# PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

## # Linear Equation :-
An equation in which the highest power of variable is one, i.e., degree of equation is one is called linear equation.

The general form of linear equation in two variables is $ax + by + c = 0$; $a \neq 0$ and $a, b, c$ are real numbers.

For e.g. :- $2x + y = 3$, $x + y = 5$, etc.

## # Solution of Linear Equations in Two Variables

The values of variables which satisfy linear equation in two variables is called solution of linear equations in two

NOTE :- A linear equation in two variables have infinite number of solutions.

# Pair of Linear Equations in Two Variables

If we consider two linear equation at a time simultaneously then, such pair is called pair of linear equations.

For e.g. :- $3x - 5 = 0$ & $x + 2y = 3$

# Nature of Graph of Linear Equation in Two Variables

If $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$ are two linear equations in two variables then nature of its graph is :

i] Intersecting lines.
ii] Parallel lines
iii] Co-incident or Overlapping lines

Q.1] Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." Represent this situation algebraically and graphically.

→ Let present age of Aftab & his daughter be $x$ years & $y$ years respectively,

7 years ago, there ages will be $x-7$ & $y-7$.

According to given condition,

$$x-7 \quad \& \quad y-7$$

$$x-7 = 7(y-7)$$

$$x-7 = 7y-49$$

$$x-7y = -42$$

| | | | |
|---|---|---|---|
| x | -7 | 0 | 7 |
| y | 5 | 6 | 7 |

# * GRAPHICAL Method of Solution of Pair of Linear Equations

## # Consistency of Linear Equation

The equations which having solutions are called consistent equation and those equations which do not have solutions are called inconsistent equation.

If the equations have solution then it has either unique solution or infinite solution.

## # Conditions for Consistency of Linear Equations

If,

$$a_1x + b_1y \cancel{z} + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

are the linear equations in two

| Compare the RATIOS | Graphical REPRESENTATION | Algebraic INTERPRETATION |
| --- | --- | --- |
| $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ | Intersecting lines | Exactly one solution |
| $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ | Coincident lines | Infinitely many solutions |
| $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ | Parallel lines | No solutions |

3) On comparing the ratios $a_1/a_2$, $b_1/b_2$ & $c_1/c_2$, find out whether the following pair of linear equations are consistent or inconsistent.

i) $3x + 2y = 5$ ; $2x - 3y = 7$

→ Compare given equation with $a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$

∴ $a_1/a_2 = 3/2$

$b_1/b_2 = 2/-3$

∴ $a_1/a_2 \neq b_1/b_2$

∴ Given equation represents unique solution i.e. they are consistent.

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(i) $x + y = 5$ ; $2x + 2y = 10$

→ Compare given equation by $a_1x + b_1y = c_1$ & $a_2x + b_2y = c_2$ ,

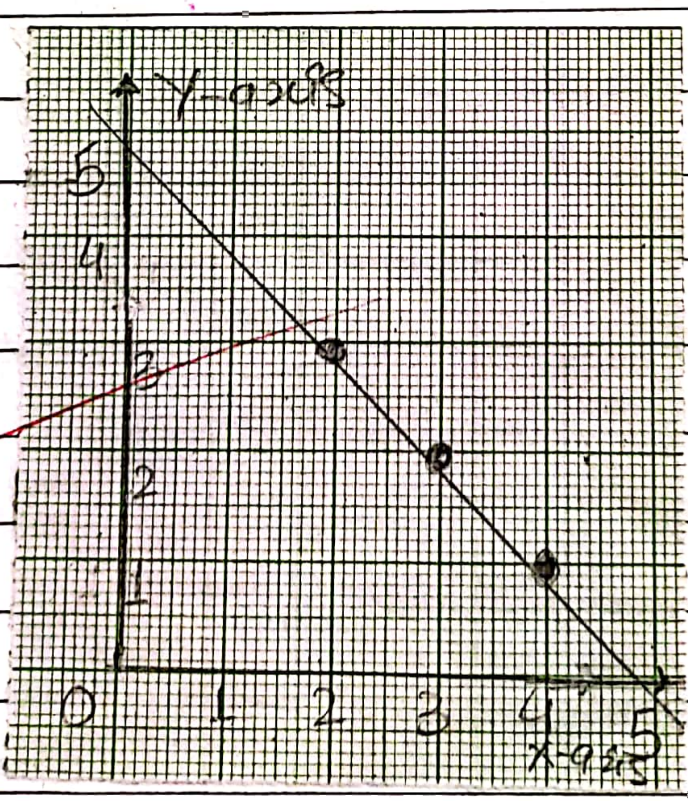
$\therefore a_1/a_2 = 1/2$

$b_1/b_2 = 1/2$

$c_1/c_2 = 5/10 = 1/2$

$\therefore a_1/a_2 = b_1/b_2 = c_1/c_2$

$\therefore$ It has infinite solution , i.e. they are consistent.



# ALGEBRAIC METHODS OF SOLVING A PAIR OF LINEAR EQUATION

The first method to find solution of linear equation in two variables is METHOD OF SUBSITUTION.

1) Solve the following pair of linear equations by the substitution method.

(i) $x+y=14$
$x-y=4$

→ Given equations are,

$$x+y=14 \quad \text{—— (1)}$$
$$x-y=4 \quad \text{—— (2)}$$

from eq. (1),

$$x=14-y \quad \text{—— (3)}$$

By putting value of $x$ in eq. (2)

$$14-y-y=4$$
$$14-2y=4$$
$$-2y=-10$$
$$y=5$$

from eq ②

$x = 14 - y$

$x = 14 - 5$

$\boxed{x = 9}$

∴ Solution of given equation is

$y = 5, \quad x = 9$

A fraction becomes 9/11, if 2 is added to both the numerator and the denominator. If 3 is added to both the num & deno, it become 5/6. Find fraction.

Let the numerator & denominator be $x$ & $y$.

$\therefore$ Fraction is $x/y$

According to condition,

$$\frac{x+2}{y+2} = \frac{9}{11}$$

$$11x + 22 = 9y + 18$$

$$11x - 9y = 18 - 22$$

$$11x - 9y = -4 \quad \text{——} \quad (1)$$

$$\frac{x+3}{y+3} = \frac{5}{6}$$

$$6x + 18 = 5y + 15$$

$$6x - 5y = 15 - 18$$

$$6x - 5y = -3 \quad \text{——} \quad (2)$$

from eq. (1),

$$11x = -4 + 9y$$

$$x = 9y - 4/11 \quad \text{——} \quad (3)$$

By putting value of x in eq. (2)

$$6 \times [9y - 4/11] - 5y = -3$$

$$\frac{54y - 24}{11} - 5y = -3$$

$$\frac{54y - 24 - 55y}{11} = -3$$

$$-y - 24 = -33$$

$$+y = \cancel{-33} + 9$$

$$y = 9$$

By putting value of y in eq. (3),

$x = 9y - 4/11$

$x = 9 \times 9 - 4/11$

$x = 81 - 4/11$

$x = 77/11$

$\boxed{x = 7}$

$\therefore$ Our fraction is $7/9 = x/y = 7/9$

# ELIMINATION METHOD

In this method we have to eliminate one of the variable i.e., either $x$ or $y$.

## Exercise 3.4

1] Solve the following pair of linear equations by the elimination method and the substitution method :

i) $x+y=5$ and $2x-3y=4$

Given equations are

$$x + y = 5 \quad \text{——— ①}$$
$$2x - 3y = 4 \quad \text{——— ②}$$

Multiply eq. ① by 3 & eq. ② by 1

$$3x + 3y = 15 \quad \text{——— ③}$$
$$2x - 3y = 4 \quad \text{——— ④}$$

Add eq. ③ & eq. ④,

$$\begin{array}{r} 3x + 3y = 15 \\ +\ 2x - 3y = 4 \\ \hline 5x \quad\ \ = 19 \end{array}$$

$$x = 19/5$$

By putting value of $x$ in eq. ①

$$x + y = 5$$
$$19/5 + y = 5$$
$$y = 5 - 19/5$$
$$y = \frac{25 - 19}{5} = 6/5$$

∴ values are, $x = 19/5$ & $y = 6/5$

# BY SUBSITUTION METHOD

$x+y=5$ & $2x-3y=4$

→ Given equations are,

$$x+y=5 \quad \text{—— (1)}$$
$$2x-3y=4 \quad \text{—— (2)}$$

from eq. (1),

$$x = 5-y \quad \text{—— (3)}$$

By putting value of $x$ in eq. (2),

$$2 \times (5-y) - 3y = 4$$
$$10 - 2y - 3y = 4$$
$$10 - 5y = 4$$
$$-5y = -6$$
$$y = \frac{+6}{+5}$$

By putting value of y in eq. (3)

$$x = 5 - y$$
$$x = 5 - \frac{6}{5}$$
$$x = \frac{25-6}{5}$$
$$x = 19/5$$

$\therefore y = 6/5$ & $x = 19/5$

2) Form the pair of linear equations in the following problems, and find their solutions by elimination method:

i) If we add 1 to the numerator and subtract 1 from the denominator fraction reduces to 1. It becomes 1/2 if we only add 1 to denominator. What is the fraction?

→ Let the num. & deno. of a fraction be $x$ & $y$.

∴ Fraction is $\frac{x}{y}$

According to given condition,

$$\frac{x+1}{y-1} = \frac{1}{1}$$

$x+1 = y-1$

$x - y = -1-1$

$x - y = -2$ —— ①

$$\frac{x}{y+1} = \frac{1}{2}$$

$2x = y+1$

$2x - y = 1$ —— ②

~~from~~ ~~eq.~~

By ~~adding~~ subtracting eq. ① & ②,

$$\begin{array}{r} 2x - \cancel{y} = 1 \\ -x \; \overset{(+)}{-} \cancel{y} = \overset{(+)}{-}2 \\ \hline x \quad\quad = 3 \end{array}$$

By putting value of $x$ in eq. ①

$3 - y = -2$

$-y = -2-3$

$\cancel{-}y = \cancel{-}5$

$\therefore$ Fraction is $3/5$

The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

Let the units place & tens place digit of 2-digit no. be $x$ & $y$.

$\therefore$ 2-digit no. $= 10 \times x + y + 1$

$= 10x + y$

According to $1^{st}$ condition,

$$x + y = 9 \quad \text{—— (1)}$$

According to $2^{nd}$ condition

$$9(10x + y) = 2(10y + x)$$
$$90x + 9y = 20y + 2x$$
$$90x - 2x + 9y - 20y = 0$$
$$88x - 11y = 0$$

Divide by 11,

$$8x - y = 0 \quad \text{—— (2)}$$

By adding eq. (1) & (2)

$$8x - y = 0$$
$$+x + y = 9$$
$$9x = 9$$
$$x = 1$$

$\therefore$ By putting value of $x$ in eq. (1)

$x + y = 9$

$1 + y = 9$

$\underline{\underline{y = 8}}$

$\therefore$ Two - digit no. is 18 i.e.

$= 10x + y$

$= 10 \times 1 + 8$

$= 18$

# CROSS MULTIPLICATION

If $a_1x + b_1y + c_1 = 0$ &
$a_2x + b_2y + c_2 = 0$

Be the pair of linear equation in two variables $x$ & $y$, then using cross multiplication method the values of $x$ & $y$ are given by,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

## Exercise 3.5

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \quad \Bigg| \quad y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

Solve the following pair of linear equations by the substitution and cross multiplication methods:

$$8x + 5y = 9$$
$$3x + 2y = 4$$

→ Given equations are,

$$8x + 5y = 9 \quad \text{--- (1)}$$
$$3x + 2y = 4 \quad \text{--- (2)}$$

By substitution method,

from eq. (2),

$$3x + 2y = 4$$
$$3x = 4 - 2y$$
$$x = \frac{4 - 2y}{3}$$

By putting value of $x$ in eq. (1)

$$8 \times \frac{4-2y}{3} + 5y = 9$$

$$\frac{32-16y}{3} + 5y = 9$$

$$\frac{32-16y+15y}{3} = 9$$

$$32 - y = 27$$

$$-y = 27 - 32$$

$$-y = -5$$

$$y = 5$$

By putting value of y in eq. (3)

$$x = \frac{4 - 2 \times 5}{3}$$

$$x = \frac{4-10}{3}$$

$$x = \frac{-6}{3}$$

$$\boxed{x = -2}$$

$\therefore x = -2, y = 5$

By cross multiplication method,

$$\frac{x}{5\times-4-2\times-9} = \frac{y}{-9\times3+4\times8} = \frac{1}{8\times2-3\times5}$$

$$\frac{x}{-20+18} = \frac{y}{-27+32} = \frac{1}{16-15}$$

$$\frac{x}{-2} = \frac{y}{5} = \frac{1}{1}$$

$$\frac{x}{-2} = 1 \quad \& \quad \frac{y}{5} = 1$$

$\therefore x = -2$ & $y = 5$

Places A and B ------------

---- What are the speeds of two cars?

Let the speed of 1st car & 2nd car be

u km/h & v km/h

Respective speed of both cars while they are travelling in same direction

$= (u - v)$ km/h

Respective speed of both cars while they are travelling in opposite direction i.e., travelling towards each other = $(u+v)$ km/h

According to given condition,

$$5(u-v) = 100$$
$$u - v = 20 \quad \text{—— (1)}$$
$$1(u+v) = 100 \quad \text{—— (2)}$$

Adding both equations,

$$u - \cancel{v} = 20$$
$$u + \cancel{v} = 100$$
$$2u = 120$$
$$u = 60 \text{ km/h}$$

Putting this value in eq. (2),

$$(60 + v) = 100$$
$$v = 40 \text{ km/h}$$

∴ Speed of one car = 60 km/h

Speed of other car = 40 km/h

The area of a rectangle - - - - - - - - - - - - - - - - Find the dimensions of rectangle.

Let length & breadth of rectangle be $x$ unit & $y$ unit.

Area $= xy$

According to given condition,

$$(x-5)(y+3) = xy - 9$$
$$3x - 5y - 6 = 0 \quad \text{--- (1)}$$

$$(x+3)(y+2) = xy + 67$$
$$2x + 3y - 61 = 0 \quad \text{--- (2)}$$

By cross multiplication,

$$\frac{x}{305-(-18)} = \frac{y}{-12-(-183)} = \frac{1}{9-(-10)}$$

$$\frac{x}{323} = \frac{y}{171} = \frac{1}{19}$$

$\therefore \frac{x}{323} = \frac{1}{19}$ & $\frac{y}{171} = \frac{1}{19}$

$x = \frac{1}{19} \times 323$

$= 17$

$y = \frac{1}{19} \times 171$

$= 9$

$\therefore$ length of rectangle = 17 units
breadth of rectangle = 9 units

# EQUATIONS REDUCIBLE TO A PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

$1/2x + 1/3y = 2$

$1/3x + 1/2y = 13/6$

Let $1/x = a$ & $1/y = b$

Above equation becomes,

$$\frac{a}{2} + \frac{b}{3} = 2 \; ; \; \frac{a}{3} + \frac{b}{2} = \frac{13}{6}$$

$$\frac{3a + 2b}{6} = 2 \; ; \; \frac{2a + 3b}{6} = \frac{13}{6}$$

$$3a + 2b = 12 \text{ — } ① \; ; \; 2a + 3b = 13 \text{ — } ②$$

By elimination method,

$$3a + 2b = 12 \text{ } ① (\times 3)$$
$$2a + 3b = 13 \text{ } ② \times 2$$

$$9a + 6b = 36$$
$$4a + 6b = 26$$

By subtracting both

$$9a + 6b = 36$$
$$\underline{-4a \underset{(-)}{\pm} 6b = 26}$$
$$5a = 10$$
$$\boxed{a = 2}$$

By putting value of a in eq.-1

$$3 \times 2 + 2b = 12$$
$$6 + 2b = 12$$
$$2b = 6$$
$$\boxed{b = 3}$$

∴ Replace the value of a & b,

| | |
|---|---|
| $1/x = a$ | $1/y = b$ |
| $1/x = 2$ | $1/y = 3$ |
| Taking reciprocal | Taking reciprocal |
| $x = 1/2$ | $y = 1/3$ |

$$\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$$

$$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$$

Let $\frac{1}{3x+y} = a$ & $\frac{1}{3x-y} = b$

Above equation becomes,

$\cancel{1}\ a \cancel{+} b = 3/4$ —— 1

$a/2 - b/2 = -1/8$

$a - b = -1/4$ —— 2

Adding both,

$$\begin{aligned} a + \cancel{b} &= 3/4 \\ + a - \cancel{b} &= -1/4 \\ \hline 2a &= 2/4 \\ 2a &= 1/2 \\ a &= 1/4 \end{aligned}$$

Putting value of a in eq. ②,

$1/4 - b = -1/4$

$\cancel{1 - b = \frac{-1}{4} \times 4}$

$\frac{1}{4} + \frac{1}{4} = b$

$2/4 = b$

$1/2 = b$

∴ Replace a & b

$\frac{1}{3x+y} = \frac{1}{4}$ & $\frac{1}{3x-y} = \frac{1}{2}$

Taking reciprocal,

$3x + y = 4$ & $3x - y = 2$

$3x + y = 4$ —— ③

$3x - y = 2$ —— ④

Adding both,

$$\begin{array}{r} 3x + y = 4 \\ + 3x - y = 2 \\ \hline 6x = 6 \end{array}$$

$x = 1$

putting value of $x$ in eq. 3,

$3 \times 1 + y = 4$

$3 + y = 4$

$\boxed{y = 1}$

$\therefore x = 1$ & $y = 1$

2) Formulate the following problems as a pair of linear equations, and hence find their solutions:

(i) Ritu can row downstream 20 km in 2 hrs, and upstream 4 km in 2 hrs. Find her speed of rowing in still water and the speed of the current.

→ Let speed of rowing ~~downstream~~ be $x$ km/h & speed of current be $y$ km/h

Downstream $= x+y$ km/h & Upstream $= x-y$ km/h

According to first condition,

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$= 20/2$$

$$= 10 \quad \text{— for eq.-①}$$

Similarly

$$\text{Speed} = \frac{4}{2}$$

$$= 2 \text{ — for eq. } ②$$

According to given condition,

$$x + y = 10 \text{ — } ①$$

$$x - y = 2 \text{ — } ②$$

~~Adding both,~~

$$x + \cancel{y} = 10$$

$$+ x - \cancel{y} = 2$$

$$2x = 12$$

$$x = 6$$

~~By putting value of x in eq. ①~~

$$\cancel{6 + y = 10}$$

$$y = 4$$

∴ Speed of rowing = 6 km/h
Speed of current = 4 km/h