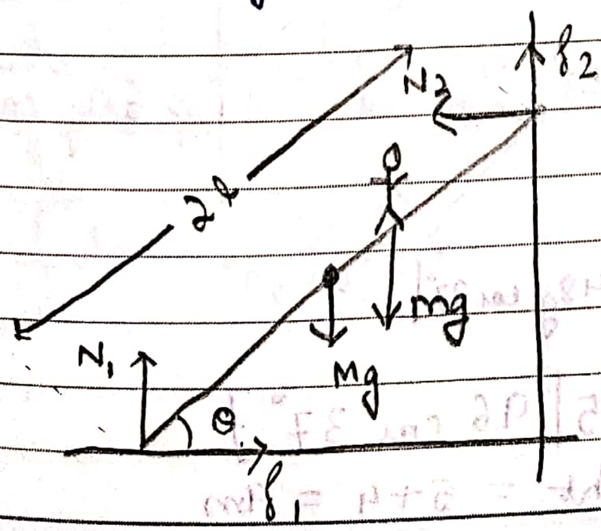
$\rightarrow N = mg\cos\theta$



$\tau_{net} = 0$

$\Rightarrow \mu N \cdot a = N \cdot x$

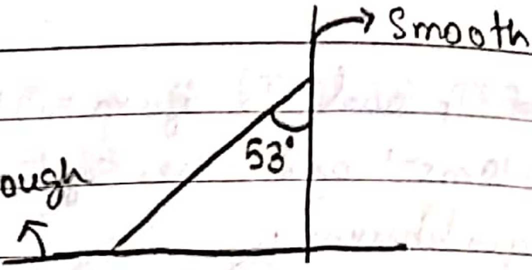
$\Rightarrow \boxed{Nx = mg\sin\theta \cdot a}$ $\Rightarrow \boxed{x = a\tan\theta}$



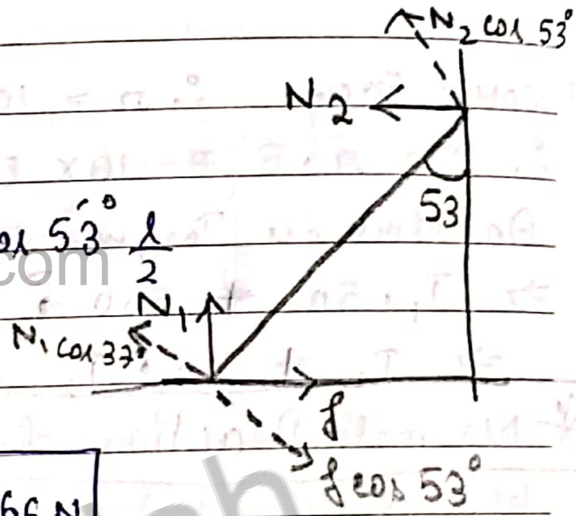
$F_{net} = 0$
$\therefore \Sigma F_x = 0$
$\Rightarrow N_2 - f_1 = 0$
$\Rightarrow N_2 = f_1$

$F_{net} = 0$
$\therefore F_y = 0$
$\Rightarrow N_1 + f_2 - (Mg + mg) = 0$
$\Rightarrow N_1 + f_2 = Mg + mg$

Q Find Normal reaction and frictional force offered by the floor to the Rough ladder. ($M = 10$ kg)



(a) 100 N, 66.66 N (b) 100 N, 33.33 N (c) 100 N, 100 N (d) 100 N, 47.47 N



$\Rightarrow f = N_2$, $\boxed{N_1 = Mg = 100\text{ N}}$

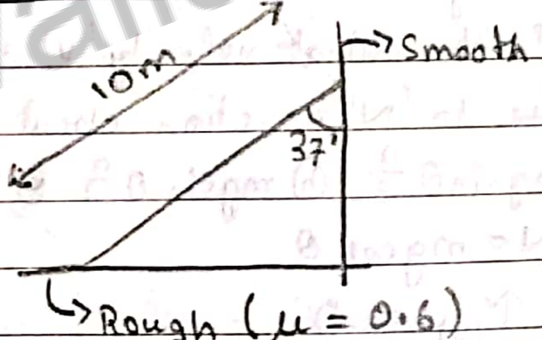
$\tau_{net} = 0$

$\Rightarrow N_1 \cos 37° \cdot \frac{l}{2} = f \cos 53° \cdot \frac{l}{2} + N_2 \cos 53° \frac{l}{2}$

$= 2f \cos 53° \cdot \frac{l}{2}$

$\Rightarrow f = \frac{N_1 \cos 37°}{2 \cos 53°} = \frac{200}{3} = \boxed{66.66\text{ N}}$

Q Upto what maximum length, the mechanic can climb up the ladder without the ladder slipping. ($L = 20$kg, Mech $= 60$kg)

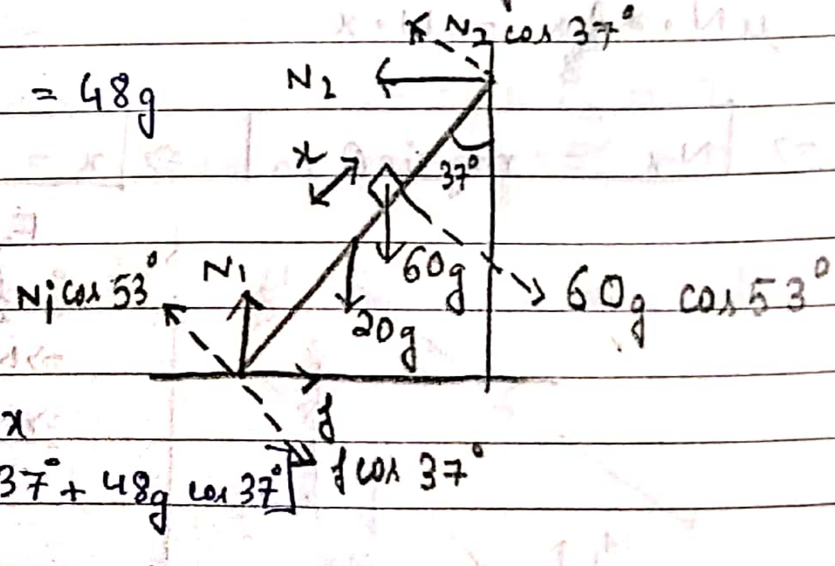


(a) 5m (b) 7m (c) 8m (d) 9m

$\rightarrow N_1 = 20g + 60g = 80g$

$f_{lim} = \mu N_1 = 80g \times 0.6 = 48g$

$f_{lim} = N_2 \Rightarrow N_2 = 48g$



$\therefore \tau_{net} = 0$

$\Rightarrow 5 \times 80g \cos 53° + 60g \cos 53° \times x$

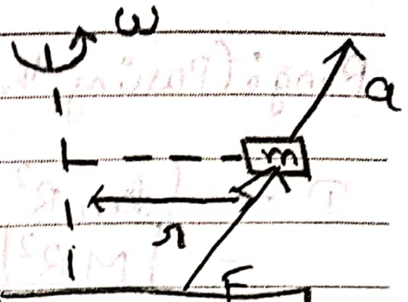
$= 5 \times [48g \cos 37° + 48g \cos 37°]$

$\Rightarrow$ ~~400g cos 53~~

$\Rightarrow 400 \cos 53° + 60 \cos 53° x = 5 [96 \cos 37°]$

$\Rightarrow x = 4$m $\therefore$ height $= 5 + 4 = 9$m

Moment Of inertia



$F = ma$

$\Rightarrow rF = mra = mr^2\omega$

$\Rightarrow \tau = I\omega$ $\boxed{\text{SI of } I = kg\,m^2}$

For a system of multiple particles,

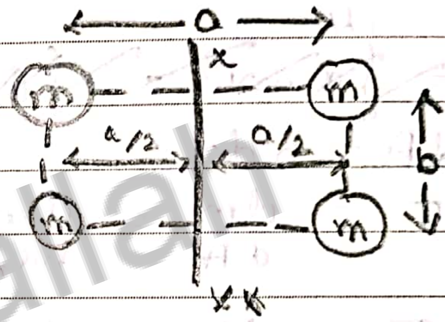
$I = \sum_{i=1}^{n} m_i r_i^2$

$\boxed{I_x = mr^2}$

Q Find I_{xx}.

(a) mb^2 (b) $2mb^2$

(c) ma^2 (d) $2ma^2$

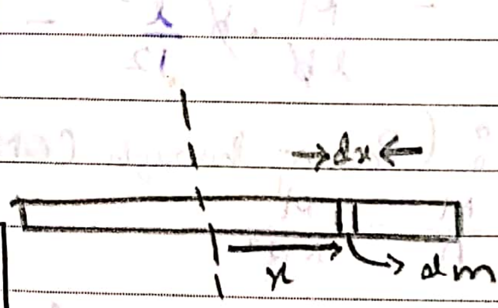


$\rightarrow I = 4 \times m\left(\frac{a}{2}\right)^2 = ma^2$

Continuous Bodies

$\boxed{I = \int r^2\,dm}$

Uniform Rod : (Passing through COM)



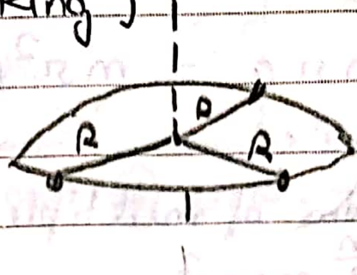
$I = \int_{-L/2}^{+L/2} dm\,x^2 = \int_{-L/2}^{+L/2} dx\,\lambda x^2 = \boxed{\frac{ML^2}{12}}$

$\boxed{dm = dx\,\lambda}$

(Passing through one end of the rod)

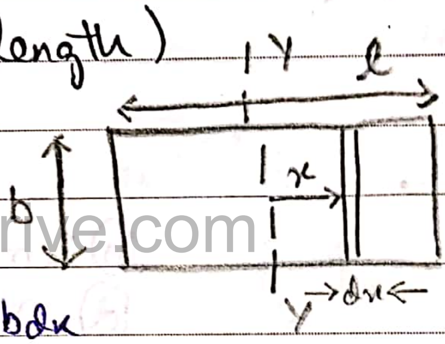
$I = \int_0^L dm\,x^2 = \lambda\int_0^L x^2\,dx = \frac{M}{L}\left[\frac{x^3}{3}\right]_0^L = \boxed{\frac{ML^2}{3}}$

Ring: (Passing through COM & Plane of Ring)



$$I = \int dm\, R^2 = R^2 \int dm$$
$$= \boxed{MR^2}$$

Rectangular lamina: ($\perp$ to its length)



$$\cancel{I = \int_{-l/2}^{+l/2} dm\, x^3 \;\; \int dx}$$

$$dA = b\,dx$$

$$\sigma = \frac{dm}{dA} = \frac{dm}{b\,dx} \Rightarrow dm = \sigma\, b\,dx$$

$$\sigma = \frac{M}{A} = \frac{M}{lb}$$

$$\therefore I = \int dm\, x^2 = \int_{-l/2}^{+l/2} \sigma\, b\,dx\, x^2 = \sigma b \int_{-l/2}^{+l/2} x^2\,dx$$

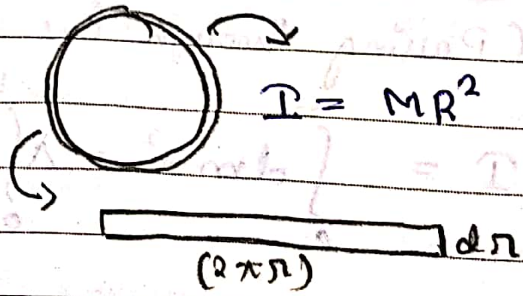
$$= \frac{M}{l\cancel{b}} \cdot \cancel{b}\, \frac{l^3}{12} = \boxed{\frac{1}{12} Ml^2}$$

Disc: (Passing through COM and $\perp$ to the plane)

$$\sigma = \frac{M}{A} = \frac{M}{\pi R^2}$$

$$\sigma = \frac{dm}{dA} \Rightarrow dm = \sigma(2\pi r\,dr)$$

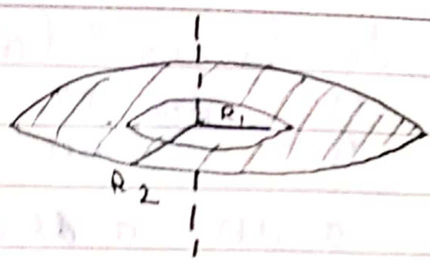
$$I = \int_0^R dm\, r^2 = \sigma 2\pi \int_0^R r^3\,dr$$



$$= \frac{M}{\cancel{\pi} R^2} \cdot 2\cancel{\pi} \left[\frac{r^4}{4}\right]_0^R$$

$$= \frac{2M}{R^2} \cdot \frac{R^4}{4} = \boxed{\frac{MR^2}{2}}$$

2D-disc (COM, $\perp$ to plane)



$$\Rightarrow \sigma = \frac{M}{\pi(R_2^2 - R_1^2)}$$

$$I = \sigma\, 2\pi \int_{R_1}^{R_2} r^3\, dr$$

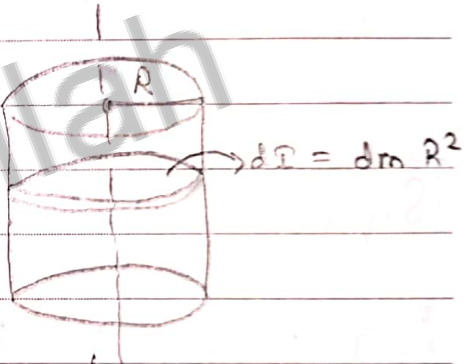
$$= \frac{2M}{(R_2^2 - R_1^2)}\left[\frac{R_2^4}{4} - \frac{R_1^4}{4}\right] = \frac{M}{2} \times \frac{1}{(R_2^2 - R_1^2)} \times (R_2^2 - R_1^2)(R_2^2 + R_1^2)$$

$$= \boxed{\frac{M(R_2^2 + R_1^2)}{2}}$$

Cylinder (Hollow): (COM, $\perp$ to circle)

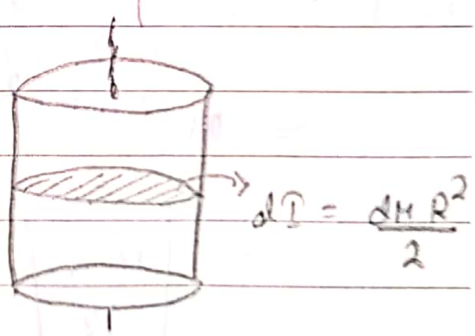
$$dI = dm\, R^2$$

$$\Rightarrow \boxed{I = MR^2}$$



Cylinder (Solid): (COM, $\perp$ to circle)

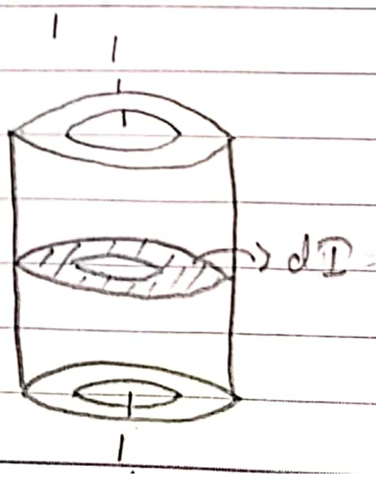
$$dI = \frac{dm\, R^2}{2} \Rightarrow \boxed{I = \frac{MR^2}{2}}$$



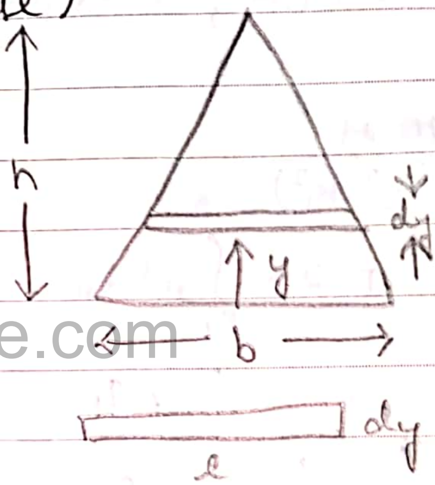
Cylinder (Partially Hollow): (COM, $\perp$ to circle)

$$dI = \frac{dm\,(R_2^2 + R_1^2)}{2}$$

$$\Rightarrow \boxed{I = \frac{M(R_2^2 + R_1^2)}{2}}$$



Triangular lamina : (about its base)



$$\sigma = \frac{M}{A} = \frac{2M}{bh} \qquad \boxed{\frac{l}{b} = \frac{h-y}{h}}$$

$$dm = \sigma\, dA = \sigma\, l\, dy$$

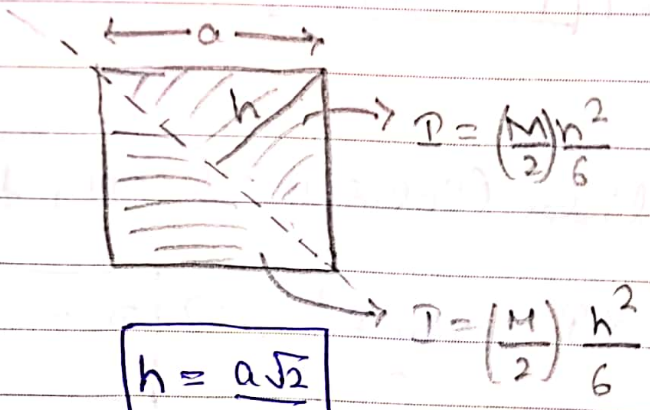
$$I = \int_0^h dm\, y^2 = \sigma b \int_0^h \frac{(h-y)}{h}\, y^2$$

$$= \frac{2M}{\cancel{b}h^2}\cdot\cancel{b}\int_0^h y^2 h - y^3$$

$$= \frac{2M}{h^2}\left\{\left[\frac{y^3}{3}\right]_0^h \cdot h - \left[\frac{y^4}{4}\right]_0^h\right\}$$

$$= \frac{2M}{h^2}\left[\frac{h^4}{3} - \frac{h^4}{4}\right] = \frac{2M}{1}\cdot\frac{h^2}{12} = \boxed{\frac{Mh^2}{6}}$$

Square : (along its diagonal)



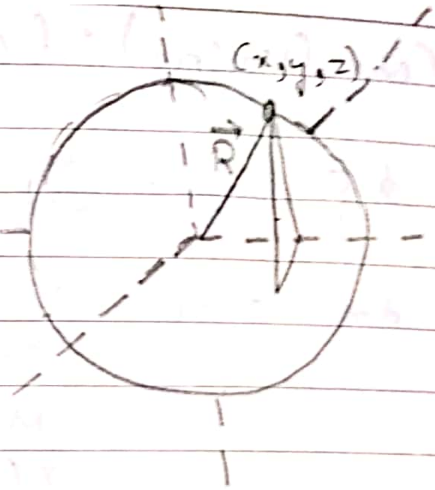
$$\boxed{h = \frac{a\sqrt{2}}{2}}$$

$$\therefore I = 2 \times \left(\frac{M}{2}\right)\frac{h^2}{6} = \frac{Mh^2}{6}$$

$$= \frac{M\left(\frac{a\sqrt{2}}{2}\right)^2}{6} = \frac{Ma^2 \times 2}{4 \times 6}$$

$$= \boxed{\frac{Ma^2}{12}}$$

Sphere (hollow) : (COM)



$\rightarrow$ $I_x = I_y = I_z$

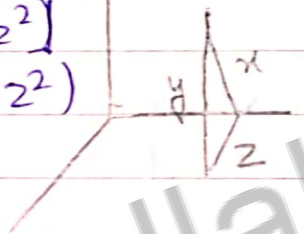
$dI_x = dm\,(\text{distance from } x\text{-axis})^2$

$= dm\,(y^2 + z^2)$

$dI_y = dm\,(\text{distance from } y\text{-axis})^2$

$= dm\,(x^2 + z^2)$

$dI_z = dm\,(x^2 + y^2)$



$x = \sqrt{y^2 + z^2}$

$y = \sqrt{z^2 + x^2}$

$z = \sqrt{x^2 + y^2}$

$\therefore dI_x + dI_y + dI_z = dm\left[2x^2 + 2y^2 + 2z^2\right]$

$= 2\,dm\,(x^2 + y^2 + z^2)$

$\Rightarrow 3\,dI = 2\,dm\,(x^2 + y^2 + z^2)$

$= 2\,dm\,R^2$

$\Rightarrow 3\int dI = 2\int dm\,R^2$

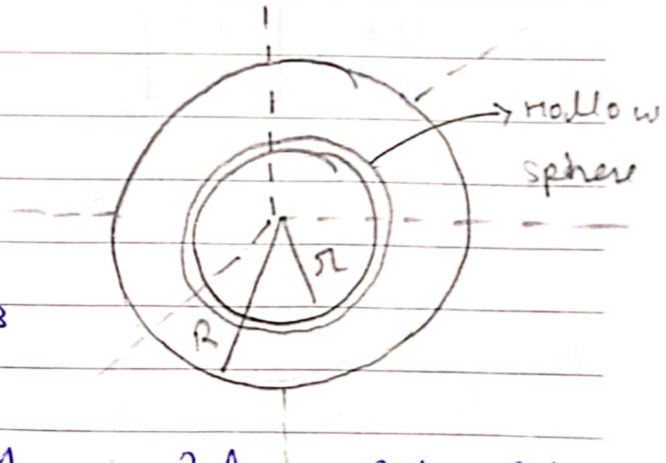
$\Rightarrow 3I = 2MR^2 \Rightarrow \boxed{I = \frac{2}{3}MR^2}$

Shortcut:

$\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$

$|R| = \sqrt{x^2 + y^2 + z^2}$

Sphere (Solid) : (COM)



$dI = \frac{2}{3}\,dm\,r^2$

$= \frac{2}{\cancel{3}} \cdot \frac{\cancel{3}M}{R^3} \cdot r^4\,dr$

$\Rightarrow \int dI = \frac{2M}{R^3}\int_0^R r^4\,dr$

$\Rightarrow I = \frac{2M}{R^3} \cdot \frac{R^5}{5}$

$\Rightarrow \boxed{I = \frac{2}{5}MR^2}$

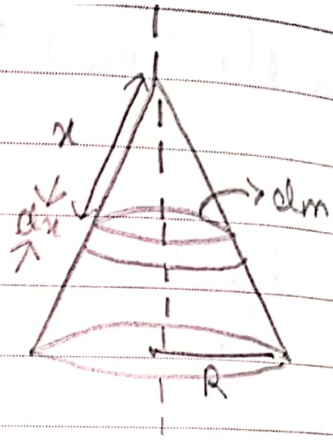
$\rho = \frac{M}{V}$

$= \frac{3M}{4\pi R^3}$

$dm = \rho\,dV$

$= \frac{3M}{4\pi R^3} \times 4\pi r^2\,dr = \frac{3M}{R^3} \cdot r^2\,dr$

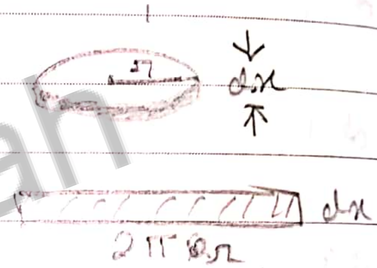
Cone (Hollow) ; (COM, axis of cone)



$d\sigma = \frac{M}{A} = \frac{M}{\pi R L}$ | $\frac{r}{R} = \frac{x}{L}$

$dm = \sigma\, dA = \frac{M}{\pi R L} \cdot 2\pi \cdot r\, dx$

$= \frac{M}{\pi R L} \cdot 2\pi \cdot \frac{xR}{L}\, dx$



$= \frac{2M}{L^2} \cdot x\, dx$

$\cancel{\therefore \int dI = \int dm\, x^2}$

$\cancel{\Rightarrow I = \frac{2M}{L}\int_0^L x^3\, dx}$

$\cancel{\Rightarrow I = \frac{2}{4} ML^2 = M}$

$\int dI = \int dm\, r^2$

$= \frac{2M}{L}\int x\, dx\; r^2$

$\Rightarrow I = \frac{2M}{L^2}\int_0^L x\, dx \cdot \left(\frac{xR}{L}\right)^2$

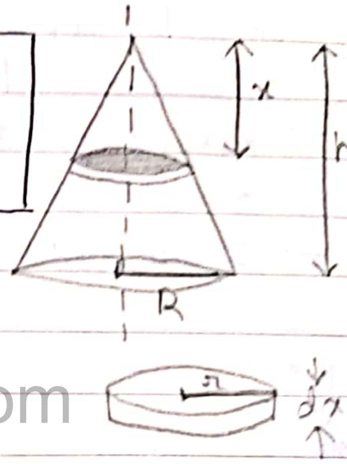
$= \frac{2MR^2}{L^4}\int_0^L x^3\, dx$

$= \frac{2MR^2}{L^3}\left[\frac{x^4}{4}\right]_0^L = \boxed{\frac{MR^2}{2}}$

Scanned by CamScanner

Cone (Solid) : (axis of cone)

$$\boxed{\frac{r}{R} = \frac{x}{h}}$$



$$\rho = \frac{M}{V} = \frac{3M}{\pi R^2 h}$$

$$dm = \rho\, dV = \rho \cdot \pi r^2 \cdot dx$$

$$= \frac{3M}{\pi R^2 h} \cdot \pi r^2 dx$$

$$= \frac{3M}{R^2 h} \cdot \left(\frac{xR}{h}\right)^2 \cdot dx$$

$$= \frac{3M}{h^3} \cdot x^2 dx$$

$$\cancel{I = \int dm\, x^2 = \frac{3M}{Rh^2}\int_0^H x^2 dx \cdot r^2 = \frac{3M}{Rh^2}}$$

$$\int dI = \int \frac{dm\, r^2}{2} \Rightarrow I = \frac{3M}{2h^3}\int x^2 dx \cdot r^2 = \frac{3M}{2h^3} \cdot \frac{R^2}{h^2}\int x^4 dx$$

$$\Rightarrow I = \frac{3MR^2}{2h^5}\left[\frac{x^5}{5}\right]_0^h = \boxed{\frac{3}{10} MR^2}$$

In hollow cone we have taken slant height and in solid cone, we have taken height because hollow cone only has material at the edge.

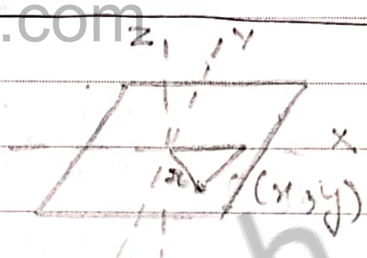
Hollow objects have higher moment of inertia because all the point masses are situated far from the axis, so distance is more and mr^2 will be more.

Perpendicular ~~and~~ axis Theorem

- Can only be applied on 2-D body.
- $\boxed{I_z = I_x + I_y}$

→ x, y, z should be mutually perpendicular
→ x and y should be in the plane of body.
→ z should be perpendicular to plane of body.

Proof:



$dI_x = dm\,y^2$

$dI_y = dm\,x^2$

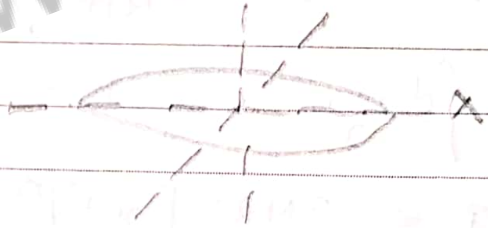
$dI_z = dm\,r^2 = dm(x^2+y^2)$

$= dI_x + dI_y$

$\Rightarrow \boxed{I_z = I_x + I_y}$

[Can be applied if axis not passing from COM]

Disc, ring:



$I_x = I_y$

$\therefore I_z = 2I_x + I_y$

$= 2I_x$

Disc:

$I_x = \frac{1}{2}\frac{MR^2}{2}$

$\boxed{I_x = \frac{MR^2}{4}}$

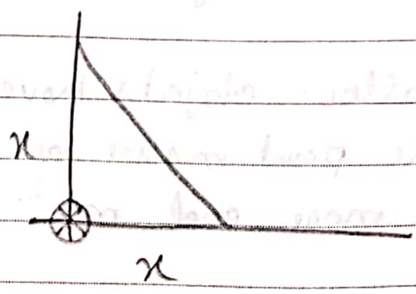
Ring:

$I_x = \frac{1}{2}MR^2$

$\Rightarrow \boxed{I_x = \frac{MR^2}{2}}$

Q. Find MOI

(a) $\frac{mx^2}{12}$ (b) $\frac{mx^2}{6}$ (c) $\frac{mx^2}{3}$ (d) $\frac{mx^2}{4}$



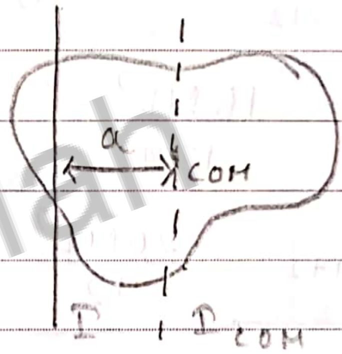
→ $I_x = \frac{mx^2}{6} = I_y$

$I_z = \frac{mx^2}{3}$

Parallel axis theorem

- $\boxed{I = I_{COM} + Ma^2}$
  - → 2 axis must be parallel
  - → One axis should pass through COM.
  - → a is the perpendicular distance between two axis
- Can be applied to 1-D, 2-D, 3-D, etc. bodies.

Proof :



$dI_{COM} = dm\, r^2$

$\therefore dI = dm(r+a)^2$

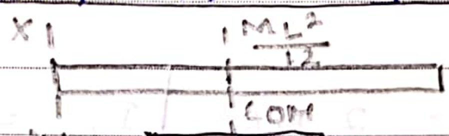
$\Rightarrow dI = dm\, r^2 + dm\, a^2 + 2a\, dm\, r$

$\Rightarrow \int dI = \int dI_{COM} + \int dm\, a^2 + \int 2a\, dm\, r$

$\Rightarrow \boxed{I = I_{COM} + Ma^2}$ $\left[\because \dfrac{\int dm\, r}{M} = 0 \Rightarrow \int dm\, r = 0\right.$

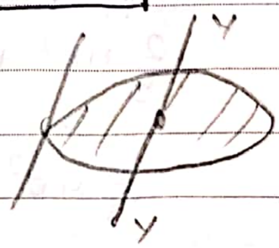
$\left.\Rightarrow 2a \int dm\, r = 0 \right]$

Q. Find I_{xx}



→ $I = I_{COM} + Ma^2 = \dfrac{ML^2}{12} + M\left(\dfrac{L}{2}\right)^2 = \boxed{\dfrac{ML^2}{3}}$

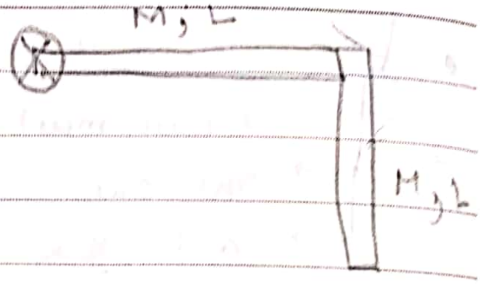
Q. Find I_{xx}.

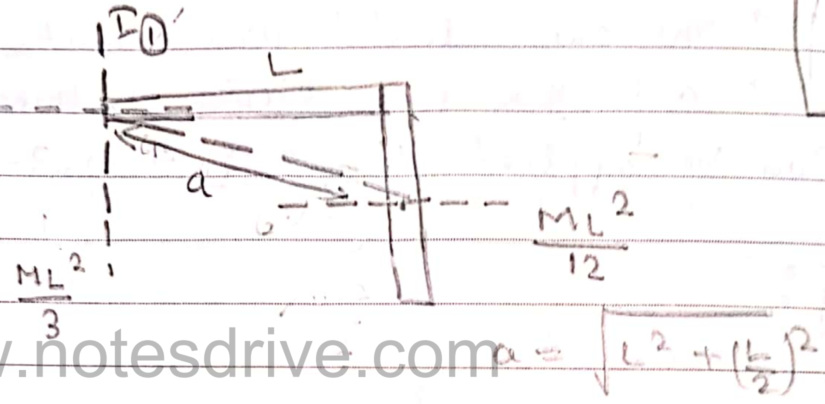


→ $I = \dfrac{MR^2}{4} + \dfrac{MR^2}{1} = \boxed{\dfrac{5}{4} MR^2}$

Q. Find I:

(a) $\frac{5}{3}ML^2$ (b) $\frac{3}{5}ML^2$ (c) $\frac{2}{5}ML^2$ (d) $\frac{5}{2}ML^2$





→ $I_1 = \frac{ML^2}{3}$

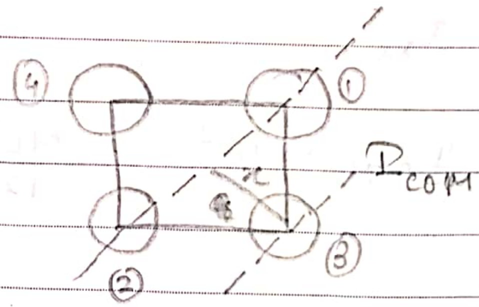
$I_2 = \frac{ML^2}{12} + M\left(L^2 + \left(\frac{L}{2}\right)^2\right)$

$= \frac{16ML^2}{12}$

$\therefore I_{1+2} = \frac{20ML^2}{12} = \frac{5}{3}ML^2$

Q. 4 solid spheres each of mass 0.5 kg and diameter $\sqrt{5}$ cm are kept at 4 corners of a square of side 4 cm, ~~Find~~ the MOI about diagonal of square is $N \times 10^{-4}$ kg m^2. The value of N is:

(a) 1 (b) ~~3~~ 3 (c) 6 (d) 9



→ $I_1 = I_2 = \frac{2}{5}MR^2$ | $I_{COM} = \frac{2}{5}MR^2$

$I_3 = I_4 = I_{COM} + \cancel{Ma^2}\, Mx^2$

$= \frac{2}{5}MR^2 + M\left(\frac{a\sqrt{2}}{2}\right)^2$

$= \frac{2}{5}MR^2 + \frac{Ma^2}{2}$

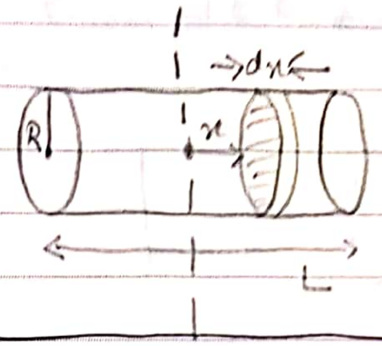
$\therefore I = I_1 + I_2 + I_3 + I_4 = \frac{8}{5}MR^2 + Ma^2$

$= \frac{8}{10} \times \frac{5}{4} + \frac{1}{2} \cdot 16 = 1 + 8 = 9$ kg cm^2

$= 9 \times 10^{-4}$ kg m^2

$\therefore \boxed{N = 9}$

Cylinder (Solid) : ($\perp$ to length , COM)



$\rightarrow$ $dI = dI_{COM} + dm x^2$

$\Rightarrow dI = \frac{dm R^2}{4} + dm\, x^2$

$\Rightarrow I = \frac{MR^2}{4} + \frac{M}{L}\int dx . x^2$

$\rho = \frac{M}{V} = \frac{M}{\pi R^2 L}$

$dm = \rho\, dm$
$= \rho \pi R^2 dx$
$= \frac{M}{L} dx$

$= \frac{MR^2}{4} + \frac{M}{L}\int_{-L/2}^{+L/2} x^2 dx$

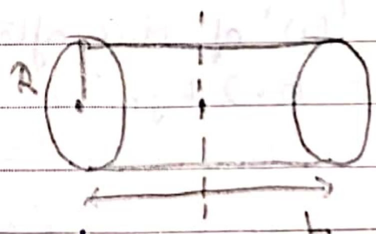
$= \frac{MR^2}{4} + \frac{M}{L}\left[\frac{x^3}{3}\right]_{-L/2}^{+L/2}$

$= \frac{MR^2}{4} + \frac{M}{L}\left(\frac{(L/2)^3}{3} + \frac{(L/2)^3}{3}\right)$

$= \frac{MR^2}{4} + \frac{M}{L}\left(\frac{2L^3}{24}\right)$

$= \boxed{\frac{MR^2}{4} + \frac{ML^2}{12}}$

Q. Find the ratio of L to R for which moment of inertia for the given axis is minimum.



$\rightarrow$ We know, $V = \pi R^2 L$ (constant) $\Rightarrow R^2 = \frac{V}{\pi L}$

For I to be minimum,
$\frac{dI}{dx} = 0$

$I = \frac{MR^2}{4} + \frac{ML^2}{12} \Rightarrow \frac{MV}{4\pi L} + \frac{ML^2}{12}$

$\frac{dI}{dx} = \frac{MV}{4\pi}\left(-L^{-2}\right) + \frac{2ML}{12} \Rightarrow 0$

$$\Rightarrow \frac{V}{\pi L^2} = \frac{2L}{3} \Rightarrow \frac{\pi R^2 L}{\pi L^2} = \frac{2L}{3} \Rightarrow \frac{R^2}{L^2} = \frac{2}{3}$$

$$\Rightarrow \boxed{\frac{L}{R} = \sqrt{\frac{2}{3}}}$$

$$\boxed{\tau = I\alpha} \quad \boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$

Q. Find 'α' and angular displacement in 't' sec.

$\rightarrow \tau = I\alpha$ | $\vec{\tau} = \vec{r} \times \vec{F}$

$\tau = r \times F$

$\Rightarrow I\alpha = \frac{L}{4} \times F$

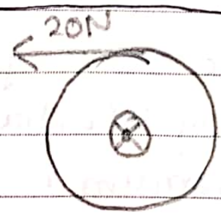
$\Rightarrow \alpha = \frac{12 F}{4 \cdot ML} = \frac{3F}{ML}$

$\therefore \theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$= 0 + \frac{1}{2} \cdot \frac{3F}{2 \cdot ML} \cdot t^2$

$= \frac{3F}{ML} \cdot t^2$

Q. Find 'ω' of disc after 5 sec. ($R = 20$cm)
($I = 0.2$ kgm^2)



$\rightarrow \tau = 0.2 \times 20 = 4$

$I = 0.2 \qquad \therefore \alpha = \frac{\tau}{I} = 20$

$\therefore \omega = 0 + \alpha t = 100$ radian sec^{-1}

Q. A wheel with MOI $2\,kgm^2$ and rotating at 50rpm is brought to rest. Find the Torque required to bring it to rest in 1 min.

(a) $\pi/6$ (b) $\pi/12$ (c) $\pi/15$ (d) $\pi/18$

$\rightarrow$ $\omega_i = 50$ rpm

$= 50 \times \frac{2\pi}{60} = \frac{5\pi}{3}$ radians s^{-1}

~~α~~ $\omega = \omega_i + \alpha t$

$\Rightarrow 0 = \frac{5\pi}{3} + \alpha \cdot 60$

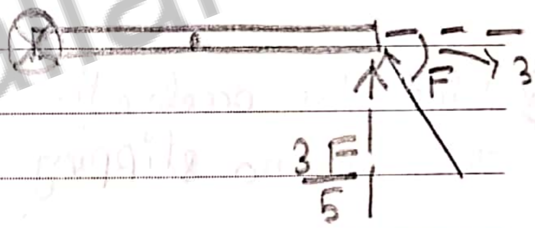
$\Rightarrow \alpha = \frac{-5\pi}{180} = \frac{-\pi}{36}$

$\tau = I\alpha$

$= 2 \times \frac{-\pi}{36}$

$= \boxed{-\frac{\pi}{18}}$

Q. Find α and a of COM immidiately after application of Force.



$\rightarrow$ $\vec{\tau} = \vec{r} \times \vec{F}$

$\Rightarrow I\alpha = L \times \frac{3F}{5}$

$\Rightarrow \boxed{\alpha = \frac{9F}{5ML}}$

$a_t = r\alpha$

$= \frac{9F}{10M}$

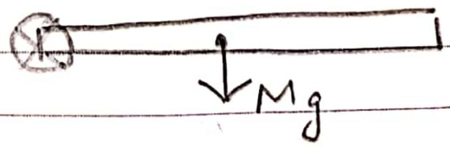
$a_c = 0$

$\therefore a = \sqrt{(a_t)^2 + (a_c)^2} = \boxed{\frac{9F}{10M}}$

Q. Find α of rod immidiately after it is released.

$\tau = rF$

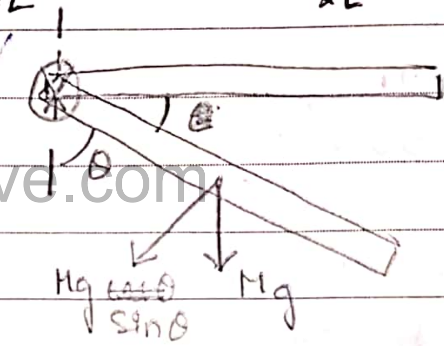
$\Rightarrow \alpha = \frac{rF}{I} = \boxed{\frac{3g}{2L}}$



Q. Find α when rod makes 'θ' with the vertical. [JEE 2017]

Released

(a) $\frac{2g}{3L}\cos\theta$ (b) $\frac{2g}{3L}\sin\theta$ (c) $\frac{3g}{2L}\cos\theta$ (d) $\frac{3g}{2L}\sin\theta$



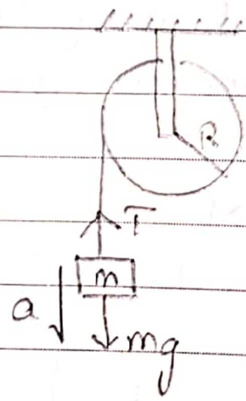
$\rightarrow$ $\tau = r \times F$

$= \frac{L}{2} \times Mg \cancel{\cos\theta} \sin\theta$

$\Rightarrow \frac{\cancel{M} L^{\cancel{2}}}{3} \times \alpha = \frac{\cancel{L}}{2} \times \cancel{M} g \sin\theta$

$\Rightarrow \boxed{\alpha = \frac{3g}{2L}\sin\theta}$

Q. Find the acceleration of the block 'm' if there is no slipping



$\tau = I\alpha$

$\Rightarrow RT = I\alpha$

$\Rightarrow T = \frac{Ia}{R^2}$

$mg - T = ma$

$\Rightarrow mg = a\left(m + \frac{I}{R^2}\right)$

& $a_t = R\alpha$

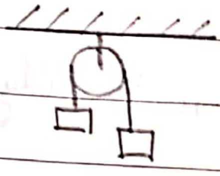
$\Rightarrow a = R\alpha$

$\Rightarrow \boxed{\alpha = \frac{a}{R}}$

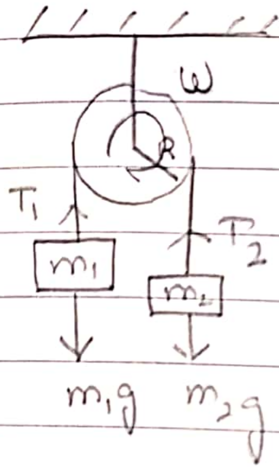
$\Rightarrow \boxed{a = \frac{mg}{m + \frac{I}{R^2}}}$ $\rightarrow$ Standard Formula

Scanned by CamScanner

Q. Find the acc. of the blocks if there is no slipping.



→



$m_2 g - T_2 = m_2 a$

$~~m~~ T_1 - m_1 g = m_1 a$

$T_1 - T_2 = \frac{Ia}{R^2}$

(+)

$(m_2 - m_1) g = a\left(m_1 + m_2 + \frac{Ia}{R^2}\right)$

$\tau = I\alpha$

$\Rightarrow \cancel{m_1 R - m}$

$\Rightarrow T_1 R - T_2 R = I\alpha$

$\Rightarrow \cancel{(T_1 R - T_2) R}$

$\Rightarrow T_1 - T_2 = \frac{Ia}{R^2}$

Q. (i) Find Angular acceleration 'α' of rod

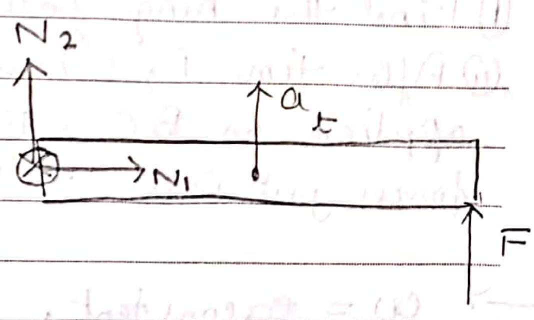
(ii) a_{COM} of rod initially

(iii) Reaction force offered by hinge to the rod.

→ (i) $\tau = I\alpha \Rightarrow r \times F = \frac{ML^2}{3} \cdot \alpha \Rightarrow \boxed{\alpha = \frac{3F}{ML}}$

(ii) $a_t = R\alpha = \frac{L}{2} \cdot \frac{3F}{ML} = \boxed{\frac{3F}{2M}}$

$\boxed{a_c = R\omega^2 = 0}$ ∴ $\boxed{a = \frac{3F}{2M}}$

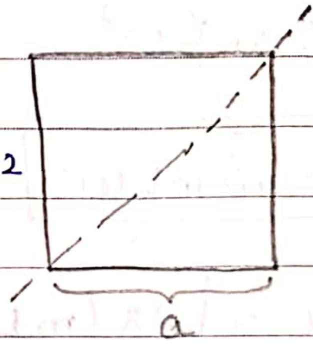


(iii) $N_2 + F = M a_t$

$\Rightarrow N_2 = \frac{3F}{2} - F = \boxed{\frac{F}{2}}$

$\boxed{N_1 = -M a_c = 0}$

Q. Find KE of lamina after 5 s.
($a = 10\,cm = 0.1\,m$, $m = 2\,kg$, $\tau = 0.1\,Nm$)



→ $I = \frac{Ma^2}{12}$, $= \frac{0.1 \times 0.1 \times 2}{1200} = \frac{1}{600}\,kg\,m^2$

$\tau = I\alpha \Rightarrow \alpha = 60\,rad\,s^{-2}$

$\therefore \omega = \omega_0 + \alpha t = 300\,rad\,s^{-1}$

$\therefore KE = \frac{1}{2} I\omega^2 = \frac{1}{\cancel{2}} \cdot \frac{1}{\cancel{600}_{2}} \times \cancel{300} \times \cancel{300}^{75} = 75\,J$

Q. A wheel $I = 3\,kg\,m^2$, $\tau = 6\,Nm$. Find the work done by Torque in 20 s. (a) 2400 J (b) 3000 J (c) 3600 J (d) 1800 J

→ $\tau = I\alpha \Rightarrow \alpha = 2\,rad\,s^{-2}$

Method 1: $\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{1}{2} \cdot 2 \cdot 20 \times 20 = 400$ radians

$\therefore W = \int \tau\, d\theta = 6 \times 400 = 2400\,J$

Method 2:

$W_{\text{all forces}} = \Delta KE$
$= KE_f - KE_i$

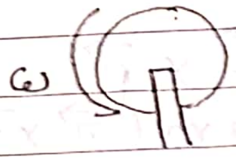
$\therefore \omega = \alpha t = 2 \times 20 = 40\,rad\,s^{-1}$

$\therefore K_i = 0$

$K_f = \frac{1}{2} I\omega^2 = 2400\,J$

$\therefore W = 2400 - 0 = 2400\,J$

Q. A flywheel is rotating with initial angular speed 60 rad s^{-1}. Its MOI about axis is 5 kg m^2. Due to friction of axle, it stops in 5 mins. Find the work done by frictional force.



ⓐ 5000 J ⓑ 9000 J ⓒ 16000 J ⓓ 24000 J

→ $\omega_f = \omega_i + \alpha t \Rightarrow 0 = 60 + \alpha \cdot 300 \Rightarrow \alpha = -\frac{1}{5}$ rad s^{-2}

Method 1:

$\therefore \tau = I\alpha = -1$ Nm

$\theta = \omega_i \cdot t + \frac{1}{2}\alpha t^2$

$= 60 \cdot 300 + \frac{1}{2}\left(-\frac{1}{5}\right) \times 300 \times 300$

$= 18000 - 9000 = 9000$ rad

$\therefore W = \int \tau \, d\theta$

$= -1 \times 9000$

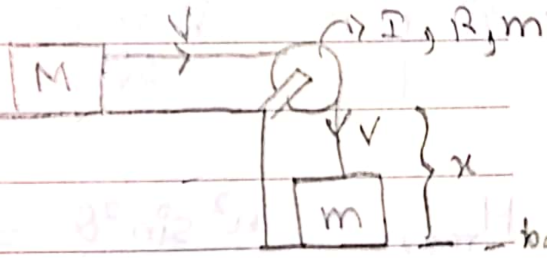
$\therefore$ Work done by frictional force $= 9000$ J

Method 2:

$W = \Delta KE = KE_f - \overset{KE_f}{0} = -\frac{1}{2} I \omega^2 = -\frac{1}{2} \cdot 5 \cdot 60 \times 60$

$= 9000$ J

Q. For no slipping between pulley and rope (massless), the system is released. Find the velocity of block 'm' when it has fallen through height 'h'.



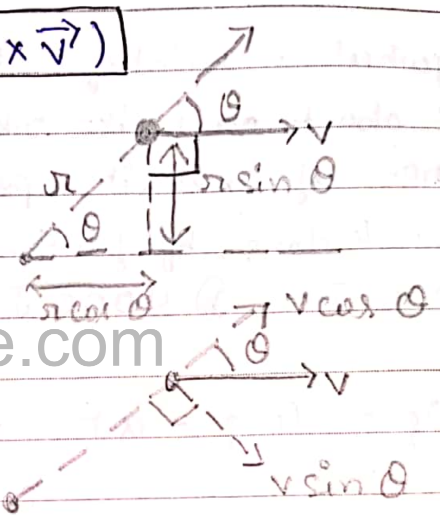
→ $U_i + K_i = U_f + K_f$

$\Rightarrow (Mgx + m'gx + 0) + (0) = (Mgx + m'gx - mgh) + \left(\frac{1}{2}Mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\right)$

$\Rightarrow 2mgh = Mv^2 + mv^2 + \frac{I v^2}{R^2} \Rightarrow \boxed{v = \frac{2mgh}{m + M + \frac{I}{R^2}}}$

Angular Momentum

$$\boxed{\vec{L} = \vec{r} \times \vec{p}} \qquad \boxed{\vec{L} = m(\vec{r} \times \vec{v})}$$



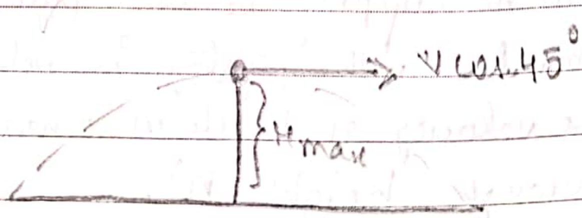
$$\vec{L} = \vec{r} \times \vec{p}$$
$$= m(\vec{r} \times \vec{v})$$
$$= mvr \sin\theta$$
$$= mv(r\sin\theta) \quad \Big| \quad = mr(v\sin\theta)$$
$$= mvr_\perp \quad \Big| \quad = mrv_\perp$$

Q. A particle of mass 'm' is moving with constant velocity 'v' ∥ to x-axis. It's angular momentum about origin is:
(a) 0 (b) Constant (c) Increases (d) Decreases

→ v is constant and $r\sin\theta$ is constant, L is also constant.

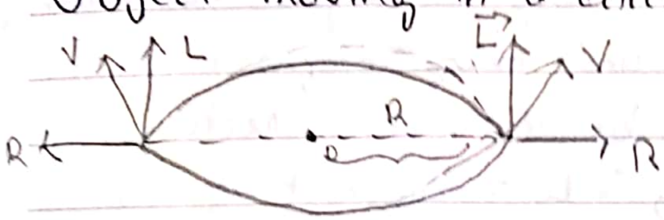
Q. A ball of mass 'm' is projected with velocity 'v' at an angle (45°) with respect to horizontal. Find $\vec{L}$ of the ball w.r.t. point of projection when ball is at it's maximum height.
(a) $\frac{mv^3}{\sqrt{2}g}$ (b) $\frac{mv^3}{2\sqrt{2}g}$ (c) $\frac{mv^3}{4\sqrt{2}g}$ (d) 0

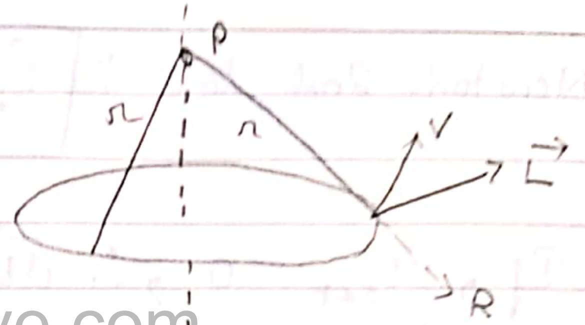
$$H_{max} = \frac{u^2 \sin^2\theta}{2g} = \frac{v^2}{4g}$$



$$\therefore L = mvr_\perp = m\frac{v}{\sqrt{2}} \cdot \frac{v^2}{4g} = \frac{mv^3}{4\sqrt{2}g}$$

Object moving in a circle :





$\vec{L} = mvr_{\perp} = mvr$

∴ If the point is in Centre of the circle, ~~angular momentum will be both~~.

$|\vec{L}| = \text{constant}$

$\hat{L} = \text{constant}$

$\vec{L} = mvr_{\perp} = mvr$

∴ If the point is not in Centre,

$|\vec{L}| = \text{constant}$

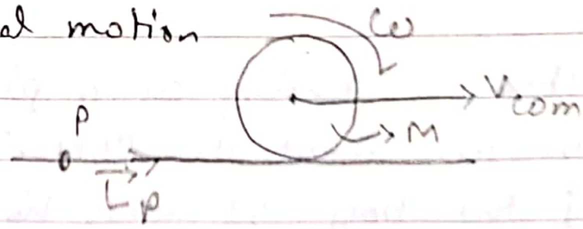
$\hat{L} \neq \text{constant}$

For a purely rotating object :

$\boxed{L = I\omega}$ → always valid

$\boxed{\vec{L} = I\vec{\omega}}$ → valid only for symmetric objects

Plane Motion = Rotational + translational motion



$L_P = \vec{L}_T + \vec{L}_R$

$\boxed{L_P = M(\vec{r} \times \vec{V}) \pm I_{com}\omega}$

Newton's 2nd law : $\boxed{\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}}$ → change in angular momentum and ~~change~~ Torque will be in same direction

If, $\tau_{ext} = 0$, $\therefore \frac{d\vec{L}}{dt} = 0$, $\therefore L$ = constant

$\therefore$ $\boxed{\vec{L_i} = \vec{L_f}}$ $\boxed{I_1\omega_1 = I_2\omega_2}$

If, $\tau_{any\ axis} = 0$ [external]

$\therefore L_{that\ axis}$ = constant

Q. A diver with MOI $6\,kgm^2$ has $\omega = 2\,rad\,s^{-1}$. He folds his body and reduces MOI to $5\,kgm^2$. Find new 'ω'.

→ $\tau_{ext} = 0$, $\therefore L_i = L_f$ ⇒ $I_1\omega_1 = I_2\omega_2$

⇒ $6 \times 2 = 5 \times \omega_2$

⇒ $\omega_2 = 2.4$ rad s^{-1}

Q. A body is standing on a platform which is free to rotate about an axis about its COM. The KE of boy and & platforms is k. If the boy streches his hands and doubles MOI. Find the new KE. (a) 2k (b) $\frac{K}{4}$ (c) $\frac{K}{2}$ (d) 4K

→ $K_i = K = \frac{1}{2}I\omega^2$

$K_f = \frac{1}{2}(2I)\left(\frac{\omega}{2}\right)^2 = \frac{I\omega^2}{4}$

$= \frac{1}{2}\left(I\omega^2\right)$

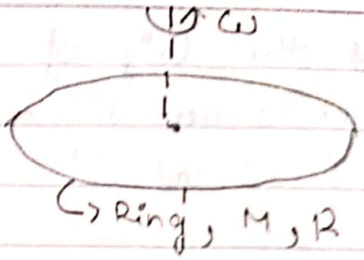
$= \boxed{\frac{K}{2}}$

$\vec{L_1} = \vec{L_2}$

⇒ $I\omega_1 = 2I\omega'$

⇒ $\omega' = \frac{\omega}{2}$

Q Two masses $\left(\frac{M}{8}\text{ each}\right)$ are kept at O. These masses can slide on the rod radially outward. At some instant, one of the mass is at $\frac{3R}{5}$ distance away from O, when angular velocity of system is $\frac{8}{9}\omega$. Find the distance of other $\frac{M}{2}$ at this instant.

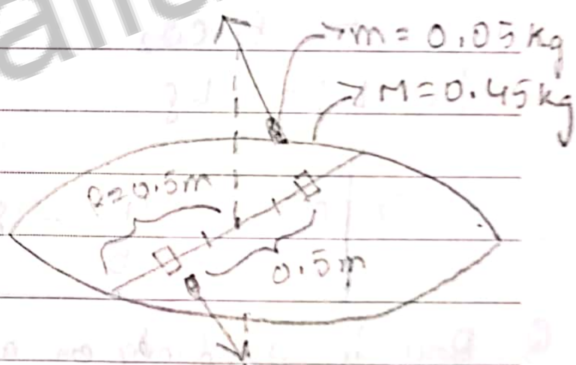


$\rightarrow$ $L_i = L_f \Rightarrow I_1\omega_1 = I_2\omega_2$

$$\Rightarrow MR^2\omega = \left[MR^2 + \frac{M}{8}\left(\frac{3R}{5}\right)^2 + \frac{M}{8}x^2\right] \times \frac{8}{9}\omega$$

$$\Rightarrow \boxed{x = \frac{4}{5}R}$$

Q When these balls leave the platform, their velocity are $9\,ms^{-1}$ each. Find the ω of platform when the balls leave the disc.



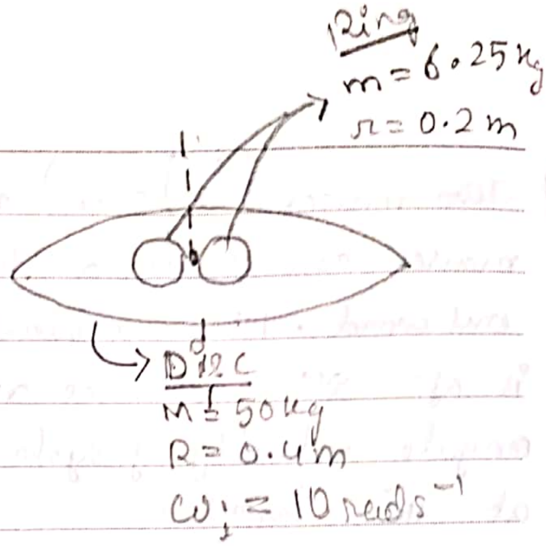
(a) $4\ rad\ s^{-1}$ (b) $6\ rad\ s^{-1}$ (c) $8\ rad\ s^{-1}$ (d) NOTA

$\rightarrow$ $L_i = 0,\ \tau_{net} = 0,\ L_f = 0$

$L_f = L_{f(balls)} + L_{Disc}$

$= mvr_\perp + I\omega$

$$\Rightarrow \omega = \frac{-mvr_\perp}{I} = \frac{2\times\cancel{0.05}\times 9\times\cancel{0.25}}{\cancel{0.45}\times\cancel{(0.5)^2}} \times 2 = \boxed{4}$$



Q Find the ω_f of the system.
(a) 1.33 rad s^{-1} (b) 4 rad s^{-1}
(c) 3.33 rad s^{-1} (d) 8 rad s^{-1}

→ $L_i = L_f$

$$L_i = I\omega_1 = \frac{MR^2}{2}\omega_1 = \frac{50\times(0.4)^2}{2}\cdot 10 = 40$$

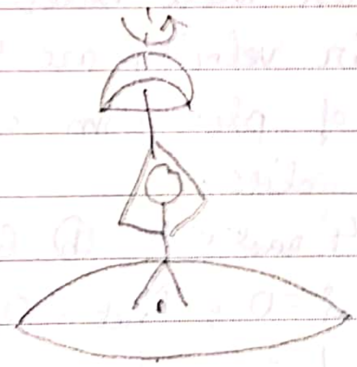
$$L_f = (2I_{\text{ring about COM of disc}} + I_{\text{disc}})\,\omega_2$$

$$= \left[2\cdot(2mr^2) + \frac{MR^2}{2}\right]\omega_2$$

$$= 5\,\omega_2$$

$\therefore\ L_i = L_f$

$$\Rightarrow \boxed{\omega_2 = \frac{40}{5} = 8}$$



Q Boy is standing on a ~~pat~~ platform which is free to rotate, holding an umbrella. If umbrella is twisted with angular speed of 2 rad s^{-1} with respect to platform. Find $\omega_{platform}$ final. $I_{B+P} = 3\times10^{-3}$, $I_U = 2\times10^{-3}$
(a) 1.33 (b) 0.4 (c) 3.33 (d) 0.8

→ $L_i = L_f = 0$, $\omega_{UP} = \omega_{UG} - \omega_{PG} = \omega_{UG} - (-\omega)$

$\Rightarrow 2 = \omega_{UG} + \omega$

$\Rightarrow \omega_{UG} = 2 - \omega$

$\therefore L_f = 0$

$\Rightarrow I_U \omega_{UG} + I_{(P+B)} \omega_{(P+B)G} = 0$

$\Rightarrow 2\times10^{-3} \times (2-\omega) + 3\times10^{-3}(-\omega) = 0$

$\Rightarrow \omega = \frac{4}{5} = \boxed{0.8}$

Q. A star is rotating (spin) about its own axis such that its period of rotation is 30 days. If due to an internal explosion, the star shrinks to nebula star (dwarf star) such that $R_i = 10^4$ km, $R_f = 3$ km, Find the new time period.

$\rightarrow L_i = L_f \Rightarrow I_1\omega_1 = I_2\omega_2 \Rightarrow \frac{2}{5}MR_i^2\omega_1 = \frac{2}{5}MR_f^2\omega_2$

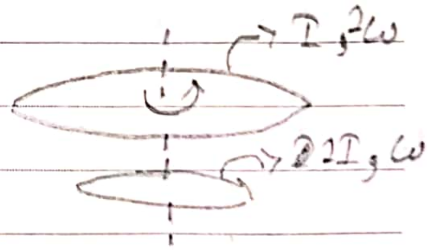
$\boxed{\omega = \frac{2\pi}{T}}$ Note

$\Rightarrow \omega_1 R_1^2 = \omega_2 R_2^2$

$\Rightarrow \frac{R_i^2}{T_i} = \frac{R_f^2}{T_f} \Rightarrow T_f = \frac{R_f^2 T_i}{R_i^2} = \frac{9\times30\times24}{10^4}$

$= 2332.8$ s

Q. Find the common angular speed when the 2 discs are put in contact



$L_i = L_f$

$\rightarrow I\,2\omega + 2I\omega = 3I\omega'$

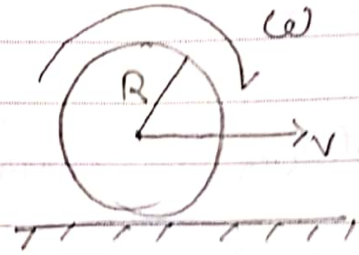
$\Rightarrow 4\omega = 3\omega'$

$\Rightarrow \boxed{\omega' = \frac{4}{3}\omega}$

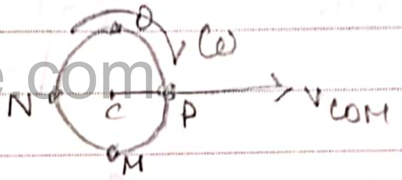
Translation + Circular motion

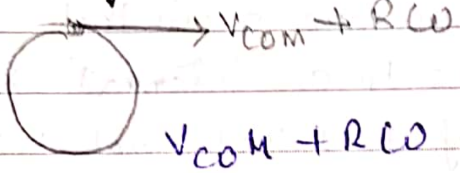
$$\boxed{\vec{S}_{PC} = \vec{S}_{PG} - \vec{S}_{CG}}$$

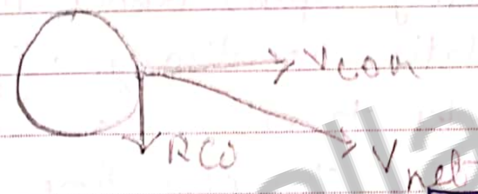
$$\boxed{\vec{V}_{PG} = \vec{V}_{CG} + \vec{V}_{PC}}$$



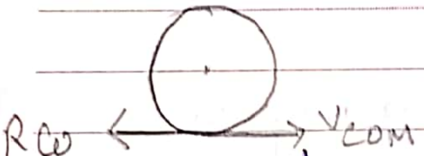
Q Find the velocities of M, N, Q, P wrt ground.





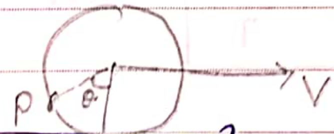


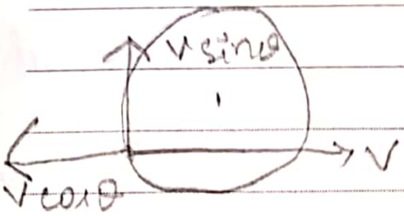
$$V_{net} = \sqrt{V_{COM}^2 + (R\omega)^2}$$



$$|V_{COM} - R\omega|$$

Q Find velocity of point P. $\left(\omega = \frac{V}{R}\right)$



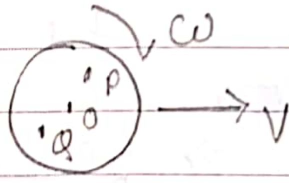


$$\therefore V_{net} = \sqrt{[v(1-\cos\theta)]^2 + [v\sin\theta]^2}$$

$$= 2v\sin\frac{\theta}{2}$$

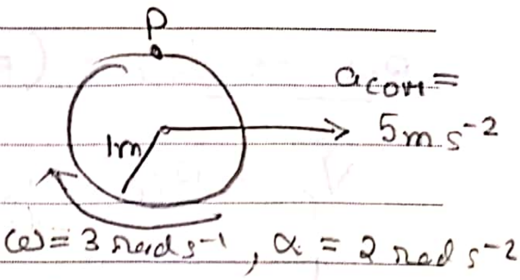
Q (a) $V_P > V_O > V_Q$ (c) $V_Q = V_P,\ V_O = \frac{V_P}{2}$

(b) $V_Q > V_O > V_P$ (d) $V_Q < V_O > V_P$



$$\boxed{\vec{a}_{PG} = \vec{a}_{CG} + \vec{a}_{PC}}$$

Q. a_{PG} : Find.

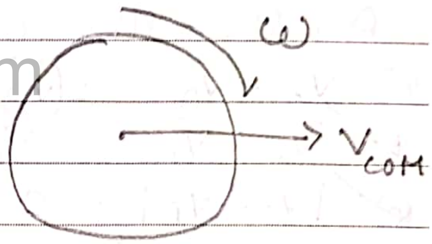


$\rightarrow \vec{a}_{PG} = \vec{a}_{COM} + \vec{a}_c + \vec{a}_t$

$= \sqrt{7^2 + 9^2}$ ~~$= \sqrt{64} = 8 rad s^{-2}$~~

$= \sqrt{130}$

$\boxed{P_{net} = M V_{COM}}$



$\vec{V}_{1C} = \vec{V}_{1G} - \vec{V}_{CG}$

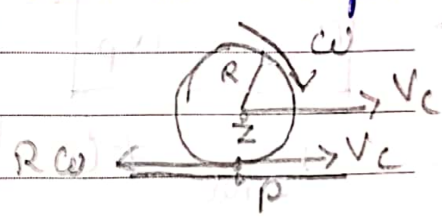
$\Rightarrow \boxed{\vec{V}_{1G} = \vec{V}_{1C} + \vec{V}_{CG}}$

$\boxed{KE_{total} = \frac{1}{2} M (V_{COM})^2 + \frac{1}{2} I \omega^2}$

$\boxed{L_{Total} = \cancel{MVr}\ M V_{COM} r + I_{COM} \omega}$

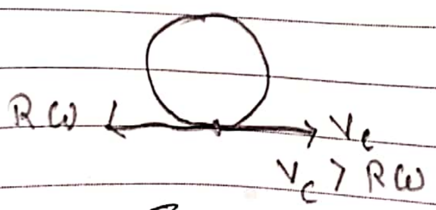
Pure Rolling

Definition – when point of contact stays at rest with respect to ground.

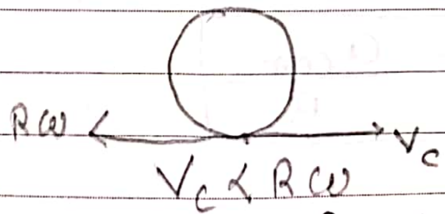


$V_z = 0 \Rightarrow V_c - R\omega = 0$

$\Rightarrow \boxed{\omega = \frac{V_c}{R}} \rightarrow$ 95% true for no slipping

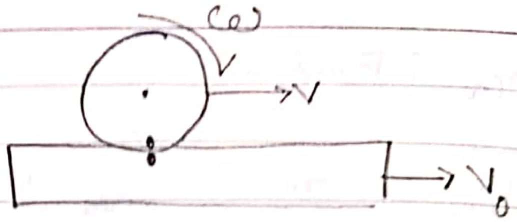


Forward slipping



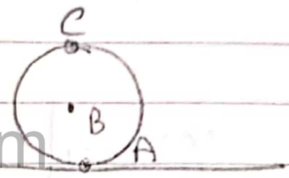
Backward slipping

Rest 5% cases: (For pure Rolling)



$$V_c - R\omega = V_0$$

$$\Rightarrow \boxed{\omega = \frac{V_c - V_0}{R}}$$



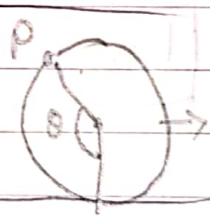
Q. A ball, rolling without slipping,
(a) $\vec{V_C} - \vec{V_A} = 2(\vec{V_B} - \vec{V_C})$
(b) $\vec{V_C} - \vec{V_B} = \vec{V_B} - \vec{V_A}$
(c) $|V_C - V_A| = 2|V_B - V_C|$
(d) $|V_C - V_A| = 4|V_B|$

[IIT JEE 2009]
[Multiple choice]

$\rightarrow V_C = V_B + R\omega = 2V_B$
$V_A = 0$

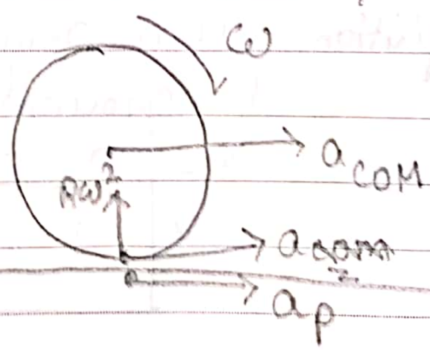
$\therefore$ Check the above options.

Speed of an point in pure rolling is $\boxed{2v\sin\frac{\theta}{2}}$



Acceleration

$\boxed{a_z = a_p}$ $\therefore \boxed{a_{zp} = 0}$

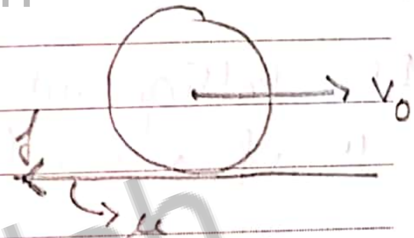


$a_{z\,net} \neq 0$ [$\because$ Centrepetal acc. will always act]

$$\boxed{\alpha = \frac{a_{com}}{R}}$$

Friction produces initial torque for the body to start rolling, after some ~~distance~~, tangential velocity decreases, angular velocity increases, after some time, tangential velocity will be equal to angular velocity, then pure rolling will start, then friction will cease to act.

Q. A disc (M, R) starts rolling on a surface with coefficient of friction (μ). What will be the ~~tangential~~ velocity of COM after the body starts pure rolling.



$\rightarrow$ $f = \mu Mg$, $a_f = -\mu g$

$v = v_0 - (\mu g)t$ — (i)

$\omega_i = 0$, $\omega_f = \omega$

$\vec{\tau} = \vec{R} \times \vec{F}$

$= R\mu Mg$

$\therefore \alpha = \frac{\tau}{I} = \frac{2\mu g}{R}$

$\omega_f = \omega_i + \alpha t$

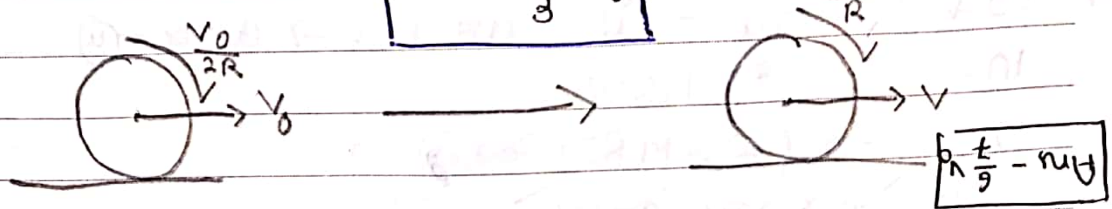
$\Rightarrow t = \frac{v_0}{2\mu g}$ — (ii)

Putting (ii) in (i),

$v = v_0 - \frac{v}{2}$ $\Rightarrow v_0 = \frac{3}{2}v$

$\Rightarrow \boxed{v = \frac{2}{3}v_0}$

Q. Find v.

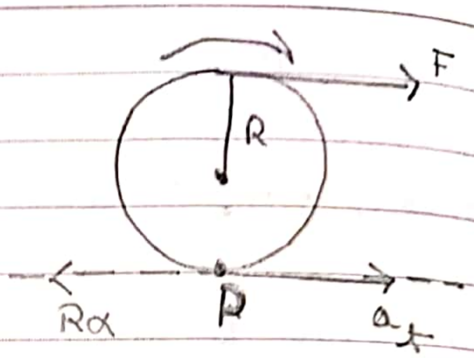


Forces in Pure Rolling

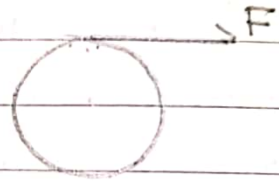
(i) $a = R\alpha$, Friction $= 0$, $a_{PG} = 0$

(ii) $a < R\alpha$, Friction (static) $\rightarrow$ aage ki taraf
$\hookrightarrow a_P = 0$

(iii) $a > R\alpha$, Friction (static) $\rightarrow$ peeche ki taraf
$\hookrightarrow a_P = 0$



Q1 For Rolling without slipping, Find a, f, α. (Disc)



$\rightarrow$ $F + f = Ma$ —— (i)

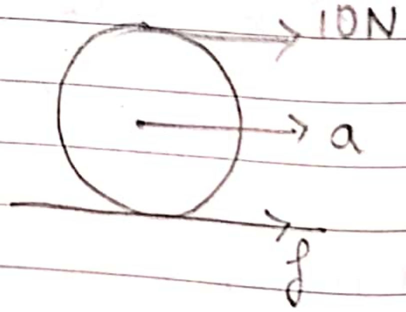
$FR - fR = I\alpha$
$= MR^2\alpha$ —— (ii)

$a = R\alpha$ —— (iii)

$F - f = Ma$

$\therefore$ $\boxed{a = \frac{F}{M}}$ $\boxed{f = 0}$ $\boxed{\alpha = \frac{F}{MR}}$

Q Solid Sphere, $M = 5kg$, $R = 1m$ (PR)



$\rightarrow$ $10 + f = 5a$ —— (i) $\quad a = R\alpha \Rightarrow a = \alpha$ —— (ii)

$10 - f = \frac{2}{5}MR^2\alpha$

$\cancel{\Rightarrow 50 - 5f = 2MR^2\alpha}$

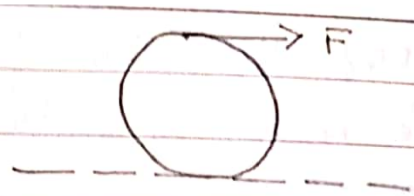
$\cancel{a = R\alpha \Rightarrow a = \alpha}$

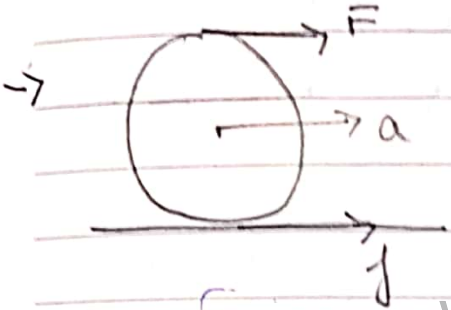
$\Rightarrow 10 - f = 2\alpha = 2a$ —— (iii)

Add in (i) and (iii),

$a = \frac{20}{7}$, $\alpha = \frac{20}{7}$, $f = \frac{30}{7}N$

Q. Solid sphere, (M, R), $a = ?$.





$$f + F = Ma \quad \text{---(i)}$$

$$FR - fR = \frac{2}{5}MR^2\alpha = \frac{2}{5}MR^2 \cdot \frac{a}{R}$$

$$\Rightarrow F - f = \frac{2}{5}Ma \quad \text{---(ii)}$$

Adding (i), (ii),

$$2F = \frac{7Ma}{5}, \quad \Rightarrow \boxed{a = \frac{10F}{7M}}$$

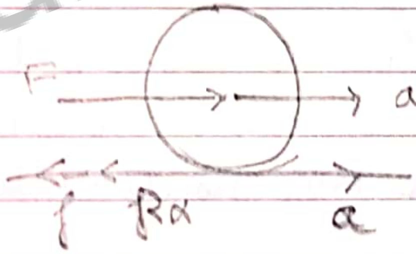
Q. Solid Sphere,

$M = 10\text{ kg}$, $R = 2\text{m}$, $\mu = 0.1$

$$\rightarrow F - f = Ma \quad \text{---(i)}$$

$$fR = \frac{2}{5}MR^2\alpha$$

$$\Rightarrow f = \frac{2}{5}MR\alpha \quad \text{---(ii)}$$



Assuming this is pure rolling, $f \leq \mu Mg$

$\therefore f = \frac{2}{5} \times 10 \times 2 \times \frac{a}{2}$

$= 4a$

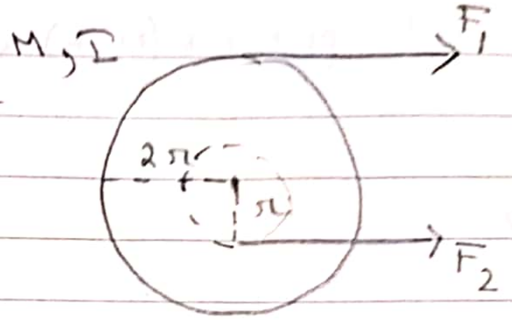
$7 - f = 10a \Rightarrow$

$\therefore f = 2\text{N}$

$\leq \mu Mg$ [which is true]

If f does not comeout to be $\leq \mu Mg$, then $f = \mu N$.

Q. Find the ratio of $\frac{F_1}{F_2}$ such that there is no need of friction for rolling without slipping.



(a) $\frac{Mr^2 + I}{Mr^2 - I}$ (b) $\frac{2Mr^2 + I}{4Mr^2 - I}$

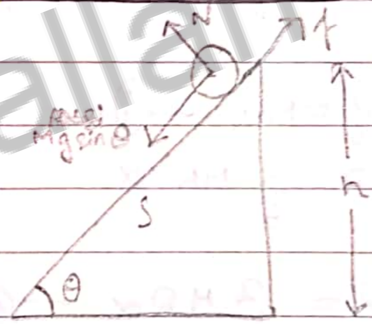
(c) $\frac{Mr^2 + I}{4Mr^2 + I}$ (d) $\frac{2Mr^2 + I}{4Mr^2 + I}$

→ $F_1 + F_2 = Ma$ —(i)

$F_1(2r) - F_2(r) = I\frac{a}{R}$ —(ii)

Solving the equations. Ans — (b)

If inclined plane is frictionless, the body will slide with translational motion and WILL NOT rotate.

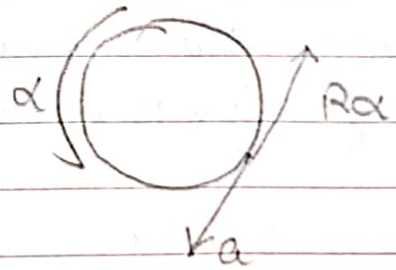


If friction is present,

$Mg\sin\theta - f = Ma$ —(i)

$fR = I\alpha$ —(ii)

$\therefore$ $\boxed{Mg\sin\theta - \frac{I\alpha}{R} = Ma}$ —(iii)



If pure rolling is occuring,

$a = r\alpha$

$\therefore$ $a = \frac{Mg\sin\theta}{M + \frac{I}{R^2}} = \boxed{\frac{g\sin\theta}{1 + \frac{I}{MR^2}}}$ → Depends upon (θ)

Q What would be the velocity of the body at the bottom?

$\rightarrow v^2 = u^2 + 2aS \Rightarrow v^2 = 0 + 2\cdot\left(\frac{g\sin\theta}{1+\frac{I}{MR^2}}\right) \times \boxed{\frac{h}{\sin\theta}}$

$\sin\theta = \frac{h}{S}$

$\Rightarrow S = \frac{h}{\sin\theta}$

$\Rightarrow \boxed{v = \sqrt{\frac{2gh}{1+\frac{I}{MR^2}}}}$ → Does not depend upon 'θ'.

Note - Static friction is present.

From (i), $f = Mg\sin\theta - Ma$

$\Rightarrow \boxed{f_{static} = Mg\sin\theta\left(\frac{I}{MR^2+I}\right)}$

↑ Required Friction

For pure rolling,

$\boxed{F_{req} < f_{lim}}$

$\boxed{\text{Work done by friction} = 0}$

$\boxed{W_{F\to T} = W_{F\to R}}$

$\boxed{\text{Conservation of ME is valid in pure rolling.}}$

↑ Because $W_F = 0$.

Q 2 solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane on a fixed height ~~on~~ at a fixed time. Cylinder P has most of its mass concentrated on its ~~eu~~ surface while Q have most of the mass ~~conce~~ concentrated on its axis. Which of the following statements is / are correct. [P[redacted]-MF]

(a) Both P & Q reach ground simultaneously.
(b) P has larger linear acceleration.
(c) Both reach with same translational kinetic energy on ground.
(d) Q reaches ground with more angular speed.