Half yearly exam 2019
Subject- mathematics
Class -12th

Time 3 hours MM 100

Instructions :-
1. All questions are compulsory.
2. Question No. 1 to 5 are objective type questions.
3. Internal option are given in Question Numbers 6 to 26.

Q.1) Choose the correct option and write it in your answer book. 1X5

i) $f(x) = x$ is defined as $f : R \to R$ then
(a) f is one one onto ✓ (b) f is many one onto
(c) f is one one but not onto (d) f is neither one one onto

ii) The number of all possible matrices of order 2X2with each entry 0 or 1
(a) 16 ✓ (b) 512
(c) 18 (d) 81

iii) A square matrix is symmetric if
(a) $A^2 = A$ (b) $A = A'$ ✓
(c) $A = -A'$ (d) $A^2 = I$

iv) If $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$ then the value of x is
(a) 2 (b) 4 (c) 2 and 4 both (d) $\pm 2\sqrt{2}$ ✓

v) Value of $\int logx dx$ is
(a) $\frac{1}{x}$ (b) $xlogx - x + c$ (c) $xlogx$ (d) $\frac{(logx)^2}{2} + c$

2) Fill in the blanks 1X5

i) if the radius of a circle are equal to 12 CM then rate of change in area with respect to r is..... ($12\pi/24\pi$).)

ii) Function $f(x)$ =sin x is increasing in interval .....($\left(0, \frac{\pi}{2}\right)$ / $\left(\frac{\pi}{2}, \pi\right)$).

iii) The gradient of normal of the curve $y = 2x^2 + 3sinx$ at X = 0 is -------- ($3/\frac{-1}{3}$).)

iv) The approximate change in the volume v of cube of side x metres cause by increasing the side by 2% is ----------($0.06x^3m^3/0.02x^3m^3$))

v) The points C which lie in the domain of function f and at which $f'(c) = 0$ is called ----------(critical point/ minima)

3) Match the column. 1X5

A	B
i) $\int a^x dx$	a) $\frac{x^{n+1}}{n+1} + c$ 2
ii) $\int x^n dx$	b) $\frac{a^x}{log_e a} + c$ 1
iii) $\int \frac{dx}{\sqrt{x^2 - a^2}}$	c) $\frac{1}{a}\tan^{-1}\frac{x}{a} + c$ 4
iv) $\int \frac{dx}{x^2 + a^2}$	d) $log\left\|x + \sqrt{x^2 - a^2}\right\| + c$ 3
v) $\int \sqrt{x^2 + a^2} dx$	e) $\frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2} log\left\|x + \sqrt{x^2 + a^2}\right\| + c$ 5

4) write true or false 1X5

i) Every differentiable function is continuous and every continuous function is differentiable .

T ii) For curve = $f(x)$, area enclosed by x axis and lines X = a and X = b is $\int_a^b f(x)dx$.

iii) The differential equation which is in the form $\frac{dy}{dx} + Py = Q$ is called linear differential equation of first order .

T iv) if $\theta = 90°$ then $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|$

v) For direction cosine $l, m, and\ n$ of any line, the relation $l^2 + m^2 + n^2 = 0$ is true .

5) Give answer in one word / one sentences 1X5

i) Write formula for finding distance between two parallel lines

ii) write the direction cosine of vector $\hat{i} + 2\hat{j} + 3\hat{k}$

iii) what is the degree of differential equation

$$(\frac{d^2y}{dx^2})^3 + (\frac{dy}{dx})^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$$

iv) write transpose for the matrix A=$\begin{bmatrix}2 & -1 & 2\\ 1 & 2 & 4\end{bmatrix}$

v) write the necessary condition for the function become invertible

6) If $x\begin{bmatrix}2\\3\end{bmatrix} + y\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}10\\5\end{bmatrix}$ then find the value of $x\ and\ y$. 2

or

$A = \begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}$, $I = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ and $A^2 = kA - 2I$ then find the value of k.

7) Find differential coefficient of e^{cosx} with respect to sin^2x. 2

or

Verify the rolle's theorem for function $f(x) = x^2 + 2x - 8, x \in [-4, 2]$

8) Evaluate 2

$$\int \frac{e^{\tan^{-1}x}}{1+x^2}\,dx$$

or

Evaluate

$$\int \frac{cos2x - cos2\alpha}{cosx - cos\alpha}\,dx$$

9) if $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5j - 2\hat{k}$ then find $\vec{a} \times \vec{b}$. 2

or

find the unit vector along the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$

10) Find the vector and cartesian equation of a line passing through the point $(5, 2, -4)$ and parallel to vector $3\hat{i} + 2\hat{j} - 8\hat{k}$ 2

or

Find the vector equation of a plane which is 7 unit far from origin and the normal of a plane is given by the vector $3\hat{i} + 5j - 6\hat{k}$

11) Find the absolute maximum value of $f(x) = 12x^{\frac{4}{3}} - 6x^{\frac{1}{3}}$ in interval $x \in [-1,1]$. 3

or

find two numbers whose sum is 24 and the product of the numbers is maximum

12) Volume of a cube increases at the rate of $8cm^3/sec$ find the rate at which surface of cube increases where edges of cube is 12 cm. 3

or

...e is dropped into a quiet lake and waves move in circles at a speed of ...c. At the instant ,when the radius of circular wave is 10 cm, how fast is enclosed area increasing ?

13) If three vectors $\vec{a}, \vec{b}, \vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 3, |\vec{b}| = 4,$ ...d $|\vec{c}| = 2$ then find the $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ 3

or

prove that the direction cosine of vector which is equally inclined with OX, OY and OZ axis are given by $\pm(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

14) ...d the angle between two plane $3x - 6y + 2z = 7$ and $2x + 2y - 2z = 5$ 3

Or

If lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular then find the value of k.

15) Three functions are $f: N \to N, g: N \to N,$ and $h: N \to N$ where $f(x) = 2x, g(y) = 3y + 4$ and $h(z) = sinz \forall x, y, z \in N$ then prove that $ho(gof) = (hog)of$ 4

or

show that in a given set $\{1, 2, 3\}$ a relation $R = \{(1, 2), (2, 1)\}$ is symmetric but neither reflexive nor transitive

16) find the value of $\tan^{-1}\{2cos(2\sin^{-1}\frac{1}{2})\}$ 4

or

prove that $\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$

17) prove that 4

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

or

Prove that

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

18) Find adjoint of matrix 4

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$$

or

using determinants, find the area of triangle formed by the vertex $(1, 0), (6, 0), (4, 3)$

19) Find the equation of the plane that contains the point, $(1, -1, 2)$ and is perpendicular to each of the plane $2x + 3y - 2z = 5$ and $x + 2y - 3z = 8$ 4

or

Find the shortest distance between lines

$\vec{r} = (6\hat{i} + 2\hat{j} + 2\hat{k}) + k(\hat{i} - 2\hat{j} + 2\hat{k})$ and

$\vec{r} = (-4\hat{i} - \hat{k}) + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$

20) Find the equation of a plane passes through the intersection of two planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ also passing through the point $(2, 2, 1)$ 4

or

Points $(1, 1, p)$ and $(-3, 0, 1)$ are at same distance from a plane $r \cdot (3i + 4j - 12k) + 13 = 0$ then find the value of p 4

21 Solve

$\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$,

or

If $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1} x\right) = 1$ then find value of x.

22) If $\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$ then find the value of x 5

or

If $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 & -6 \end{bmatrix}$ then verify $(AB)' = B'A'$

23) Is $f(x) = \begin{cases} x + 5 & \text{if } x \le 1 \\ x - 5 & x > 1 \end{cases}$ a continuous function, show it 5

Or

Find the value of a and b where $f(x)$ which defined as

$$f(x) = \begin{cases} 5 & \text{if } x \le 2 \\ ax + b & 2 < x < 10 \\ 21 & x \ge 10 \end{cases}$$

is a continuous function .

24) Evalute

$\int_0^{\frac{\pi}{2}} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a - x}} dx$ 5

or

Prove that

$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \frac{\pi}{4}$

25) Find the area enclosed by the circle $x^2 + y^2 = a^2$

or 5

Find the area of the region bounded by the two parabola $y = x^2$ and $y^2 = x$.

26) Find the general solution of differential equation

$COS^2 x \frac{dy}{dx} + y = \tan x (0 \le x < \frac{\pi}{2})$ 5

Or

Find the general solution of differential equation

$(x^2 + xy)dy = (x^2 + y^2)dx$