C-445 HIGHER MATHEMATICS 2014

Time : 3 Hours] **Class : 12th** **[M. M. : 100**

Instructions :- (1) All questions are compulsory. (2) Read the instructions of question paper carefully and write answrs of them. (3) There are two Sections-Section A and Section B in the question paper. (4) In Section A Question Nos. 1 to 5 are objective type which contain Choose the correct answers, True/False, Fill in the blanks, Match the columns and Answer in one word/sentence. Each question carries 5 marks. (5) Internal options are given in Question Nos. 6 to 24 of Section B. (6) Q. Nos. 6 to 10 carry 2 marks each. (7) Q. Nos. 11 to 17 carry 4 marks each. (8) Q. Nos. 18 to 22 carry 5 marks each. (9) Q. Nos. 23 and 24 carry 6 marks each.

(Section–A)

1. Choose the correct answer from the given options of each objective type question :

(i) Fraction $\frac{2x+3}{(x+1)(x-3)} = \frac{a}{x+1} + \frac{b}{x-3}$, $\tan a + b =$

(a) 0 (b) 1

(c) 2 (d) 3

(ii) $\sin\left[\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2}\right]$ is equal to :

(a) 1 (b) 2

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

(iii) Equation of plane parallel to z–axis :

(a) $ax + by + d = 0$ (b) $bx + cz + d = 0$

(c) $by + cz + d = 0$ (d) None of these

(iv) If $y = \log[\log(\log x)]$, then value of $\frac{dy}{dx}$ is :

(a) $\frac{1}{x \log x}$ (b) $\frac{1}{x \log x \log(\log x)}$

(c) $\frac{1}{x \log(\log x)}$ (d) $\frac{1}{x}$

(v) Coordinates of centre of sphere $6x^2 + 6y^2 + 6z^2 - 16x + 9z - 6 = 0$ are :

(a) $\left(\frac{3}{4}, 0, \frac{4}{3}\right)$ (b) $(-16, 0, 9)$

(c) $\left(-8, 0, \frac{9}{2}\right)$ (d) $\left(\frac{4}{3}, 0, \frac{-3}{4}\right)$

2. Write True/False in the following statements :

(i) Distance from origin to the plane $6x - 3y + 2z + 14 = 0$ is 2.

(ii) The value of coefficient of correlation is always 2.

(iii) Integration of constant is zero.

(iv) Unit vector in the direction of vector $\vec{a}$ is $\frac{\vec{a}}{|\vec{a}|}$.

http://www.mpboardonline.com

(v) Position vectors of points P and Q are $\hat{i}+3\hat{j}-7\hat{k}$ and $5\hat{i}-2\hat{j}+4\hat{k}$ respectively, then value of |PQ| is $9\sqrt{2}$.

3. Fill in the blanks :

(i) nth derivative of e^{ax} is ..... .

(ii) In perfect correlation both regression lines be ..... .

(iii) Projection of $\vec{b}$ in the direction of $\vec{a}$ ..... .

(iv) If $xy = 6$, then minimum value of $2x + 3y$ is ..... .

(v) Direction cosines of Y-axis are ..... .

4. Match the correct pairs :

'A'	'B'
(a) Newton-Raphson's formula	(i) $\int_a^b f(x)\,dx = \frac{h}{3}[y_0 + y_n) + 4(y_1 + y_3 + ... + y_{n-1}) + 2(y_2 + y_4 + ... + y_{n-2})]$
(b) Simpson's rule formula	(ii) 2.667
(c) Cube root of 2 upto 3 place of decimal	(iii) 1.258
(d) Trapezoidal rule	(iv) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
(e) Approximate value of $\int_0^2 x^2 dx$ by Simpson's rule when n = 4	(v) 1.25
	(vi) $\int_a^b f(x)\,dx = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + ... y_{n-1})]$. Where $h = \left(\frac{b-a}{n}\right)$

5. Write answers in one word/one sentence each :

(i) Write the value of $\int e^x (\sin x + \cos x)\,dx$.

(ii) Write the value of $\int_0^{\frac{\pi}{2}} \sin x\,dx$.

(iii) Write the value of $\int_0^{\frac{\pi}{2}} \log(\sin x)\,dx$.

http://www.mpboardonline.com

(iv) Write the value of $\int_0^{\pi} |\cos x|\, dx$.

(v) Write the value of $\int \frac{\cot x}{\log(\sin x)} dx$.

(Section-B)

6. AC and BD are the diagonals of a quadrilateral ABCD, prove that :

$\vec{AB} + \vec{DC} = \vec{AC} + \vec{DB}$

(Or) If $\vec{a} = 2\hat{i} - 5\hat{j} + 8\hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} - \hat{k}$, $\vec{c} = -3\hat{i} + 2\hat{j} - \hat{k}$, then find $|\vec{a} + \vec{b} + \vec{c}|$.

7. Find the Vector and Cartesian equation of the sphere whose centre $(2, -3, 4)$ and radius is 5.

(Or) Find the vector equation of the sphere concentric with the sphere $|\vec{r} + (i - 2j - 3k)| = 5$ and its radius is two times of that sphere.

8. If $\vec{a} = 2\hat{i} + 5\hat{j} + 8\hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} + 7\hat{k}$ and $\vec{c} = 3\hat{i} + 2\hat{j} + \hat{k}$, then find the value of $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$.

(Or) A particle is acted upon by a force $\vec{F} = 4\hat{i} + \hat{j} - 3\hat{k}$ it is displaced from the point $(1, 2, 3)$ to the point $(5, 4, 1)$. Find the work done.

9. Evaluate :

$\int \sqrt{\frac{1-x}{1+x}}\, dx$.

(Or) Evaluate :

$\int \frac{1}{1 - \sin x} dx$.

10. Evaluate :

$\int \log_e x\, dx$.

(Or) Solve :

$\int \frac{\sec x}{(\sec x - \tan x)} dx$.

http://www.mpboardonline.com

11. Resolve the following fraction into partial fraction :

$$\frac{x^2+7x}{x^2+7x-8}.$$

(Or) If $\frac{1}{x^3-2x^2-x+2}=\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{x-2}$, then find the value of A + B + C.

12. Solve the equation :

$$\tan^{-1}x+2\cot^{-1}x=\frac{2\pi}{3}.$$

(Or) Prove that :

$\sec^2(\tan^{-1}2)+\operatorname{cosec}^2(\cot^{-1}3)=15.$

13. If $y=\sin(2\sin^{-1}x)$, then prove that :

$$\frac{dy}{dx}=2\sqrt{\frac{1-y^2}{1-x^2}}$$

(Or) If $y=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$,

then find $\frac{dy}{dx}$.

14. Differentiate sec x by first principle.

(Or) If $y=\log\sqrt{\frac{1-\cos3x}{1+\cos3x}}$, find $\frac{dy}{dx}$.

15. The profit function is $P(x)=41+24x-18x^2$, calculate the maximum profit of the company.

(Or) The radius of a spherical balloon is incerasing at the constant rate of 10 cm/sec. At what rate is the surface area increasing when the radius is 15 cm?

16. Calculate coefficient of correlation from the following data :

x	y
3	15
10	17
8	4
6	5
8	4

http://www.mpboardonline.com

(Or) If r is a coefficient of correlation of two variables x and y, then prove that :

$$r = \frac{\sigma_x^2 + \sigma_y^2 - \sigma_{x-y}^2}{2\sigma_x\sigma_y}$$

Where σ_x^2, σ_y^2 and σ_{x-y}^2 are the variance of x, y and x - y respectively.

17. Find the lines of regression from the following data :

x	y
2	6
4	5
6	4
8	3
10	2

(Or) Find the value of y from the following data when x = 70 and coefficient of correlation is 0.8 :

	x	y
Mean	18	100
S.D.	14	20

18. Find the angle between the two lines whose direction cosines are given by equations $l + m + n = 0$ and $2l + 2m - mn = 0$.

(Or) Find the equations of planes passing through the intersection of the planes $x + 3y + 6 = 0$ and $3x - y + 4z = 0$ whose distance from origin is 1.

19. If

$$f(x) = \begin{cases} \dfrac{1 - \cos 4x}{x^2} & x \neq 0 \\ \\ 4 & x = 0 \end{cases}$$

then discuss the continuity of $f(x)$ at $x = 0$.

(Or) Evaluate :

$$\lim_{x \to 0} \frac{6^x - 1}{\sqrt{3 - x} - \sqrt{3}}$$

http://www.mpboardonline.com

20. Find the area of the ellipse :

$$\frac{x^2}{4}+\frac{y^2}{9}=1.$$

(Or) Evaluate :

$$\int \frac{\sin x}{\sin x+\cos x}\,dx .$$

21. Solve the differential equation :

$$(1+x^2)\frac{dy}{dx}+2xy=4x^2.$$

(Or) Solve the homogeneous differential equation :

$(x^2+y^2)\,dx+2xy\,dy=0.$

22. Two cubical dice are thrown simultaneously. Find the probability of getting an odd number on the first dice or getting the sum 9 on the two dice.

(Or) A coin is tossed twice. Find the probability distribution of the number of heads.

23. Find the equation of the sphere passing through the points $(3,0,0)$, $(0,-1,0)$ and $(0,0,-2)$ and having the centre on the plane $3x+2y+4z=1$

(Or) Prove that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ are intersecting to each other. Find their point of intersection.

24. Prove by vector method :

$\sin(\alpha-\beta)=\sin\alpha\ .\ \cos\beta-\cos\alpha\ \sin\beta.$

(Or) If D, E, F are the mid point of the sides BC, CA, AB of the triangle ABC, then prove by vector method that :

$$\Delta DEF=\frac{1}{4}\Delta ABC.$$

❑ ❑ ❑

http://www.mpboardonline.com