CH-2 Units & Measurements

Dimensional Analysis

Mass → M　　Current → A
length → L　　Temperature → θ
Time → T　　$Q = I \times t$ (charge = current × time)

→ velocity $= \frac{Disp}{Time} = \frac{L}{T} = LT^{-1}$

here there is No term of M so M^0

velocity $= M^0L^1T^{-1}$

The power of it is called Dimensional

→ Momentum $= P = MV$
$= M \times \frac{disp}{time}$
$= \frac{ML}{T} = M^1L^1T^{-1}$

→ Acceleration $= \frac{\Delta V}{\Delta t} = \frac{disp/time}{time} = \frac{L}{T \times T} = L^1T^{-2}$
$= M^0L^1T^{-2}$

→ Force = mass × acc $= M \times a = M \times \frac{disp/time}{time}$
$= M^1L^1T^{-2}$

→ Pressure $= \frac{F}{A} = \frac{M^1L^1T^{-2}}{L^2} = M^1L^{-1}T^{-2}$

→ Work = F × disp $= MLT^{-2} \times L = ML^2T^{-2}$

(same)

→ Energy $= ML^2T^{-2}$

→ kinetic Energy $= ML^2T^{-2}$

→ potential Energy $= ML^2T^{-2}$

→ Moment of Inertia

$I = M \times d^2$
$= M^1 L^2 T^0$

→ Surface Tension

$T = \frac{\text{Force}}{\text{unit length}} = \frac{F}{l} = \frac{MLT^{-2}}{L}$

$= MT^{-2} = M^1 L^0 T^{-2}$

→ Torque $= F \times \perp d$

$= MLT^{-2} \times L$
$= ML^2T^{-2}$

→ Stress $= \frac{F}{A} = \frac{MLT^{-2}}{L^2}$

$= M^1 L^{-1} T^{-2}$

→ Strain $= \frac{\text{change in length}}{\text{original length}} = \frac{\Delta l}{l}$

$= \frac{L}{L} = L^0 = M^0 L^0 T^0$

↓

This quantity is called Dimensional-le[ss]

→ Refractive Index → $\frac{\text{Speed in } 1^{st} \text{ medium}}{\text{Speed in } 2^{nd} \text{ medium}}$

$= \frac{V_1}{V_2} = M^0 L^0 T^0$

→ Mechanical Advantage $= \frac{\text{Load}}{\text{Effort}} = \frac{L}{E} = \frac{F}{F}$

$= M^0 L^0 T^0$

→ Potential Diff $= \frac{\text{Work done}}{\text{unit charge}} = \frac{W}{Q} = \frac{F \times d}{I \times t}$

$V = \frac{MLT^{-2} \times L}{A \times T} = ML^2 T^{-3} A^{-1}$

* IIT and JEE imp question

1) Boltzmann constant $= K$

$PV = KT$ — Temperature

Pressure, Volume, Bolzmann constant

$K = \frac{PV}{T} = \frac{\frac{F \times A \times l}{A}}{\theta}$ $(\because V = A \times l)$

$= \frac{MLT^{-2}L}{\theta} = \frac{M^1L^2T^{-2}}{\theta}$

* Another imp question of IIT & JEE

2) Planck constant $= h$

$E = hf$ — frequency

Energy, Planck constant

$\Rightarrow h = \frac{E}{f} = \frac{W}{\frac{1}{T}} = F \times disp \times T$

$= MLT^{-2} \times L \times T$

$= M^1L^2T^{-1}$

* 3) Gravitational constant - G.

$F = \frac{Gm_1m_2}{d^2}$ $(\because m_1, m_2 = \text{mass}, d = \text{distance})$

$G = \frac{Fd^2}{m_1 \times m_2}$

$= \frac{MLT^{-2} \times L^2}{M \times M}$

$= M^{-1}L^3T^{-2}$

→ Angular Velocity = ω = $\frac{Velocity}{radius}$

ω (ohmega)

$\frac{v}{r} = \frac{disp/time}{L} = \frac{L}{TL} = L^0T^{-1}$

$= M^0L^0T^{-1}$ = Also of frequency

→ Angular Momentum

= MVr

= $\frac{M \times disp}{time} \times L$

= $\frac{M \times L \times L}{T} = M^1L^2T^{-1}$

→ Resistance

= $\frac{V}{I}$ = $\frac{potential}{current}$

= $\frac{Work\ done}{charge \times current}$ = $\frac{W}{Q \times I}$

= $\frac{W}{I \times t \times I}$ = $\frac{F \times disp}{I \times I \times T}$ = $\frac{MLT^{-2} \times L}{A^2T}$

→ density = $\frac{Mass}{volume}$ = $\frac{M}{L^3}$ = $M^1L^{-3}T^0$

Angle → Dimensionless
→ Unit → radian

Imp Question :- Name one physical quantity which has unit but no Dimension

Ans = Angle

Solid Angle → Dimensionless
→ unit → steradian

Page No.
Date:

Principle of homogenity

↳ Only those physical quantities can be added or subtracted which have same dimension

or

The dimensions of all the terms written on both sides of a correct equation should be same

$A + B = C + D$

for e.g

Q- $v = a + bt$

$v \rightarrow$ velocity
$t \rightarrow$ time
find dimension of a & b

$v - a - bt = 0$

dim of a = dim of v

$= \frac{L}{T} = LT^{-1} = M^0 L^1 T^{-1}$

Page No.
Date:

dim of $b \times t$ = dim of v

$b \times t = \frac{L}{T}$

$b = \frac{L}{T^2} = LT^{-2} = M^0 L^1 T^{-2}$

Q $x = a + \frac{c}{m}$

x = disp
M = mass
find dimension of a & c

Ans $x - a - \frac{c}{M} = 0$

dim of a = dim of x
= L

$= M^0 L^1 T^0$

dim of $\frac{c}{M}$ = dim of x

$\frac{c}{M} = L \qquad = c = M^1 L^1 T^0$

Page No.
Date :

Q $\mu = A + \frac{B}{\lambda^2}$

$\mu \to$ refractive index
$\hookrightarrow$ dimensionless quantity
$\lambda \to$ wavelength

Ans $\mu - A - \frac{B}{\lambda^2} = 0$

dim of A = dim of μ
$= M^0 L^0 T^0$

dim of $\frac{B}{\lambda^2}$ = dim of μ

$\frac{B}{L^2} = M^0 L^0 T^0$

$B = M^0 L^2 T^0$

Page No.
Date :

Q- find dim of x y

$y = \tan(xt)$

t = time
y = ?

Note $\tan(xt)$ is consider as $\tan\theta$
and θ = Angle and Angle = dimensionless

Ans ⇒ dim of $xt = M^0 L^0 T^0$

$X \times T = M^0 L^0 T^0$

$x = M^0 L^0 T^{-1}$

Q $P = a^{(r^2 x)}$

r = radius
a = ?
P = ?

Note Power of any → Dimensionless

Ans dim of $r^2 x = M^0 L^0 T^0$

$L^2 x = M^0 L^0 T^0$
$x = M^0 L^{-2} T^0$

Page No.
Date :

Application
↓

1) check to see if an eq^n is correct
if formula is correct then
Dimensions on LHS = RHS

1) $S = vt$

$S \to$ Disp
$v \to$ velocity
$t \to$ time

LHS RHS

$S = L$ $v \times t$

$\frac{L \times T}{T}$

$= L$

LHS = RHS

$L = L$

So eq^n is correct

2) $P = \frac{4T}{R}$?

P = Pressure
T = surface Tension
R = radius

LHS

$P = \frac{F}{A} = \frac{MLT^{-2}}{L \times L} = ML^{-1}T^{-2}$

RHS

$\frac{4T}{R} = \frac{F}{l \times R} = \frac{MLT^{-2}}{L \times L} = ML^{-1}T^{-2}$

{ i.e. surface Tension = force / unit length }

LHS = RHS

Page No.
Date:

3) $E = mc^2$

E = energy
m = mass
c = speed of light

⇒ LHS

E = Work = F × disp = $MLT^{-2} \times L$
$= ML^2T^{-2}$

RHS

mc^2 = Mass × $\left(\frac{disp}{Time}\right)^2 = M\left(\frac{L}{T}\right)^2$

$= ML^2T^{-2}$

4) $h = \frac{r\rho g}{2T\cos\theta}$

h = height
r = radius
ρ = density (ρ = Mass / volume $= \frac{M}{L^3}$)
g = acc. due to gravity
T = surface Tension

LHS

$h = L$

RHS $\frac{r\rho g}{2T\cos\theta} = \frac{L \times \frac{M}{L^3} \times \frac{\Delta V}{\Delta t}}{\frac{F}{l}}$

Page No.
Date:

$= \frac{L \times \frac{M}{L^3} \times \frac{\frac{disp}{T}}{T}}{\frac{MLT^{-2}}{L}}$

$= \frac{\frac{L \times M}{L^3} \times \frac{disp}{T \times T}}{\frac{MLT^{-2}}{L}}$

$= \frac{\frac{L \times M}{L^3} \times \frac{L}{T \times T}}{\frac{MLT^{-2}}{L}}$

$= \frac{L}{L^3 T^{-2}} \times \frac{L}{T^{-2}}$

$= \frac{L^2}{L^3} = \frac{1}{L}$

RHS $= \frac{1}{L}$

Now que is that if it is incorrect than write correct formula so if $\frac{RHS}{\rho g} = \frac{2 T \cos\theta}{r \rho g}$

than RHS = L

so, LHS = RHS

To write a formula just by knowing the factors on which a given quantity depend

=) Let Force depends upons mass & acc

$F \propto m^x$

$F \propto a^y$

$F = K m^x a^y$

$F = M^1 L^1 T^{-2}$

$M = M$

$a = \frac{\Delta v}{\Delta t} = \frac{disp}{t \times t} = \frac{L}{T^2}$

$M L T^{-2} = (M)^x \left(\frac{L}{T^2}\right)^y$

$M^1 L^1 T^{-2} = M^x L^y T^{-2y}$

=) $x = 1$, $y = 1$ $\quad -2 = -2y$

$y = 1$

=> $K M^1 a^1$

so, $F = KMa$

* Energy of a particle executing SHM depend upon Mass, Amplitude, angular velocity of particle

(M) (A) (ω)→(omega)

=) $E \propto M^x$

$E \propto A^y$

$E \propto \omega^z$

$E = K M^x A^y \omega^z$

E = energy

= work

= F × dis

$= MLT^{-2} \times L$

$= ML^2T^{-2}$

$M = M$

$A = L$

$\omega = \frac{1}{T}$

$M L^2 T^{-2} = M^x L^y \left(\frac{1}{T}\right)^z$

$M^1 L^2 T^{-2} = M^x L^y T^{-z}$

$x = 1$, $y = 2$, $-2 = -z$

$z = 2$

$E = K M^1 A^2 \omega^2$

* Find expression for force on a falling spherical object of radius 'r', in fluid of ~~density ρ^x~~ & viscosity coefficient 'η' with speed v ~~→~~ (Terminal velocity)

⇒ $F \propto r^x$ $\quad F = k r^x \eta^y v^z$

$F \propto \eta^y$

$F \propto v^z$

$F = MLT^{-2}$

$r = L$

$\eta = M^1L^{-1}T^{-1}$

$v = \frac{disp}{Time} = \frac{L}{T}$

for η (eta) = $F = \eta A \frac{\Delta velocity}{\Delta distance}$

$F = \eta A \frac{\Delta V}{\Delta Z}$

$\eta = \frac{F \Delta Z}{A \Delta V} = \frac{MLT^{-2} \times L}{L^2 \times \frac{L}{T}} = M^1L^{-1}T^{-1}$

$= ML^{-1}T^{-1}$

Now $F = k r^x \eta^y v^z$

$MLT^{-2} = L^x \times (M^1L^{-1}T^{-1})^y \times \left(\frac{L}{T}\right)^z$

$M^1L^1T^{-2} = L^{x-y+z} M^y T^{-y-z}$

$y = 1$, $\quad -2 = -y - z$, $\quad 1 = x - y + z = x - 1 + 1$

$-2 = -1 - z$ $\quad x = 1$

$z = -1 + 2 = 1$

$z = 1$

so, $x = 1$, $y = 1$ $z = 1$

$F = k r^x \eta^y v^z$

$F = k r^1 \eta^1 v^1$

$\rightarrow (T)$

* Time period of pendulum depend upon length (l), acc due to gravity (g) & mass of bob (M)

$\Rightarrow$ $T \propto l^x$, $T \propto g^y$, $T \propto M^z$

$T = k\, l^x g^y M^z$

$T = T$
$l = L$
$g = \frac{L}{T^2}$
$M = M$

$T = L^x \left(\frac{L}{T^2}\right)^y M^z$

$M^0 L^0 T^1 = L^{x+y}\, T^{-2y}\, M^z$

$z = 0$, $x + y = 0$, $-2y = 1$

$y = -\frac{1}{2}$

$x = \frac{1}{2}$ $y = -\frac{1}{2}$ $z = 0$

$T = k\, l^x g^y M^z$

$= k\, l^{\frac{1}{2}} g^{-\frac{1}{2}} M^0$

$= \frac{k\, l^{\frac{1}{2}}}{g^{\frac{1}{2}}} = k\sqrt{\frac{l}{g}}$

NEET / JEE / AIPMT

IMP Question

2014 Neet

Q- F, V, T $\rightarrow$ fundamental unit

Mass dimension ?

a) $F^1 V^1 T^{-1}$

b) $F^1 V^1 T^{-2}$

c) $F^1 V^{-1} T^{-1}$

d) $F^1 V^{-1} T^1$

$\Rightarrow$ $M \propto F^x$, $M \propto V^y$, $M \propto T^z$

$M = k\, F^x V^y T^z$

$M = M$
$F = MLT^{-2}$
$V = \frac{L}{T}$
$T = T$

$M = (MLT^{-2})^x \left(\frac{L}{T}\right)^y T^z$

$M^1 L^0 T^0 = M^x L^{x+y} T^{-2x-y+z}$

$x = 1$	$x + y = 0$ $y = -x$ $y = -1$	$-2x - y + z = 0$ $-2(1) - (-1) + z = 0$	$\rightarrow -2 + 1 + z = 0$ $z = 1$

Page No.
Date:

$x=1, y=-1, z=1$

$M = kF^x v^y T^z$

$M = kF^1 v^{-1} T^1 = D$ Ans

2015 AIPMT

E, V, T → fundamental quantity,

dimensional formula → surface tension (S.T)

$S.T \propto E^x$
$S.T \propto V^y$
$S.T \propto T^z$

a) $EV^{-2}T^{-1}$
b) $EV^{-1}T^{-2}$
c) $EV^{-2}T^{-2}$
d) $E^{-2}V^{-1}T^{-3}$

$S.T = kE^x V^y T^z$

$MT^{-2} = (ML^2T^{-2})^x \left(\frac{L}{T}\right)^y T^z$

$M^1L^0T^{-2} = M^x L^{2x+y} T^{-2x+z-y}$

$x=1$	$2x+y=0$	$-2x+z-y=-2$
	$2+y=0$	$-2+z+2=-2$
	$y=-2$	$z=-2$

$S.T = \frac{F}{L}$

$= \frac{MLT^{-2}}{L} = MT^{-2}$

$E = F \times dis$
$= MLT^{-2} \times L$
$= ML^2T^{-2}$

$V = \frac{L}{T}$, $T = T$

Page No.
Date:

$S.T = kE^x V^y T^z$

$S.T = kE^1 V^{-2} T^{-2} = C$ Ans

2015 NEET

Q. critical velocity $v_c \rightarrow \eta^x \rho^y r^z$

a) 1, -1, -1
b) -1, -1, 1
c) -1, -1, -1
d) 1, 1, 1

$\Rightarrow v_c = k\eta^x \rho^y r^z$

$\frac{L}{T} = (M^1L^{-1}T^{-1})^x \left(\frac{M}{L^3}\right)^y (L)^z$

$M^0L^1T^{-1} = M^{x+y} L^{-x-3y+z} T^{-x}$

$-x=-1$	$x+y=0$	$-x-3y+z=1$
$x=1$	$y=-1$	$(-1)-3(-1)+z=1$
		$-1-3-1+z=1$
		$-1+3+z=1$
		$z=-1$

v_c = velocity

η → coefficient of viscosity

ρ → density

r → radius

2017 NEET

Ans = y - 2
x = z = 1/2

Q- Length → $\frac{e^2}{4\pi\epsilon_0}$, c, G

$L \propto \left(\frac{e^2}{4\pi\epsilon_0}\right)^x$

$L \propto (c)^y$

$L \propto (G)^z$

$\left(\frac{e^2}{4\pi\epsilon_0}\right)^x = F \times d^2$
$= M^1L^1T^{-2}L^2$
$= M^1L^3T^{-2}$

$(c)^y = \frac{L}{T} = LT^{-1}$

$(G)^z = M^{-1}L^3T^{-2}$

2017 IIT JEE Main

Q- Time (T) velocity (c) Angular momentum (h)

Mass = ?

a) $T^{-1}c^{-2}h^1$ = Ans = a
b) $T^{-1}c^2h^1$
c) $T^{-1}c^{-2}h^{-1}$
d) $T^{-1}c^{-2}h^1$

⇒ $M \propto T^x$
$M \propto c^y$
$M \propto h^z$

$M = kT^xc^yh^z$

$M = (T)^x(LT^{-1})^y(ML^2T^{-1})^z$

$M^1L^0T^0 = M^zL^{y+2z}T^{x-1y-z}$

z = 1	y+2z = 0	x-1y-z = 0
	y+2 = 0	x+2-1 = 0
	y = -2	x = -1

M = M
T = T

$c = \frac{dis}{Time} = \frac{L}{T} = LT^{-1}$

$h = mvr$
$= M \times \frac{L}{T} \times L$
$= M^1L^2T^{-1}$

2016

IIT JEE

Mains

Q: Current → I, Electrical conductivity (σ)

a) $ML^{-3}T^{-3}I^{2}$
b) $M^{-1}L^{3}T^{3}I$
c) $M^{-1}L^{-3}T^{3}I^{2}$
d) $M^{-3}L^{-3}T^{-3}I$

$J = \sigma E$

$\frac{\text{current}}{\text{Area}} = \text{conductivity} \times \frac{\text{Force}}{\text{unit Charge}}$

$\frac{i}{A} = \sigma \times \frac{F}{Q}$

$\sigma = \frac{i \times Q}{F \times A} = \frac{I^2 \times T}{MLT^{-2} \times L^2}$

$= M^{-1}L^{-3}T^{3}I^{2}$

2016

JEE IIT

Q: A, B, C, D → 4 physical quantities (different dimensions)

$AD = C\log(BD)$ → correct
↳ X

a) $A^2 - B^2C^2$
b) $\frac{A-C}{D}$
c) $\frac{A}{B} - C$
d) $\frac{C}{BD} - \frac{AD^2}{C}$

⇒ $A \times D = C(\;)$

↓ A → y, ↓ D → x

$C = yx$

Error Analysis

- systematic
  - ↓ remove
  - ↓ add / subtract
- Random
  - ↓ cause unknown
  - ↓ no. of Measurements / observation

measurement → Accurate, Precise

Accurate		Precise
5.10		6.10
5.20	Actual = 5	6.11
4.90	length	6.12
5.00		6.02
4.80		

Page No.
Date :

Q Error :-

x_1	x_2	x_3	x_4	x_5
5.1cm	5.2cm	5.3cm	5.4cm	5.0

=> Accurate value = mean

$$\bar{x} = \frac{5.0 + 5.1 + 5.2 + 5.3 + 5.4}{5}$$

$\bar{x} = 5.2$ cm

Absolute Error = $\bar{x} - x$

5.2 − 5.1 = 0.1cm

5.2 − 5.2 = 0.0cm

5.2 − 5.3 = −0.1cm

5.2 − 5.0 = 0.2cm

5.2 − 5.4 = −0.2cm

Mean Absolute Error $= \frac{0.1 + 0.0 + 0.1 + 0.2 + 0.2}{5}$

$\Delta\bar{x} = |\Delta\bar{x}|$

$= \frac{0.8}{5} = 0.16\,cm$

Representation $\bar{x} \pm \Delta\bar{x}$

5.2 ± 0.16

↓ Mean ↓ Mean Absolute Error

F. e $= \frac{0.16}{5.2} = 0.03$

% e $= \frac{0.16}{5.2} \times 100 = 3.0$

Fractional error :- $\rightarrow \frac{\Delta\bar{x}}{\bar{x}}$

% error $\rightarrow \frac{\Delta\bar{x}}{\bar{x}} \times 100$

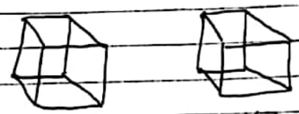
eg Rod $l = \underset{\bar{x}}{6} \pm \underset{\Delta\bar{x}}{0.6}\,cm$

f.e $= \frac{0.6}{6} = 0.1$

% e $= \frac{0.6}{6} \times 100 = 10\%$

Propogation of Error

$2cm^3 \pm 0.02$ $\quad$ $4cm^3 \pm 0.04$



V_1 $\quad$ V_2

1) Add / Sub

Total vol. $= V_1 + V_2 = 6cm^3 \pm 0.06cm^3$

Diff $= V_2 - V_1 = 2cm^3 \pm 0.02cm^3$ ✗

↓

$0.06cm^3$

because error is always added

2) Division / Multiplication

1) $y = \frac{a}{b}$

$\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$

2) $y = \frac{a^2 b^3}{c^4}$

$\frac{\Delta Y}{Y} = 2\frac{\Delta a}{a} + 3\frac{\Delta b}{b} + 4\frac{\Delta c}{c}$

eg $R_1 = 5 \pm 0.05\,\Omega$ $\quad$ $R_2 = 2 \pm 0.01\,\Omega$

Series, Re

$\Rightarrow$ $Re = R_1 + R_2 = 7 \pm 0.06\,\Omega$

eg $y = \frac{a^m b^n}{c^p}$

$\frac{\Delta Y}{Y} = m\frac{\Delta a}{a} + n\frac{\Delta b}{b} + p\frac{\Delta c}{c}$

eg $l = 4 \pm 0.04\,m$

$b = 2 \pm 0.02\,m$

$A = l \times b$

$= 4 \times 2 = 8\,m^2$

Area $= 8 \pm \Delta A$

$A = l \times b$

$= \frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta b}{b}$

$= \frac{\Delta A}{8} = \frac{0.04}{4} + \frac{0.02}{2}$ $\quad = 0.02 \times 8 = 0.16$

$= 8 \pm 0.16$ Ans

eg A piece of material 'x' is given to us Mass 40g with error of 0.4g & its volume is 10 cm³ with error in 1 cm³. Find density.

$M \pm \Delta M$
40g 0.4g

$V \pm \Delta V$
$10 cm^3$ $1 cm^3$

ρ = density $= \frac{M}{V} = \frac{40 g}{10} = 4 g/cm^3$

$\rho = \frac{M}{V}$

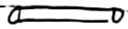
$\frac{\Delta \rho}{\rho} = \frac{\Delta M}{M} + \frac{\Delta V}{V}$

$\frac{\Delta \rho}{4} = \frac{0.4}{4} + \frac{1}{10}$

$\Delta \rho = 0.2 \times 8 = 1.6$

Ans = $4 g/cm^3 \pm 1.6 g/cm^3$

eg length of wire is 4cm ± 0.04cm radius 1cm ± 0.01cm Find volume of wire



$V = \pi r^2 l = \frac{22}{7} \times 1 \times 1 \times 4 = \frac{88}{7} cm$

$\frac{\Delta V}{V} = 2\frac{\Delta r}{r} + 1\frac{\Delta l}{l}$

$\frac{\Delta V}{\frac{88}{7}} = \frac{2 \times 0.01}{1} + \frac{0.04}{4}$ = Ans